Engineering Physics-I

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8.   Fibre Optics and Applications   **8.1–8.59**

PREFACE

Today's technological developments are the result of the joint efforts of physicists and engineers. A study of physics is, therefore, indispensable for students of engineering colleges in order to excel in their respective fields. This book has been written with a view to provide a comprehensive text on Engineering Physics for the first-year BE/BTech degree students studying in the engineering colleges affiliated to Anna University.

About the Book

This book covers topics like crystal physics, properties of matter, thermal physics, quantum physics, acoustics, ultrasonics, lasers and fiber optics. **Chapter 1** elucidates crystal physics. Properties of matter are discussed in **Chapter 2**. **Chapter 3** comprises thermal physics. **Chapter 4** discusses quantum physics and its applications. Concepts of acoustics are presented in **Chapter 5**. Ultrasonic concepts have been explained in detail in **Chapter 6**. Various types of lasers along with their applications are covered in **Chapter 7**. **Chapter 8** describes the basics of various types of fibers and their applications.

The text presents the fundamental principles of physics and their applications in a simple language. Numerous solved problems, including problems from the latest question papers, review questions and exercise problems have been given at the end of each unit. Questions that appeared in various examination papers have been marked to notify their respective year of appearance.

Salient Features

- Complete syllabus coverage
- Simple and lucid writing style

- Rich pedagogy
  - ➢ 47 Solved Examples
  - ➢ 19 Exercise Questions
  - ➢ 541 Practice Questions
  - ➢ 220 Short-Answer Questions

Acknowledgements

We hope that this book will be well received by students and teachers alike. We invite criticism and suggestions from the readers for making further improvements to the text.

K Tamilarasan

K Prabu

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UNIT 1

CRYSTAL PHYSICS

1.1 INTRODUCTION

The matter is usually found to exist in solids, liquids and gases. All these materials consist of atoms or molecules. A vast majority of commonly used materials are solids in nature. Solids differ by their properties such as brittle, ductile, malleable, strong, weak, good conductors of heat and electricity, bad conductors of heat and electricity, magnetic, nonmagnetic and so on. The basic reason for these different properties of solids is their structure. The behaviour of solid materials is closely related to the structure of the materials. Thus the study of the geometry of the crystal helps us to understand the different behaviour of solids in their mechanical, metallurgical, electrical, magnetic and optical properties. Solids can be either crystalline or noncrystalline. Crystallography is the science of study of the geometric from and other physical properties of crystalline solids by X-rays, electron beams, neutron beams, etc. Crystals, in turn, can be classified as single crystals and polycrystals.
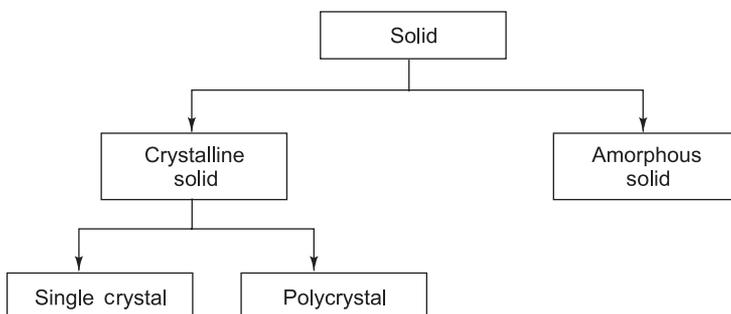
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                    ┌──────────┐
                    │  Solid   │
                    └──────────┘
              ┌───────────┴────────────┐
     ┌────────────────┐         ┌────────────────┐
     │  Crystalline   │         │   Amorphous    │
     │     solid      │         │     solid      │
     └────────────────┘         └────────────────┘
         ┌─────┴──────┐
 ┌────────────────┐ ┌────────────────┐
 │ Single crystal │ │  Polycrystal   │
 └────────────────┘ └────────────────┘
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A crystal is defined as a solid in which atoms or molecules are regularly arranged. That is, the crystal is said to possess a long-range order.

Single Crystal   A single crystal is a solid in which atoms or molecules are regularly arranged throughout its entire volume. It is schematically shown in Fig. 1.1.

Atoms/Molecules regularly arranged
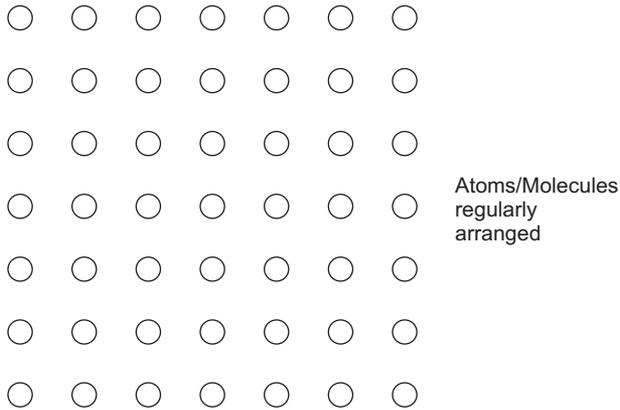
Fig. 1.1 Schematic of a Single Crystal

Examples   Ruby, quartz, GaAs, etc.

Polycrystal   A polycrystal is a solid made up of a large number of single crystals with different orientations. It is schematically shown in Fig. 1.2.
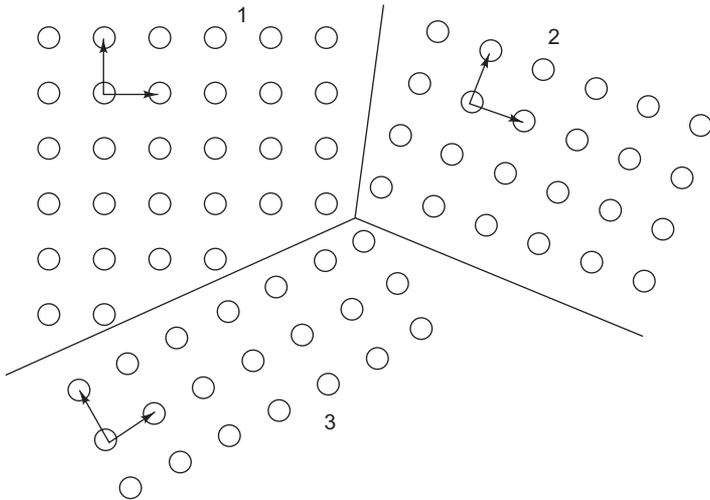
1, 2, 3-Single crystals with different orientations

Fig. 1.2 Schematic of a Polycrystal

Examples   Cu, Al, steel, brass, etc.

Amorphous Material   Amorphous material is a solid with no regular arrangement of atoms or molecules. It has a short range order of about 1 to 1.5 nm. An amorphous solid is schematically shown in Fig. 1.3.

Atoms/Molecules
randomly
arranged
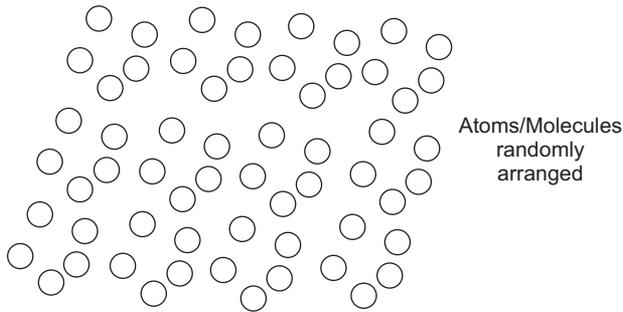
Fig. 1.3   Schematic of an Amorphous Solid

Examples   Window glass, polyethylene, PVC, Teflon, Bakelite, etc.

Lattice   It is defined as an array of points which are imaginarily kept to represent the position of atoms in the crystal such that every lattice point has got the same environment as that of the other. Hence, one lattice point cannot be distinguished from the other lattice point.

1.2   BASIC CONCEPTS IN CRYSTAL PHYSICS

In the study of crystal physics, the concepts such as space lattice, basis, crystal structure, unit cell, Bravais lattices, lattice planes, and Miller indices are employed. Now, we will discuss these concepts in detail.

1.2.1   Space Lattice

The structure of a crystal may be described in terms of an idealised geometrical concept called space lattice. Space lattice is defined as an infinite three-dimensional array of points in which every point has an identical environment as any other point in the array.

Consider a two-dimensional array of points (Fig. 1.4). Let O be the arbitrarily chosen origin. Let $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the lattice points A and B, respectively.

r - Resultant vector
r_1, r_2 - Radial vectors
a, b - Translational
vectors
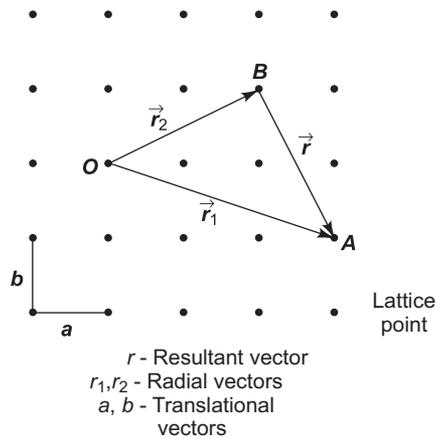
Fig. 1.4 Space Lattice

The given array of points is considered to be a two-dimensional space lattice, if the difference $\vec{r}$ of the two vectors $\vec{r}_1$ and $\vec{r}_2$ satisfies the relation

$$\vec{r} = n_1 \vec{a} + n_2 \vec{b} \tag{1.1}$$

where n_1 and n_2 are integers; $\vec{a}$ and $\vec{b}$ are fundamental translational vectors characteristic of the array.

For a three-dimensional space lattice, Eq. (1.1) is modified as

$$\vec{r} = n_1 \vec{a} + n_2 \vec{b} + n_3 \vec{c} \tag{1.2}$$

It should be noted that a space lattice represents the geometry of a set of points in space, while the crystal structure represents the actual ordering of its constituent atoms, molecules or ions in space.

1.2.2 Basis and Crystal Structure

For a lattice to represent a crystal structure, every lattice point is associated with a unit assembly of atoms or molecules identical in composition called **basis** or **pattern** or **structural motif.** When the basis is repeated with correct periodicity in all directions, the crystal structure is obtained (Fig. 1.5). That is,

Lattice + Basis = Crystal Structure
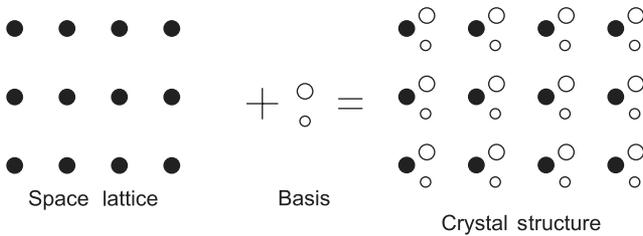
Fig. 1.5 ┃ Crystal Structure

The basis representing each lattice point of the given crystal structure is shown in Fig. 1.6. It consists of three different atoms. It may be noted that the basis is identical in composition, arrangement and orientation of atoms.
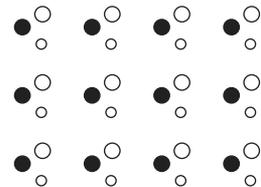
Fig. 1.6 ┃ Basis

A basis may consist of one atom or a group of atoms. In the crystals of Al and Ba, the basis consists of one atom. But, in the crystals of NaCl and KCl, the basis consists of two atoms.

1.2.3   Unit Cell

Consider a two-dimensional crystal lattice (Fig. 1.7). Let $ABCD$ be a parallelogram of sides a and b. When it is rotated through any integral multiples of vectors $\vec{a}$ and $\vec{b}$, the entire crystal lattice is constructed. The fundamental unit $ABCD$ is called a **unit cell**.
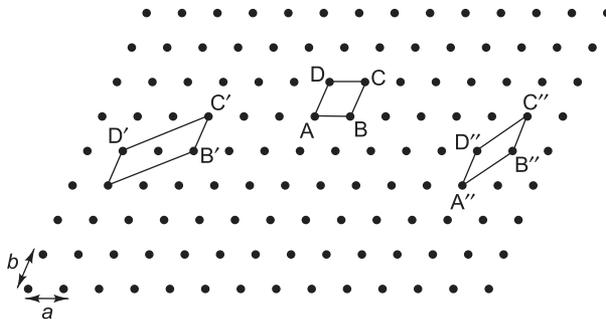
Fig. 1.7 ┃ Unit Cell

The choice of unit cell is not unique. That is, the crystal structure can also be constructed by the repetition of unit cells $A'B'C'D'$, $A''B''C''D''$, etc. The unit cell of a crystal may be defined in three different ways.

Definition 1 A unit cell is defined as the smallest geometric figure, the repetition of which gives the actual crystal structure.

Definition 2 A unit cell is defined as the elementary pattern of minimum number of atoms, molecules or group of molecules which represents all the characteristics of the crystal.

Definition 3 A unit cell is defined as that volume of a solid from which the entire crystal structure may be constructed by translational repetition in three dimensions.

Parameters of a Unit Cell A unit cell is characterised by its primitives a, b, c and interfacial angles α, β, γ (Fig. 1.8). Let X, Y, Z be the crystallographic axes.
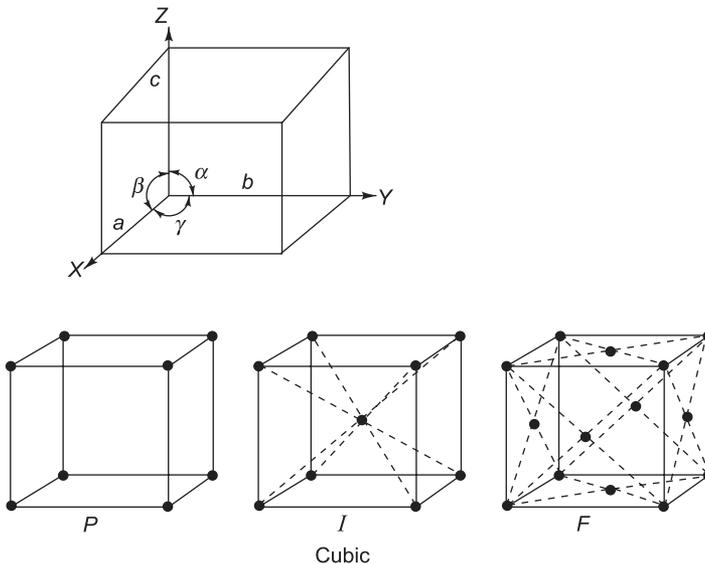
Fig. 1.8 Unit Cell Parameters

The intercepts of the unit cell on the crystallographic axes are called the primitives. The unit cell formed by the primitives is called the primitive cell. A primitive cell has only one lattice point. The angles between the crystallographic axes are called the interfacial angles.

1.3 CRYSTAL SYSTEMS

Based on the values of primitives and interfacial angles of the unit cell, the crystals are classified into seven systems, namely, cubic, tetragonal,

orthorhombic (or rhombic), monoclinic, triclinic, rhombohedral (or trigonal) and hexagonal crystal systems. The unit cells of various crystal systems are schematically shown in Fig. 1.9. The unit cell parameters of various crystal systems are given in Table 1.1.
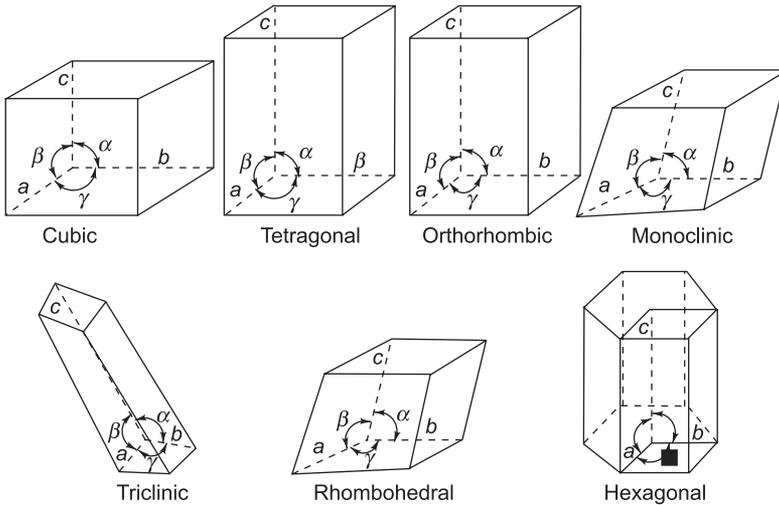
Fig. 1.9 Crystal Systems

Table 1.1 Crystal Systems

Crystal system	Lattice parameters		Example
Cubic	$a = b = c$;	$\alpha = \beta = \gamma = 90°$	NaCl, Po
Tetragonal	$a = b \neq c$;	$\alpha = \beta = \gamma = 90°$	$NiSO_4$, NO_2
Orthorhombic	$a \neq b \neq c$;	$\alpha = \beta = \gamma = 90°$	KNO_3, $BaSO_4$
Monoclinic	$a \neq b \neq c$;	$\alpha = \beta = 90° \neq \gamma$	Na_2SO_3, $FeSO_4$
Triclinic	$a \neq b \neq c$;	$\alpha \neq \beta \neq \gamma \neq 90°$	$CuSO_4$
Rhombohedral	$a = b = c$;	$\alpha \neq \beta \neq \gamma \neq 90°$	$CaSO_4$
	$\alpha, \beta, \gamma < 120°$		
Hexagonal	$a = b \neq c$;	$\alpha = \beta = 90°$	SiO_2, AgI
	$\gamma = 120°$		

As seen from Table 1.1, cubic crystals have the largest symmetry while triclinic crystals have the least symmetry. In fact, more than half of the elements in nature crystallise into cubic systems.

1.3.1   Types of Lattices

Based on the arrangement of lattice points, space lattices are classified into four types, namely, primitive lattice (or *P* lattice), body-centred lattice (or *I* lattice), face-centred lattice (or *F* lattice) and base-centred lattice (or *C* lattice).

Primitive Lattice   There is one lattice point at each corner of the unit cell.

Body-Centred Lattice   There is one lattice point at each corner of the unit cell. In addition, there is a lattice point at the centre of the unit cell.

Face-Centred Lattice   There is one lattice point at each corner of the unit cell. In addition, there is one lattice point at the centre of each face of the unit cell.

Base-Centred Lattice   There is one lattice point at each corner of the unit cell. And, there is one lattice point at the centre of the two bases of the unit cell.

1.3.2   Bravais Space Lattices

There are only 14 ways of arranging lattice points in space so that each point has an identical environment. The 14 types of arrangements are called Bravais lattices. The fourteen Bravais lattices are illustrated in Fig. 1.10.

As seen in Fig. 1.10, there are more than one type of lattice arrangements in the crystal systems like cubic, tetragonal, monoclinic and orthorhombic.

1.3.3   Lattice Planes and Miller Indices

The crystal lattice may be considered to be made up of a set of parallel equidistant planes passing through the lattice points called **lattice planes**. For a given lattice, there are a number of ways of choosing the lattice planes (Fig. 1.11).

The choice of latice planes is not unique. Figure 1.11 shows four different sets of lattice planes *A*, *B*, *C* and *D* for the given crystal lattice. A plane in a crystal is designated by three integers (*h k l*) called **Miller indices**.

P   I   F
Cubic

P   I
Tetragonal

P   I   F   C
Orthorhombic

P   C   P
Monoclinic   Triclinic

P   P
Rhombohedral   Hexagonal
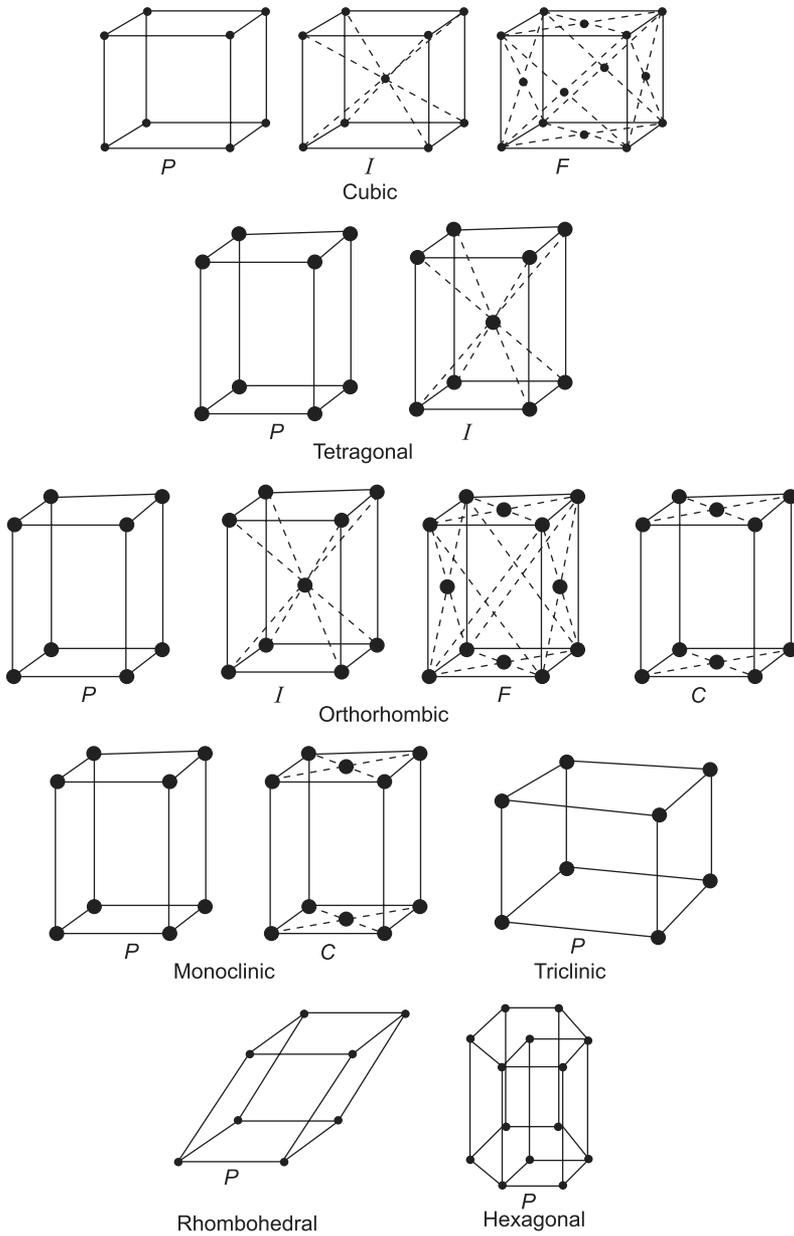
Fig. 1.10 Bravais Lattices
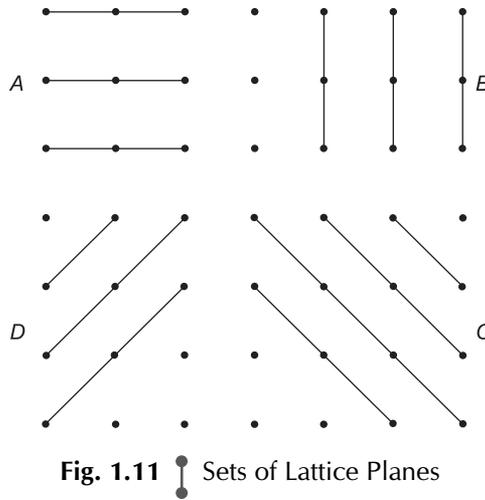
Fig. 1.11 Sets of Lattice Planes

1.3.4   Determination of Miller Indices

The Miller indices of a lattice plane are determined by the following procedure:

1. Find the intercepts of the given plane on the three crystallographic axes.
2. Express the intercepts in terms of the lattice parameters. Let these be *pa, qb, rc*.
3. Find the reciprocals of the coefficients *p, q, r* of the intercepts.
4. Reduce the reciprocals 1/*p*, 1/*q*, 1/*r* to the smallest integers through multiplication by their LCM.
5. Enclose the integers in brackets to get the Miller indices (*h k l*) of the given plane.

Figure 1.12 illustrates some selected lattice planes in a cubic crystal.

Illustrative Example   *Consider the case of finding the Miller indices of a lattice plane with intercepts 5a and 2c on the x and z axes. The plane is parallel to the y axis. Let a, b, c be the primitive vectors of the lattice.*

For the given plane, we have the intercepts as

$$pa,\ qb,\ rc = 5a,\ \infty b,\ 2c$$

i.e., $$\frac{1}{p}, \frac{1}{q}, \frac{1}{r} = \frac{1}{5}, \frac{1}{\infty}, \frac{1}{2}$$

or $$\frac{1}{p}, \frac{1}{q}, \frac{1}{r} = \frac{1}{5}, 0, \frac{1}{2}$$

Multiplying the reciprocals by their LCM, we get the Miller indices as (2, 0, 5).
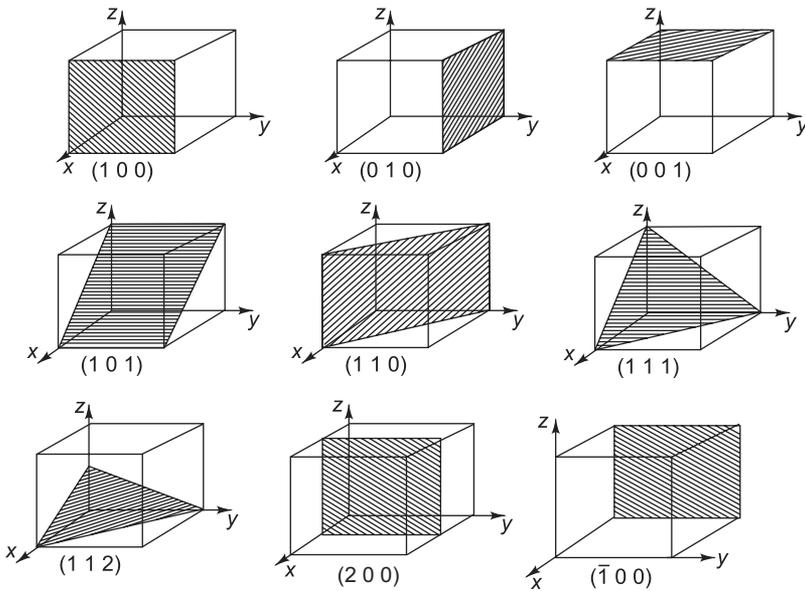
Fig. 1.12 Lattice Planes in a Cubic Crystal

Characteristics of Miller Indices

- For a plane parallel to a crystallographic axis, the Miller index for the given axis is zero.
- Equidistant parallel planes have the same Miller indices.
- Miller indices do not define a particular plane but a set of parallel planes.
- A bar put on a Miller index means that the corresponding intercept has a negative value.
- A plane passing through the origin of the crystallographic axis is identified with a parallel plane having a nonzero intercept.
- The normal to the plane with Miller indices ($h\,k\,l$) is the direction [$h\,k\,l$].
- The distance between adjacent planes of a set of parallel planes of Miller indices ($h\,k\,l$) in a cubic crystal is given by

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

where a is the cube edge.

- The angle between the normals to the two planes $(h_1 k_1 l_1)$ and $(h_2 k_2 l_2)$ in a cubic system is

$$\cos\theta = \frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{h_1^2 + k_1^2 + l_1^2}\ \sqrt{h_2^2 + k_2^2 + l_2^2}}$$

- In general, the planes that determine the physical and chemical properties of solids are those with low Miller index numbers.

1.3.5   Interplanar Distance in a Cubic Crystal

Consider the plane ABC in a cubic crystal of side 'a' (Fig. 1.13). Let it be designated by Miller indices ($h\ k\ l$). Let OM be the normal to the plane ABC passing through the origin O. Let the normal OM make angles θ_1, θ_2, θ_3 with x, y, z axes, respectively. Let $d' \equiv OM$ be the perpendicular distance between the plane ABC and the origin O. Then, we get

$$\cos\theta_1 = \frac{d'}{OA} = \frac{d'}{a/h} = \frac{d'\cdot h}{a}$$

$$\cos\theta_2 = \frac{d'}{OB} = \frac{d'}{a/k} = \frac{d'.k}{a} \qquad (1.3)$$

$$\cos\theta_3 = \frac{d'}{OC} = \frac{d'}{a/l} = \frac{d'.l}{a}$$
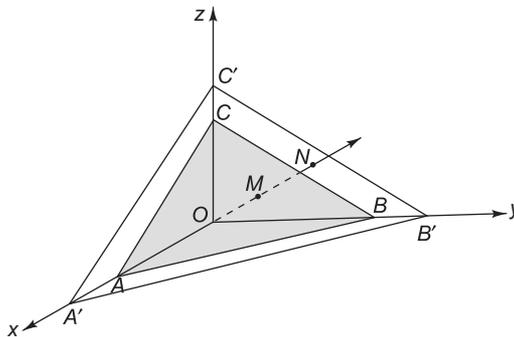
Fig. 1.13 ⁞ Planes in a Cubic Crystal

On the other hand, we have

$$\cos^2 \theta_1 + \cos^2 \theta_2 + \cos^2 \theta_3 = 1 \tag{1.4}$$

Substitution of Eq. (1.3) into Eq. (1.4) gives

$$\left(\frac{d' \cdot h}{a}\right)^2 + \left(\frac{d' \cdot k}{a}\right)^2 + \left(\frac{d' \cdot l}{a}\right)^2 = 1$$

or $OM \equiv d' = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}}$

Consider the neighbouring plane $A'B'C'$ parallel to the plane ABC. Let ON be the normal to the plane $A'B'C'$. Let the plane $A'B'C'$ be located at a distance $2d' \equiv ON$ from the origin. Then, the intercepts of the plane $A'B'C'$ are $2a/h$, $2a/k$, $2a/l$. Therefore, we get

$$ON \equiv 2d' = \frac{2a}{\sqrt{h^2 + k^2 + l^2}}$$

The interplanar distance between the planes ABC and $A'B'C'$ is then given by

$$d = ON - OM$$

i.e., $d = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}}$ \hfill (1.5)

1.3.6   Simple Crystal Structures

It has been found that most of the common metals possess cubic or hexagonal structures only. The important simple crystal structures are

1.   Simple Cubic (SC) structure
2.   Body-Centred Cubic (BCC) structure
3.   Face Centred Cubic (FCC) structure
4.   Hexagonal Close Packed (HCP) structure

1. Simple Cubic (SC) Structure

The unit cell of a simple cubic structure has one atom at each corner. So there are eight corner atoms. The arrangement of lattice points in a SC cell is shown in Fig. 1.14(a).

(a) Atomic side model   (b) Hard sphere model
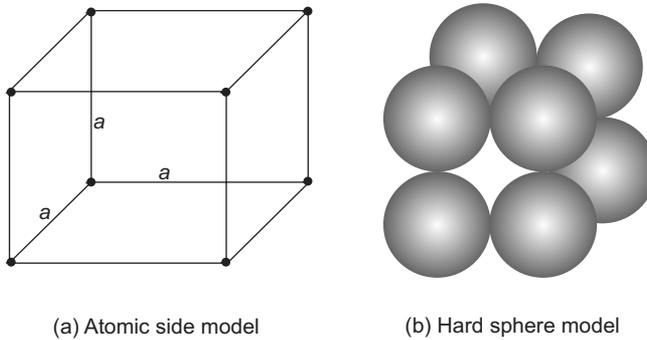
Fig. 1.14 Simple Cubic Cell

2. Body-Centred Cubic (BCC) Structure

The unit cell of a BCC has an atom at each corner. Also it has one atom at the centre of the body. So we have eight corner atoms and one centre atom. The arrangement of lattice points in a BCC cell is shown in Fig. 1.15 (a).

(a) Atomic side model   (b) Hard sphere model
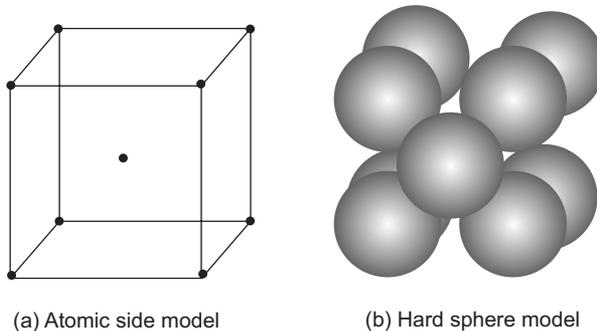
Fig. 1.15 Body-Centred Cubic Cell

Example   Tungsten, vanadium, molybdenum, chromium, alkali metals, α-iron (below 910°C), δ-iron (1400°C to 1539°C), etc.

3. Face-Centred Cubic (BCC) Structure

The unit cell of an FCC has one atom at each corner. It has also one atom at the centre of each face. So it has eight corner atoms and six face-centred atoms. The arrangement of lattice points in an FCC cell is shown in Fig. 1.16(a).

(a) Atomic side model   (b) Hard sphere model repeating structure
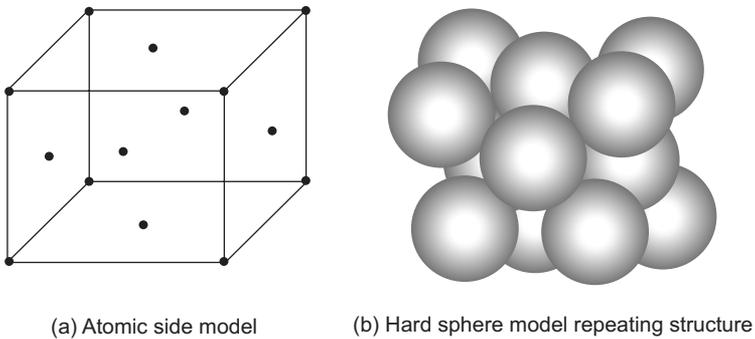
Fig. 1.16 Face-Centred Cubic Cell

Example Copper, silver, gold, aluminium, nickel, lead, platinum, γ-iron (910°C to 1400°C), etc.

4. Hexagonal Close Packed (HCP) Structure

The unit cell an HCP contains one atom at each corner of the hexagonal prism, one atom each at the centre of the hexagonal faces and three more atoms within the body of the cell. The arrangement of lattice points in HCP cells is shown in Fig. 1.17 (a).

(a) HCP structure   (b) Sequence of layers
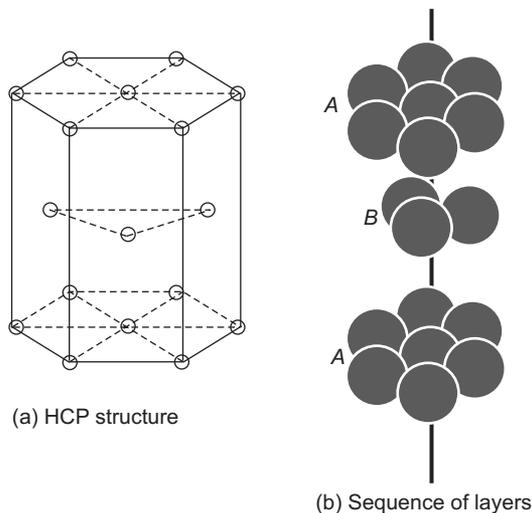
Fig. 1.17 HCP Structures

Example Magnesium, zinc, titanium, zirconium, beryllium, cadmium, etc.

1.4 NUMBER OF ATOMS PER UNIT CELL

1.4.1 Simple Cubic (SC) Lattice

In an SC lattice, there are eight atoms, one at each corner of the unit cell [Fig. 1.18 (a, b)]. Each corner atom is shared by eight unit cells.

(a)   (b)
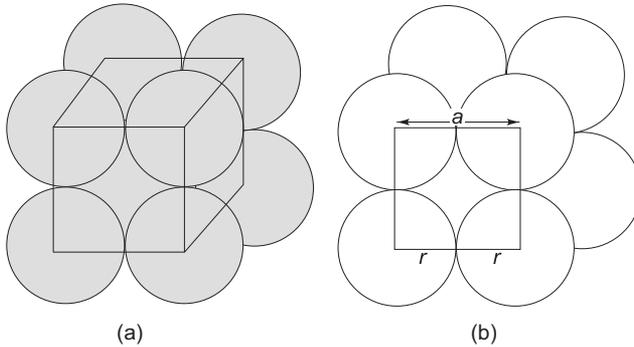
Fig. 1.18 SC Lattice

Therefore, the contribution of a corner atom to the given unit cell is equal to one-eighth of the atom. Thus, the total number of atoms in one unit cell is

$$n = \frac{1}{8} \times 8$$

i.e., $n = 1$

1.4.2 Body-Centred Cubic (BCC) Lattice

In a BCC lattice, there are eight atoms one at each corner of the unit cell and one atom at the centre of the unit cell (Fig. 1.19). Each corner atom is shared by eight unit cells. Hence, the contribution of the corner atom to the given unit cell is equal to one-eighth of the atom. The body-centred atom exclusively belongs to the given unit cell.
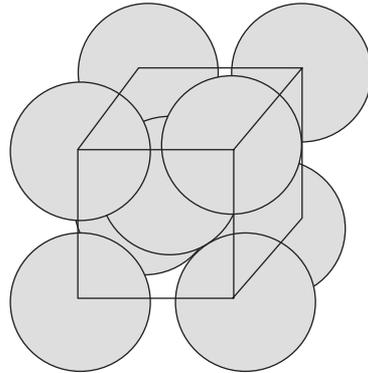
Fig. 1.19 BCC Lattice

Thus, the total number of atoms per unit cell is

$$n = \left(\frac{1}{8} \times 8\right) + 1$$

i.e., $n = 2$

1.4.3   Face-Centred Cubic (FCC) Lattice

In an FCC lattice, there are eight corner atoms, one at each corner of the unit cell and six face-centred atoms one at the centre of each face of the unit cell [Fig. 1.20(a), (b)]. Each corner atom is shared by eight unit cells.

Hence, the contribution of each corner atom to the given unit cell is equal to one-eighth of the atom. Each face-centred atom is shared by two unit cells. Therefore, the contribution of each face-centred atom to the given unit cell is equal to half of the atom. Thus, the total number of atoms per unit cell is

$$n = \left(\frac{1}{8} \times 8\right) + \left(\frac{1}{2} \times 6\right)$$

i.e., $n = 4$

(a)   (b)
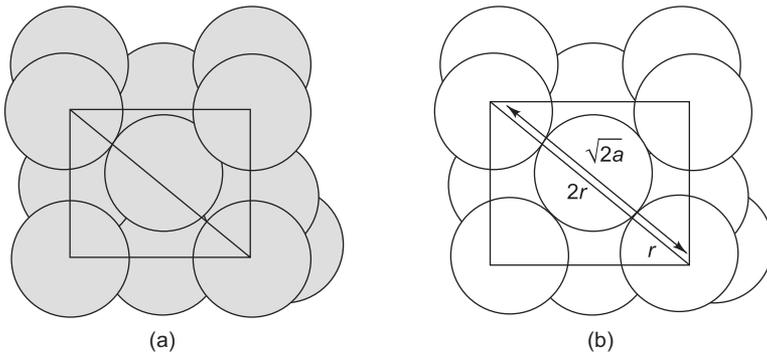
Fig. 1.20   FCC Lattice

1.4.4   Hexagonal Close Packed (HCP) Lattice

The HCP lattice has three layers of atoms. The top and bottom layers of unit cells have six corner atoms, one at each corner of the unit cell and one face-centred atom. The central layer has three atoms [Fig. 1.21(a), (b)]. Each corner atom is shared by six unit cells. Therefore, the contribution of each corner atom to the given unit cell is equal to

one-sixth of the atom. The face-centred atom is shared by two unit cells.

(a)   (b)
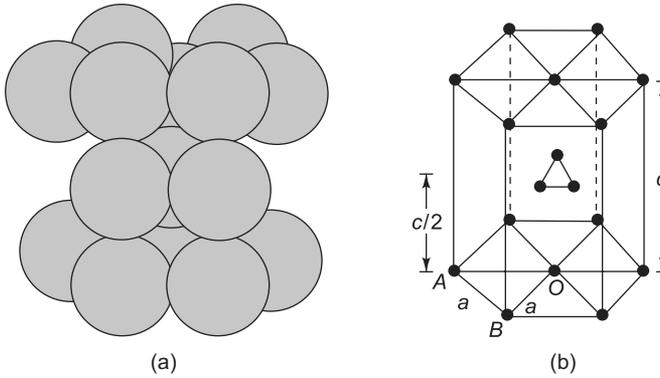
Fig. 1.21 HCP Lattice

Hence, the contribution of each face-centred atom to the given unit cell is equal to half of the atom. The atoms of the central layer exclusively belong to the given unit cell. Thus, the total number of atoms per unit cell is

$$n = \left(\frac{1}{6} \times 12\right) + \left(\frac{1}{2} \times 2\right) + 3$$

i.e., $n = 6$

1.5 COORDINATION NUMBER

An atom in a crystal is surrounded by other atoms. As a measure of how densely the crystal structure is packed, a concept called the coordination number is introduced. **Coordination number is the number of equidistant neighbours of an atom in a crystal**. Higher values of coordination number indicate that the given crystal structure is more closely packed.

1.5.1 Simple Cubic (SC) Structure

In an SC structure, the corner atom has four nearest neighbours in the same plane and two nearest neighbours in a vertical plane (one exactly above and the other exactly below).

Hence, the coordination number is

$$N_C = 4 + 2 = 6$$

1.5.2 Body-Centred Cubic (BCC) Structure

In a BCC structure, the body-centred atom is surrounded by eight equidistant corner atoms. Therefore, the coordination number is

$$N_C = 8$$

1.5.3 Face-Centred Cubic (FCC) Structure

In an FCC structure, the corner atom is surrounded by four face-centred atoms in its plane, by four face-centred atoms below its plane and by four face-centred atoms above its plane. Hence, the coordination number is

$$N_C = 4 + 4 + 4 = 12$$

1.5.4 Hexagonal Close Packed (HCP) Structure

In an HCP structure, the central atom at the top layer is surrounded by six equidistant corner atoms in its plane, three equidistant atoms below its plane and three equidistant atoms above its plane. Therefore, the coordination number is

$$N_C = 6 + 3 + 3 = 12$$

1.6 ATOMIC RADIUS

It is defined as half the distance between the nearest neighbouring atoms in a crystal. Assuming that the atoms are spherical, the atomic radius is expressed in terms of cube edge.

1.6.1 Simple Cubic (SC) Structure

In an SC structure (Fig. 1.22), the corner atoms touch each other along the cube edge. As seen in Fig. 1.22, the atomic radius r and the cube edge a are related by

$$2r = a$$

i.e., $r = a/2$ (1.6)
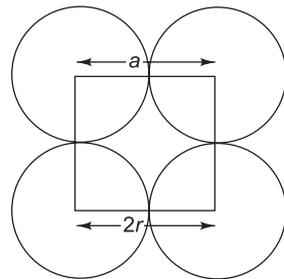
Fig. 1.22 Front View of SC Structure

1.6.2 Body-Centred Cubic (BCC) Structure

In a BCC structure, the atoms touch along the body diagonal (Fig. 1.23). Let r be the atomic radius and a be the cube edge.

From $\triangle ABC$, we get

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$(AC)^2 = a^2 + a^2 + 2a^2 \qquad (1.7)$$

From $\triangle ACD$, we get

$$(AD)^2 = (AC)^2 + (CD)^2$$

Hence, using Eq. (1.7), we get

$$(4r)^2 = 2a^2 + a^2 = 3a^2$$

i.e. $\qquad r = \dfrac{\sqrt{3}\,a}{4} \qquad (1.8)$
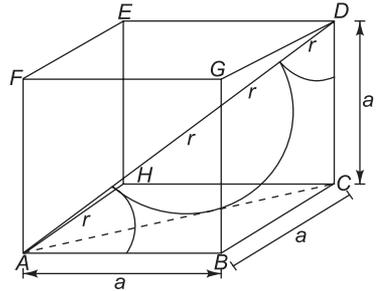
Fig. 1.23 BCC Structure

1.6.3 Face-Centred Cubic (FCC) Structure

In an FCC structure, the face-centred atom touches the corner atoms along the face diagonal (Fig. 1.24). Let r be the atomic radius and a be the cube edge. From $\triangle ABC$, we get

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$(4r)^2 = a^2 + a^2 = 2a^2$$

i.e., $\qquad r = \dfrac{a}{2\sqrt{2}} \qquad (1.9)$
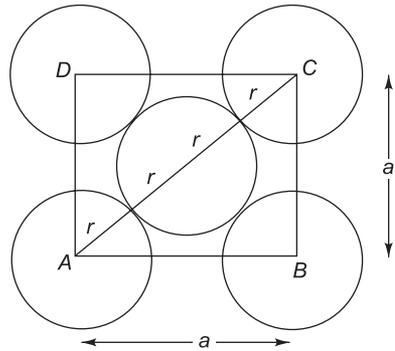
Fig. 1.24 Front View of FCC Structure

1.6.4 Hexagonal Close Packed (HCP) Structure

In an HCP structure, the corner atoms touch each other along the edge of the hexagonal layer (Fig. 1.25).

Therefore, the atomic radius r and the edge of the unit cell a are related by

$$2r = a$$

i.e., $\qquad r = \dfrac{a}{2} \qquad (1.10)$
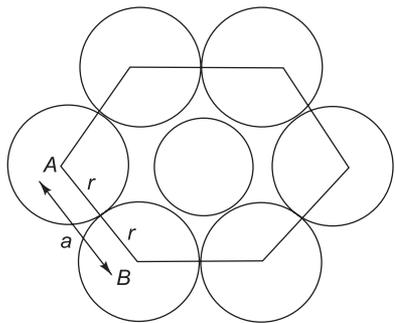
Fig. 1.25 Hexagonal Layer of HCP Structure

1.7   NEAREST NEIGHBOUR DISTANCE

It is the distance between the centres of two neighbouring atoms.
Therefore, the nearest neighbour distance is given as

$$d = 2r \tag{1.11}$$

where r is the atomic radius.

1.8   ATOMIC PACKING FACTOR

Atomic packing factor is defined as the ratio of volume occupied by atoms in the unit cell to the volume of the unit cell. It is also known as the packing fraction.

1.8.1   Simple Cubic (SC) Structure

Consider the atoms to be spheres of radius r. Then, volume of an atom is

$$v = \frac{4}{3}\pi r^3$$

Number of atoms per unit cell is $n = 1$

Volume of the unit cell is $V = a^3$

Atomic radius is $r = \dfrac{a}{2}$

Therefore, the atomic packing factor is

$$PF = \frac{nv}{V} = \frac{1 \times \frac{4}{3}\pi r^3}{a^3}$$

$$PF = \frac{\frac{4}{3}\pi \left(\frac{a}{2}\right)^3}{a^3}$$

i.e., $$PF = \frac{\pi}{6} = 0.52 \tag{1.12}$$

Thus, the atomic packing factor of the SC structure is 52%. That is, only 52% of the unit cell volume is occupied by atoms. Hence, the SC structure is a loosely packed one. Polonium crystallises into a SC structure.

1.8.2   Body-Centred Cubic (BCC) Structure

Consider the atoms to be spheres of radius r. Then, volume of an atom is

$$v = \frac{4}{3}\pi r^3$$

Number of atoms per unit cell is $n = 2$

Volume of the unit cell is $V = a^3$

Atomic radius is $r = \dfrac{\sqrt{3}\,a}{4}$

Hence, the atomic packing factor is

$$PF = \frac{nv}{V} = \frac{2 \times \dfrac{4}{3}\pi r^3}{a^3}$$

$$PF = \frac{\dfrac{8}{3}\pi \left(\dfrac{\sqrt{3}\,a}{4}\right)^3}{a^3}$$

i.e., $$PF = \frac{\sqrt{3}\,\pi}{8} = 0.68 \tag{1.13}$$

Thus, the atomic packing factor of the BCC structure is 68%. That is, 68% of the unit cell volume is occupied by atoms. Cesium, rubidium, potassium, sodium, lithium, tungsten, chromium, barium and iron crystallise into an BCC structures.

1.8.3   Face-Centred Cubic (FCC) Structure

Considering the atoms to be spheres of radius r, we have the volume of an atom as

$$v = \frac{4}{3}\pi r^3$$

Number of atoms per unit cell is $n = 4$

Volume of the unit cell is $V = a^3$

Atomic radius is $r = \dfrac{a}{2\sqrt{2}}$

Hence, the atomic packing factor is

$$PF = \frac{nv}{V} = \frac{4 \times \frac{4}{3}\pi r^3}{a^3}$$

$$PF = \frac{\frac{16}{3}\pi\left(\frac{a}{2\sqrt{2}}\right)^3}{a^3}$$

i.e., $PF = \dfrac{\pi}{3\sqrt{2}} = 0.74$ (1.14)

Thus, the atomic packing factor of the FCC structure is 74%. That is, 74% of the unit cell volume is occupied by atoms. In comparison with SC and BCC structures, the FCC structure has higher atomic packing factor. Therefore, the FCC structure is a densely packed one. Aluminium, copper, gold, lead, platinum, nickel and palladium crystallise into FCC structures.

1.8.4 Hexagonal Close Packed (HCP) Structure

It is a hexagonal structure having 12 corner atoms one at each corner, two face-centred atoms at the centres of the two hexagonal faces (at the top and bottom faces) and three atoms in the middle layer within the structure [Fig. 1.21(a), (b)]. The bottom layer of the HCP structure and its constituent ΔAOB are shown in Fig. 1.26 and in Fig. 1.27, respectively.
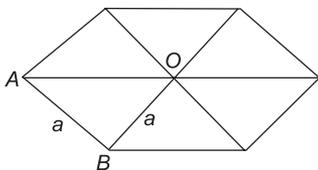
Fig. 1.26 Bottom Layer of HCP Structure
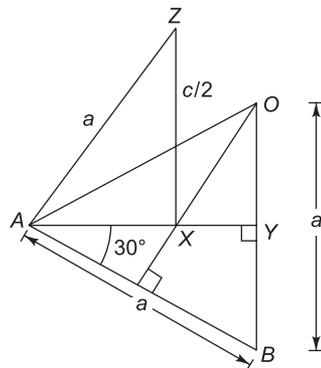
Fig. 1.27 Constituent Triangle

Consider the atoms to be spheres of radius r. Then, volume of an atom is

$$v = \frac{4}{3}\pi r^3$$

Number of atoms per unit cell is $n = 6$

Atomic radius is $r = a/2$

Considering the ΔAOB (Fig. 1.27), we get

Area of $\Delta AOB = \frac{1}{2} \times OB \times AY$

On the other hand, from the ΔAYB (Fig. 1.27), we get

$$AY = a\cos 30° = \frac{a\sqrt{3}}{2} \tag{1.15}$$

$$\text{Area of } \Delta AOB = \frac{1}{2} \times a \times \frac{a\sqrt{3}}{2}$$

i.e., $\quad$ Area of $\Delta AOB = \frac{\sqrt{3}}{4}a^2$

From the ΔAXZ (Fig. 1.27), we get

$$(AZ)^2 = (AX)^2 + (XZ)^2$$

Since X is the orthocentre of the ΔAOB, we have $AX = \frac{2}{3}AY$. Therefore, we get

$$(AZ)^2 = \left(\frac{2}{3}AY\right)^2 + (XZ)^2$$

i.e., $\quad a^2 = \left(\frac{a}{\sqrt{3}}\right)^2 + \left(\frac{c}{2}\right)^2$

$$\frac{2a^2}{3} = \frac{c^2}{4}$$

i.e., $\quad c = \sqrt{\frac{8}{3}}\,a \tag{1.16}$

In an HCP structure, unit cell volume is

$$V = \text{Base area} \times \text{Height}$$

$$V = 6 \times \text{Area of } \Delta AOB \times \text{Height}$$

$$V = 6 \times \frac{\sqrt{3}}{4} a^2 \times c \tag{1.17}$$

Substituting Eq. (1.16) into Eq. (1.17), we get

$$V = 3\sqrt{2}\, a^3$$

Hence, the atomic packing factor is

$$PF = \frac{nv}{V} = \frac{6 \times \frac{4}{3}\pi r^3}{3\sqrt{2}\, a^3}$$

$$PF = \frac{8\pi \left(\frac{a}{2}\right)^3}{3\sqrt{2}\, a^3}$$

i.e., $$PF = \frac{\pi}{3\sqrt{2}} = 0.74 \tag{1.18}$$

Thus, the atomic packing factor of the HCP structure is 74%. That is, 74% of the unit cell volume is occupied by atoms. Hence, the HCP structure is closely packed. Cadmium, zinc, beryllium, magnesium, titanium, zirconium and thallium crystallise into HCP structures.

Table 1.2 Properties of Crystal Structures

Properties	*SC*	*BCC*	*FCC*	*HCP*
Number of atoms per unit cell	1	2	4	6
Atomic radius	$a/2$	$\frac{\sqrt{3}\,a}{4}$	$\frac{a}{2\sqrt{2}}$	$a/2$
Unit cell volume	a^3	a^3	a^3	$3\sqrt{2}\, a^3$
Nearest neighbour	a	$\frac{\sqrt{3}\,a}{2}$	$\frac{a}{\sqrt{2}}$	a
Coordination number	6	8	12	12
Packing factor	0.52	0.68	0.74	0.74
Examples	Po	Cs	Al	Mg
		Rb, Li	Cu	Zn
		K, Cr	Ag, Pb	Cd

1.9 DENSITY OF A MATERIAL

Now, we can correlate the macroscopic property of a material such as its density to the microscopic property such as unit cell parameters. Consider a material consisting of cubic unit cells. Let 'a' be the cube edge. Let n be the number of atoms in the unit cell. Let M be the molecular mass (or atomic weight) of the material. Let ρ be its density. Then, density of the material is given by

$$\rho = \frac{\text{Mass of the unit cell}}{\text{Volume of the unit cell}}$$

i.e., $\quad \rho = \frac{\text{Number of atoms in the unit cell} \times \text{Mass of one atom}}{\text{Volume of the unit cell}}$

or $\quad \rho = \frac{nM}{N_A a^3}$ \hfill (1.19)

where N_A is Avogadro's number.

Equation (1.19) correlates the density of the material to the parameters of unit cell.

Table 1.3 Summary of Characteristics of the Unit Cells

S No.	Characteristics	Unit cells			
		SC	BCC	FCC	HCP
1.	Atoms per unit cell, Z	1	2	4	6
2.	Atomic radius, r	a/2	$\frac{a\sqrt{3}}{4}$	$\frac{a}{2\sqrt{2}}$	a/2
3.	Nearest neighbour	a	$\frac{a\sqrt{3}}{2}$	$\frac{a}{\sqrt{2}}$	a
4.	Coordination number, CN	6	8	12	12
5.	Atomic packing fraction, APF	$\pi/6$	$\frac{\pi\sqrt{3}}{8}$	$\frac{\pi}{3\sqrt{2}}$	$\frac{\pi}{3\sqrt{2}}$
6.	Void space	48%	32%	26%	26%
7	Density	$\frac{M}{N_A a^3}$	$\frac{2M}{N_A a^3}$	$\frac{4M}{N_A a^3}$	$\frac{\sqrt{2}M}{N_A a^3}$
8.	Unit cell volume	a^3	a^3	a^3	$\frac{3\sqrt{3}}{2}a^2 c$
9	Examples	Polonium	Fe, Ba, Cr	Al, Cu, Pb, gold	Zn, Mg

1.10   CRYSTAL IMPERFECTIONS

1.10.1   Introduction

In crystals, the atoms are ions arranged in a regular and periodic manner in three-dimensions. But the crystals are not always perfect and there are some deviations from the regular arrangements. Any derivations in a crystal from the regular and periodic arrangement of atoms is known as crystal imperfections. Mechanical strength, ductility, magnetic hysteresis, conducting properties, dielectric strength are greatly affected by the presence of impurities or defects in crystals.

(i)   Point defect
(ii)   Line defect
(iii)   Surface and grain boundary defect
(iv)   Volume defect

1.10.2   Point Defects

The defects which takes place due to imperfect packing of atoms during crystallisation are known as point defects. They are also known as zero-dimensional imperfections.

Vacancy refers to a missing or vacant atomic site as shown in Fig. 1.28. Vacancy may occur as a result of imperfect packing during the crystallisation. It also occurs

Fig. 1.28   Lattice Vacancy

from the thermal vibrations of atoms at elevated temperatures. It may also occur if an atom leaves its own site and is dissolved intertially into the structure. The vacancy concentration is given by
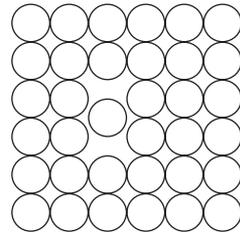
$$n = N \exp\left(\frac{-E_v}{kT}\right) \qquad (1.20)$$

where E_v is the average energy required to create such a vacancy, N is the number of atoms per unit volume, T is the temperature and k is the Boltzmann's constant.

Interstitial Defect   When an extra atom occupies interstitial position, i.e., voids or normally unoccupied state, interstitial defect occurs.

Self-Interstitial Defect   When an atom is displaced from its normal position within the crystal to an interstitial position, self-interstitial defect occurs.

Substitutional Defect   Whenever a foreign atom substitutes or replaces the position of the parent atom in the crystal lattice, substitutional defect occurs. Substitutional atoms may be smaller or larger than the atom being replaced.

Schottky Defect   Whenever a pair of positive and negative ions are missing from a crystal, the defect occurs is known as Schottky defect as shown in Fig. 1.29(a). The crystal is electrically neutral in this type of defect.

Frenkel Defect   An atom may leave its regular site and may occupy a nearby interstitial site of the matrix giving rise to two defects, i.e., one vacancy and the other self-interstitial defect simultaneously. These two defects are together is known as Frenkel defect, as shown in Fig. 1.29(b).

Missing pair of ions

Ion displaced from its structurally correct position

(a)   (b)
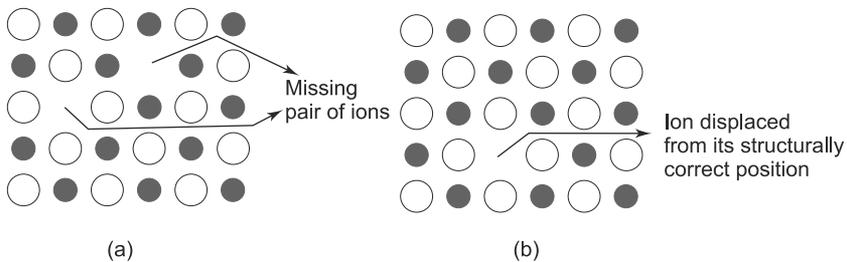
Fig. 1.29   (a) Schottky defect (b) Frenkel defect

1.10.3   Line Imperfection

Dislocation or distortion is the most important line imperfection. Dislocation is defined as a disturbed region between two perfect parts of a crystal. When an extra line of atoms is introduced from the upper portion of the crystal and this line of atoms terminates in the middle of the crystal. This type of imperfection along a line is known as line imperfection. Edge dislocation and screw dislocation are the two types of line imperfection. The dislocation is a boundary between the slipped region and the unslipped region and lies in the slip plane. The structure and behaviour of dislocations affect many properties of engineering materials.

Edge Dislocation   Edge dislocation is represented in Fig. 1.30. This location line can be considered as an imperfection due to the insertion of an extra half plane of atoms. If the extra plane of atoms is inserted above the slip plane then the line imperfection is called positive dislocation. It is represented by the symbol ⊥. When the extra plane of atoms is inserted below the slip plane then the defect is called negative dislocation. It is represented by the symbol τ. A line drawn through the dislocated region is called the dislocation line. The horizontal line in the symbol represents the dislocation line and the vertical line represents the incomplete atomic plane. The atoms lying above the dislocation line are compressed, whereas the atoms that are lying below the dislocation line exhibit tension. Thus, a stress field is acting around the dislocation line.
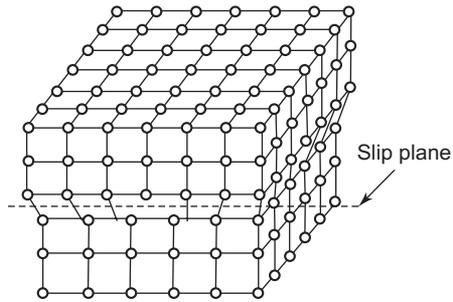
Fig. 1.30   A Positive Edge Dislocation

Burgers Vector   The dislocation line is described by Burgers vector (*b*). The vector that indicates the direction and magnitude of the shift of the lattice on the slip plane is called a Burgers vector. Burgers circuit or loop is formed by taking integral lattice translations in each pair of parallel sides in the region where dislocation is to be found out and the circuit will close itself in a perfect region of the crystal, i.e., a region with no dislocation.

Let us consider a point *S* as the starting point. If we traverse, by atom, to atom, right by 3 steps and then down by 3 steps, by 3 steps to the left and again 3 steps up, we reach the starting points. As the circuit closes, the start *S* and finish *F* of the circuit meet at the same atom. Thus we conclude bv Burgers loop that there is no dislocation. On the other hand, if the circuit fails to close itself in the region, we can conclude there is a dislocation.
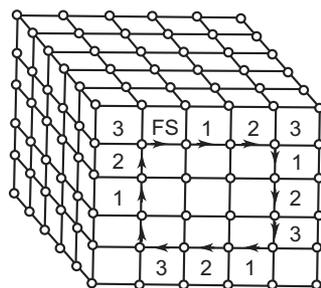
Fig. 1.31   Burgers Vector

Screw Dislocation Screw dislocation arises from a displacement of the atoms in one part of a crystal relative to the rest of the crystal, forming a spiral ramp around the dislocation line. In Fig. 1.32 the upper portion of the crystal is sheared through a distance of one atomic distance. The line separating the dislocated portion and undislocated portion is known as the dislocation line A. A circular arrow (∪) is the symbolic representation of screw dislocation. In some crystals, the edge and screw dislocations may occur simultaneously. By means of Burgers circuit for screw dislocation, we can determine the magnitude and direction of the screw dislocation.
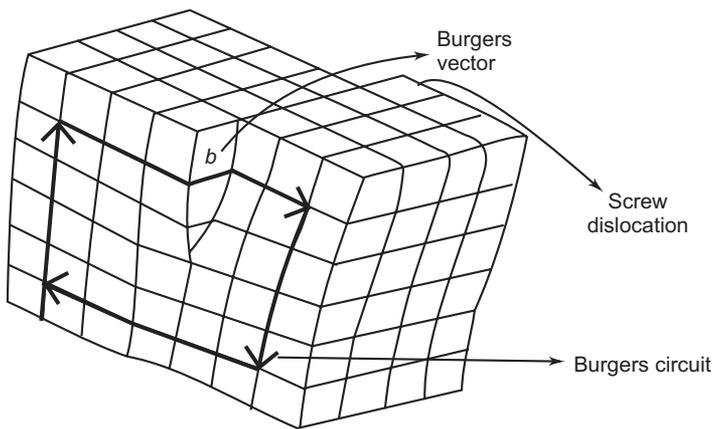
Fig. 1.32 Screw Dislocation

1.10.4 Surface Defects

The defects which take place on the surface of a material are known as surface or plane defects. They are also known as two dimensional surface imperfections. There are four different types of surface defects namely, grain boundaries, tilt boundaries, twin boundaries and stacking fault.

1.10.5 Grain Boundary

Grain boundaries are the defects which separate grains of different orientation from each other in a polycrystalline material. There is formed when two growing grain surfaces meet. Figure 1.33 shows the two-dimensional analogy of grain boundaries. They are observed during the solidification of polycrystalline material. Their area depends on the grain size of the material and increases with decrease

in grain size and vice versa. There are two types of grain boundaries. One is high angle boundary if the orientation difference between two crystals is greater than 10–15° and another is low angle boundary, if the orientation difference between two crystals is less than 10°.
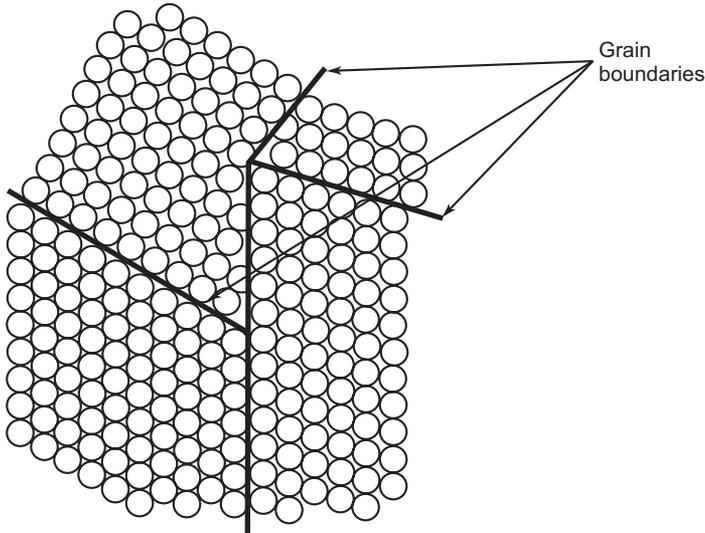
Fig. 1.33 Grain Boundary

1.10.6 Tilt Boundary

If the angle of dislocation in a crystal is less than 10°, then the defect is said to be **tilt boundary**. Usually, this defect is accompanied by edge dislocations (Fig. 1.34).

It is observed that there is a tilting of the upper and the lower portion of the crystal due to the presence of edge dislocation. Hence, this type of defect is said to be tilt boundary.

$$h = \frac{b}{\theta}$$
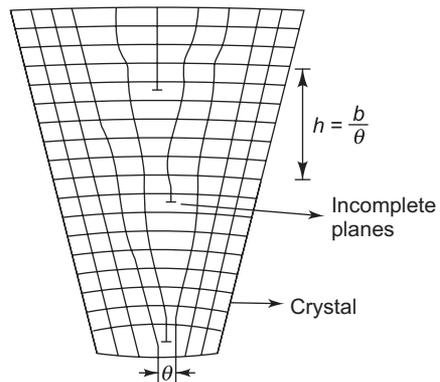
Fig. 1.34 Tilt Boundary

1.10.7 Twin Boundaries

These are the boundaries in the grains at which the atomic arrangement on one side of the boundary is the mirror image of the atoms on the

other side. The twin boundary is illustrated in the Fig. 1.35.The volume of material which has an orientation similar to the mirror image of the matrix operation is called a twin. The mirror is called the twinning plane. Twin boundary can also be produced during recrystallisation or as a result of annealing after plastic deformation. Hence they are classified into two types, namely, mechanical twins and annealing twins.
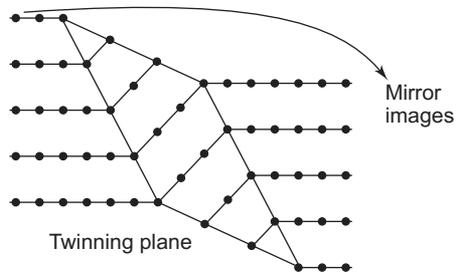
Fig. 1.35 Twin Boundary

1.10.8 Stacking Faults

Most of the metals crystallise either in FCC or HCP structures. For example, in the series *ABCABCBCABC*..., a layer *A* is missing or an extra layer *A* has been inserted as in series *ABCABCABACABC*... . In some crystals, the stacking faults extend through the entire crystal and in some crystals they occupy only a part of the plane.

An HCP unit cell has the stacking sequence of *ABABAB*... and so on. It may also have a stacking fault, if an extra layer *C* is introduced or can be substituted for a layer of type *B*.
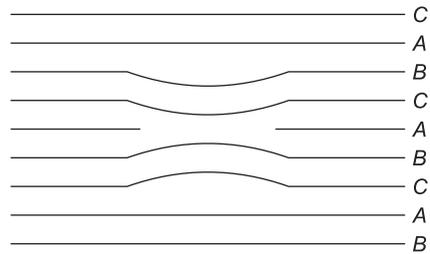
Fig. 1.36 Stacking Fault

1.10.9 Volume Defect

Cracks may arise when there is only a small dissimilarity between the stacking sequences of close packed planes in metals. These are known as volume defects. In addition, whenever a cluster of atoms are missing, large vacancy or void is created. This is also a volume defect. Foreign particle inclusions or noncrystalline regions which have the dimensions of the order of 20 Å are also known as volume defects.

1.11 DIAMOND CUBIC STRUCTURE

The diamond lattice can be identified to be formed by inter penetrating two FCC lattices along the body diagonal by one-fourth cube edge.

One sublattice has its origin at the point (0, 0, 0) and the other at a point quarter along the way of the body diagonal (at the point ($a/4$, $a/4$, $a/4$). In Fig. 1.37 the basic diamond lattice and the atomic positions in the cubic cell of diamond projected on the cube face are shown. The fractions indicate height about the base in units of the cube's edge. The point at 0 and 1/2 are on the FCC lattice, those at 1/4 and 3/4 are on a similar lattice displaced among the body diagonals by 1/4 of the cube's edge.
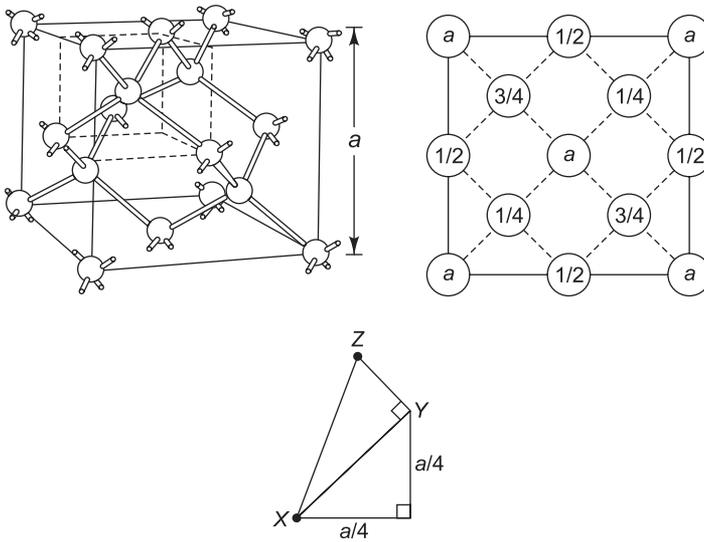
Fig. 1.37 Diamond Structure

The packing factor of this structure is calculated as follows:

$$(XY)^2 = \left(\frac{a}{4}\right)^2 + \left(\frac{a}{4}\right)^2 = \frac{a^2}{8}$$

$$(XY)^2 = (XY)^2 + (XY)^2 = \frac{a^2}{8} + \frac{a^2}{16} = \frac{3a^2}{16}$$

But, $XZ = 2r$

Therefore, $(2r)^2 = \dfrac{3a^2}{16}$

(or) the nearest neighbour distance, $2r = \dfrac{\sqrt{3}a}{4}$

Lattice constant, $a = \dfrac{8r}{\sqrt{3}}$

Packing factor $= v/V = \dfrac{8 \times \dfrac{4}{3}\pi r^3}{a^3} = \dfrac{32}{3}\dfrac{\pi r^3 3\sqrt{3}}{(8r)^3}$

$\qquad\qquad = \dfrac{\pi\sqrt{3}}{16} = 0.34$, (or) 34%

Thus it is a loosely packed structure. Carbon, silicon and germanium crystallise in this structure.

1.12   GRAPHITE STRUCTURE

1.12.1   Cubic Zinc Sulphide or Zinc Blende Structure

Zinc blende structure is almost identical to the diamond structure expect that the two interpenetrating FCC sublattices are of different atoms and displaced from each other by one-quarter of the body diagonal.

The cubic zinc sulphide structure results when zinc atoms are placed on the FCC lattice and S atoms on the other FCC lattice as shown in Fig. 1.38. There are four molecules of Zns per cell. In each atom there are four equally distant atoms of the opposite kind arranged at the corners of a tetrahedron. CuF, Zns, Cds, InSb, AgI, etc., also have ZnS structure.

○ Zn
● S
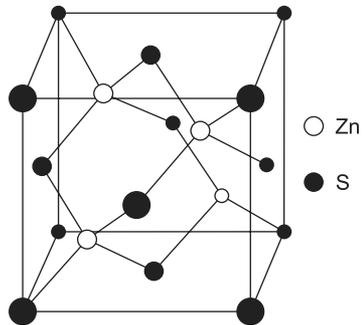
Fig. 1.38 Zinc Blende Structure

1.12.2   Sodium Chloride Structure

Sodium chloride (NaCl) is an ionic crystal. It posses an FCC structure with a basis of one sodium ion and one chlorine ion in an alternate fashion as shown in Fig. 1.39. The Cl ions are located at the corners as well as the centres of the faces of the cube. The Na ions are located exactly at the mid-point of the axial length of the unit cell along each axis. Most of the alkali halides and sulphides exhibit these type of

structures. Thus NaCl crystals can be identified to be composed of two FCC sublattices with one of Cl ion having the origin (0,0,0) and the other of Na ion having the origin (1/2,0,0)

○ Na^{++} ● Cl^{-}
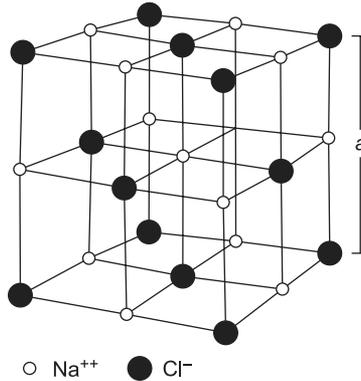
Fig. 1.39 Sodium Chloride Structure

1.13   ALLOTROPY AND POLYMORPHISM

Most substances exist in more than one stable crystalline form. The various forms have the same composition but the crystal structures are different. This occurs due to a change in temperature or pressure or both. Polymorphism is defined as the ability of a material to exist in two or more crystal structures. If the change in structures is reversible, then the polymorphic change is known as allotropy. It indicates that there is a change only in physical properties and not in chemical properties. Cobalt posses an HCP structure at ordinary temperature and when heated above 477°C, it changes to an FCC structure. Iron is a good example of polymorphism and allotropy. When iron is heated to 912°C, the structure changes to FCC and when heated above 1400°C it changes to a BCC structure. When it is allowed to cool, above 912°C it possesses an FCC structure and a BCC structure below 912 °C.

1.14   CRYSTAL GROWTH TECHNIQUES

Crystals are widely used in device applications. There are three types of crystal growth techniques. They are
   (i)   Growth from solution
   (ii) Growth from melt
   (iii) Growth from vapour

1.15 SOLUTION GROWTH

Crystals which possess high solubility and have variation in solubility with temperature in a particular solvent are grown by the solution-growth technique. In general, organic crystals are grown by this method. Solution growth is generally simple and inexpensive.

Growth of crystals comprises the following three general steps:
  (i)   Creation of supersaturation/supercooling
  (ii) Producing crystal nuclei of microscopic size
  (iii) Consecutive growth of crystals to yield distinct faces

Depending upon the solvents (water or organic liquid or molten salt) and solubility of the solute, there are two methods in solution growth.
  (i) Low-temperature solution growth
       (growth from aqueous solution, gel growth)
  (ii) High-temperature solution growth
       (flux growth, hydrothermal growth)

1.15.1   Growth from Aqueous Solution

At temperatures few degrees above the ambient temperature and at normal pressure, crystal growth is carried out from aqueous or organic media.

1.15.2   Gel Growth

Gel is a semi-solid possessing high viscosity and is a porous material consisting of a semirigid network. Due to the complication such as decomposition before melting or non-availability of suitable flux by conventional techniques, gel growth method is the most suitable method.

Gel growth by single crystals can be classified as follows.
  (i) Reaction method
  (ii) Complex dilution method
  (ii) Reduction of solubility method

1.15.3   Hydrothermal Growth

Hydrothermal refers to high temperatures and pressures. Crystal growth is carried out at high temperatures and pressures. This method is preferred for crystal materials which don't have sufficient solubility in water or other solvents at normal temperatures and pressures.

1.15.4 Flux Growth

In the absence of a proper solvent for low-temperature solution growth, crystal growth is done by utilizing high-temperature molten salts.

1.16 MELT GROWTH

Melt growth is suitable for materials that melt congruently and do not have any phase transition below melting point; these can be grown by melt technique. Czochralski and Bridgman techniques are the most commonly used melt-growth techniques.

1.16.1 Bridgman Technique

This growth technique was developed by Bridgman in 1925. It is one of the simplest techniques for growth of crystals from melts. This technique is preferred for the materials which melt congruently, do not decompose before melting and do not undergo phase transformation between the melting point and room temperature; these can be grown as single crystals. There are two different types of Bridgman technique

   (i)   Horizontal Bridgman technique (Fig. 1.40)
  (ii)   Vertical Bridgman technique (Fig. 1.41)
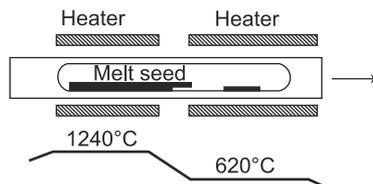
Fig. 1.40 Schematic Representation of Horizontal Bridgman Technique

    The material to be grown is encapsulated in a glass or quartz tube and suspended in the furnace having suitable gradient for growth. The conical-shaped ampoule tip enhances nucleation of a single crystal. After melting of the substance, the growth ampoule is dragged from hot zone to cold zone gradually. The lower region of the ampoule has a capillary tip in which the melt is filled and during lowering in the gradient, seed initiates and gradually grows up the entire melt region of the ampoule. The lowering rate of the ampoule, required to produce a crystal of reasonable quality, varies for different materials

and it also depends largely on materials of molecular and crystalline complexity.
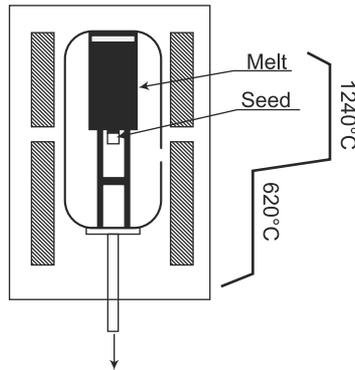
Fig. 1.41 | Schematic Representation of Vertical Bridgman Technique

This method has merits such as producing crystals with good dimensional tolerance quickly, fast growth and employ relatively simple technology.

1.16.2   Czochralski Technique

This growth technique was developed by Czochralski in 1916. The material to be grown is taken in a crucible and is heated by a heater which results in a clear melt with a free liquid surface at the top.

A monocrystalline seed with the required orientation is attached to a pulling rod and is mounted above the melt surface. The rod can be vertically moved and rotated during growth. The temperature in the heater coil is suitably adjusted so that the required portion of the dipped seed is remelted and a melt meniscus is formed. The whole assembly is enclosed in a vessel filled with inert gas for semiconductors and oxygen/air for oxides.

The rod is then rotated and slowly moved upwards. As the melt freezes on the crystal, it does so in the same in the same orientation as the seed and a single crystal is grown. The desired shape and diameter of the crystal is controlled by the pulling rate of the rod, rotation rate of the rod and heater temperature. This technique is known as necking procedure. This technique is applied to grow crystals with wide range of materials.
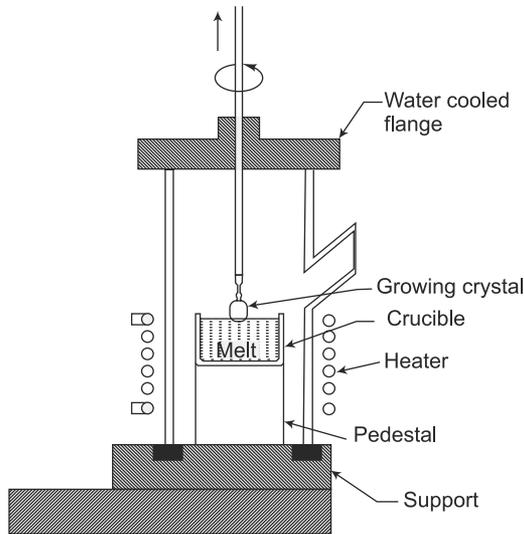
Fig. 1.42 Schematic Representation of Czochralski Technique

1.17   VAPOUR GROWTH

In this technique, the material to be grown is supplied in the form of vapour. The powders of the material are atomized by exposing to high temperature. A seed is brought near the melting point so that the incoming atoms or molecules will have high mobility on the growing surface. A crystal of high purity can be grown by this method at very low temperature. Synthetic rubies and sapphires are grown by this technique. The growth of a crystal from vapour can be classified as follows.

1. Physical vapour deposition (PVD)
2. Chemical vapour deposition (CVD)

1. Physical Vapour Deposition

In this technique, there is a transfer of atoms/molecules by saturated carrier gas to a colder growth zone. This process involves no chemical reaction.

2. Chemical Vapour Deposition

In this technique, gaseous compounds of the materials are send to a substrate where a thermal reaction takes place. Reaction byproducts are then removed from the system.

SUMMARY

▶▶ Solids are classified into three groups on the basis of atomic arrangement. They are single crystals, polycrystals and amorphous substances.

▶▶ Lattice is defined as an array of points which are imaginarily kept to represent the position of atoms in the crystal, such that every lattice point has got the same environment as that of the other. Hence one lattice point cannot be distinguished from the other lattice point.

▶▶ Atomic arrangement in crystalline solids is described by space lattice.

▶▶ A space lattice is a three-dimensional array of points, each of which has an identical environment.

▶▶ A unit cell is the small cell in a crystal structure that carries the characteristics of the crystal. The repetition of the unit cell in three dimensions generates the whole crystal structure.

▶▶ A unit cell is characterised by three axial lengths (a, b, c) and three axial angles (α, β, γ).

▶▶ A crystal structure is developed by the combination of space lattices and its basis.

▶▶ The single crystals are classified into seven crystal systems.

▶▶ There are only 14 ways of arranging lattice points in space so that each point has an identical environment. The 14 types of arrangements are called Bravais lattices.

▶▶ Miller indices are used to designate a plane in a crystal lattice.

▶▶ $d = a/\sqrt{h^2 + k^2 + l^2}$ is the interplanar distance between two neighborhood planes in a cubic crystal.

▶▶ The cubic system has three types of unit cells namely simple cubic, body-centered cubic and face-centered cubic.

▶▶ Only one element polonium exhibits a simple cubic structure and its atomic packing factor value is 52%.

▶▶ BCC structure is exhibited by about 20 metals and its atomic packing factor value is 68%.

▶▶ About 29 metals crystallise in this structure and their atomic packing factor value is 74%.

▶▶ HCP structure has an atomic packing factor value of 74%.

▶▶ FCC and HCP are closely packed structures.

▶▶ Polymorphism is defined as the ability of a material to exist in two or more crystal structures.

▶ If the change in structures reversible, then the polymeric change is known as allotropy.

▶ Any deviations in a crystal from regular and periodic arrangement of atoms are known as crystal imperfections.

▶ Point defects, line defects, surface and grain boundary defects and volume defects are the various types of crystal imperfections.

▶ Growth from solution, melt and vapour are the methods to grow c crystals according to the requirements.

✓ SOLVED PROBLEMS

1.1 Copper crystallising into FCC structure has an atomic radius of 1.278 Å and an atomic weight of 63.5. Determine the density of copper. Avogadro's number is 6.023×10^{23}/mol.

Solution   Given Data

$$r = 1.278 \text{ Å} = 1.278 = 1.278 \times 10^{-10} \text{ m}$$

$$M = 63.5 \text{ g/mol} = 63.5 \times 10^{-3} \text{ g/mol}$$

$$N_A = 6.023 \times 10^{23} \text{ mol}^{-1} \qquad \rho = ?$$

Density of the material is given by

$$\rho = \frac{nM}{N_A a^3} \tag{1}$$

where a is the cube edge and n is the number of atoms per unit cell. In FCC structure, the number of atoms per unit cell is $n = 4$.

Moreover, in FCC structure, we have

$$\rho = 2\sqrt{2}\, r$$

Hence, Eq. (1) is rewritten as

$$\rho = \frac{nM}{N_A (2\sqrt{2}r)^3} \tag{2}$$

Substitution of the given data into Eq. (2) gives

$$\rho = \frac{4 \times 63.5 \times 10^{-3}}{6.023 \times 10^{23} \times (2\sqrt{2})^3 \times (1.278 \times 10^{-10})^3}$$

i.e., $\rho = \mathbf{8928 \ kg/m^3}$

1.2 Calculate the number of atoms per unit cell of a metal having the lattice parameter of 2.9 Å and the density of 7.87 g/cm³. Atomic weight of the metal is 55.85 and Avogadro's number is 6.023×10^{23}/mol.

Solution   Given Data

$$a = 2.9 \text{ Å} = 2.9 \times 10^{-10} \text{ m}$$
$$\rho = 7.87 \text{ g/cm}^3 = 7870 \text{ kg/m}^3$$
$$M = 55.85 \text{ g/mol} = 55.85 \times 10^3 \text{ kg/mol}$$
$$N_A = 6.023 \times 10^{23} \text{ mol}^{-1} \qquad n = ?$$

Density of the metal is given by

$$\rho = \frac{nM}{N_A \, a^3}$$

or, $$n = \frac{\rho N_A \, a^3}{M} \qquad\qquad\qquad (1)$$

Substitution of the given data into Eq. (1) gives n

$$n = \frac{7870 \times 6.023 \times 10^{23} \times (2.9 \times 10^{-10})^3}{55.85 \times 10^{-3}}$$

i.e.,   **$n = 2$**

1.3 Zinc has an HCP structure. Height of the unit cell is 4.94 Å. The nearest neighbour distance is 2.7 Å. Atomic weight of zinc is 65.37. Calculate the density of zinc. Avogadro's number is 6.023×10^{23} mol^{-1}.

Solution   Given Data

$$c = 4.94 \text{ Å} = 4.94 \times 10^{-10} \text{ m}$$
$$a \equiv 2r = 2.7 \text{ Å} = 2.7 \times 10^{10} \text{ m}$$
$$M = 65.37 \text{ g/mol} = 65.37 \times 10^{-3} \text{ kg/mol}$$
$$N_A = 6.023 \times 10^{23} \text{ mol}^{-1} \qquad n = ?$$

The density of a crystal is given by

$$\rho = \frac{\text{Unit cell mass}}{\text{Unit cell volume}}$$

$$\rho = \frac{\text{Number of atoms in unit cell} \times \text{Mass of one atom}}{\text{Unit cell volume}}$$

$$\rho = \frac{nM}{N_A V} \tag{1}$$

where V is the unit cell volume and n is the number of atoms in the unit cell. In an HCP structure, we have

$$V = \frac{3\sqrt{3}\, a^2 c}{2}$$

$$n = 6$$

Hence, Eq.(1) is rewritten as

$$\rho = \frac{2nM}{N_A\, 3\sqrt{3}\, a^2 c} \tag{2}$$

Substitution of the given data into Eq. (2) gives

$$\rho = \frac{2 \times 6 \times 65.37 \times 10^{-3}}{6.023 \times 10^{23} \times 3\sqrt{3} \times (2.7 \times 10^{-10})^2 \times 4.94 \times 10^{-10}}$$

i.e., $p = \mathbf{6960\ kg/m^3}$

1.4 Lead crystallises into an FCC structure and it has the nearest neighbour distance of 3.499 Å. Calculate the number of lead atoms per square millimetre on the plane (1 0 0).

Solution Given Data

$$2r = 3.499\ \text{Å} = 3.499 \times 10^{-10}\ \text{m}$$
$$(h\ k\ l) = (1\ 0\ 0) \qquad N = ?$$
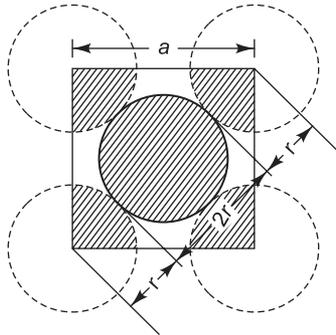
Fig. 1.43

As seen (Fig. 1.40), the number of atoms in the plane (1 0 0) is 2. Area of the plane (1 0 0) is a^2, where a is the cube edge.

In an fcc structure, we have

$$a = 2\sqrt{2}\,r$$

Thus, the number of atoms per unit area on the plane (1 0 0) is N'

$$N' = \frac{\text{Number of atoms on the plane}(1\ 0\ 0)}{\text{Area of the plane}(1\ 0\ 0)}$$

$$N' = \frac{2}{a^2}$$

i.e., $$N' = \frac{2}{(2\sqrt{2}\,r)^2}$$

or $$N' = \frac{1}{(2r)^2} \qquad (1)$$

Substitution of the given data into Eq. (1) gives

$$N' = \frac{1}{(3.499 \times 10^{-10})^2}$$

$$N' = 0.08168 \times 10^{20}/\text{m}^2$$

Therefore, the number of atoms per mm^2 on the plane (1 0 0) is

$$N = N' \times 10^{-6}/\text{mm}^2$$

i.e., $$N = 8.168 \times 10^{12} \text{ atoms/mm}^2$$

1.5 Copper crystallises into an FCC structure. Its atomic radius is 1.278 Å. Calculate the interplanar spacing for (2 1 2) and (1 1 0) planes in copper crystal.

Solution Given Data

$$r = 1.278 \text{ Å} = 1.278 \times 10^{-10} \text{ m}$$
$$(h_1\ k_1\ l_1) = (2\ 1\ 2)$$
$$(h_2\ k_2\ l_2) = (1\ 1\ 0) \quad d_{212} = ?; \quad d_{110} = ?$$

The interplanar spacing in a cubic crystal is given by d

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \tag{1}$$

where a is the cube edge.

Moreover, in an FCC structure, we have

$$a = 2\sqrt{2}\, r$$

Therefore, Eq. (1) is rewritten as

$$d = \frac{2\sqrt{2}\, r}{\sqrt{h^2 + k^2 + l^2}} \tag{2}$$

Substitution of the given data for the plane (2 1 2) into Eq. (2) gives

$$d_{212} = \frac{2\sqrt{2} \times 1.278 \times 10^{-10}}{\sqrt{4 + 1 + 4}}$$

$$d_{212} = 1.205 \times 10^{-10}\, m$$

i.e., $\quad$ **d_{212} = 1.205 Å**

Similarly, we get

$$d_{110} = \frac{2\sqrt{2} \times 1.278 \times 10^{-10}}{\sqrt{1 + 1 + 0}}$$

$$d_{110} = 2.556 \times 10^{-10}\, m$$

i.e., $\quad$ **d_{110} = 2.556 Å**

1.6 Find the Miller indices of a plane making intercepts a, $b/2$, $3c$ in an SC structure.

Solution Given Data

$\quad$ Intercepts = a, $b/2$, $3c$

$\quad$ (h k l) = ?

Coefficients of the intercepts are 1, 1/2, 3. Hence, their reciprocals are 1, 2, 1/3

Multiplication by LCM gives the Miller indices as

$$(h\ k\ l) = (3,\ 6,\ 1)$$

Show that for a simple cubic system.

$$d_{100} : d_{110} : d_{111} = \sqrt{6} : \sqrt{3} : \sqrt{2}$$

Solution   Given Data

$(h_1 \ k_1 \ l_1) = (1\ 0\ 0)$
$(h_2 \ k_2 \ l_2) = (1\ 1\ 0)$
$(h_3 \ k_3 \ l_3) = (1\ 1\ 1)$ $d_{100} : d_{110} : d_{111} = ?$

Interplanar spacing in a cubic crystal is

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \tag{1}$$

where *a* is the cube edge.

Substitution of the given data into Eq. (1) gives

$$d_{100} = \frac{a}{\sqrt{1+0+0}} = a$$

$$d_{110} = \frac{a}{\sqrt{1+1+0}} = \frac{a}{\sqrt{2}}$$

$$d_{111} = \frac{a}{\sqrt{1+1+1}} = \frac{a}{\sqrt{3}}$$

Therefore, we have

$$d_{100} : d_{110} : d_{111} = a : \frac{a}{\sqrt{2}} : \frac{a}{\sqrt{3}}$$

i.e., $$\mathbf{d_{100} : d_{110} : d_{111} = \sqrt{6} : \sqrt{3} : \sqrt{2}}$$

Draw the planes (1 1 1) and (1 0 1) in a cubic crystal.

Solution   Given Data

$(h_1 \ k_1 \ l_1) = (1\ 1\ 1)$
$(h_2 \ k_2 \ l_2) = (1\ 0\ 1)$

Plane (1 1 1): In this case, $h = 1$, $k = 1$, $l = 1$. Therefore, the reciprocals of h, k, l (i.e., intercepts) are

$$\frac{1}{1}, \frac{1}{1}, \frac{1}{1} = 1,\ 1,\ 1$$

Now, a plane with intercepts 1, 1, 1 along the x, y, z axis is sketched (Fig. 1.41).
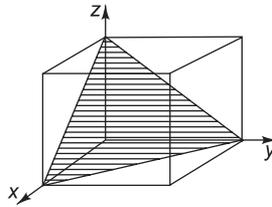
Fig. 1.44

Plane (1 0 1): In this case, $h = 1, k = 0, l = 1$. Hence, the reciprocals of h, k, l (i.e., intercepts) are

$$\frac{1}{1}, \frac{1}{0}, \frac{1}{1} = 1, \infty, 1$$

Now, a plane with the intercepts 1, ∞, 1 along the x, y, z axes is sketched (Fig. 1.42).
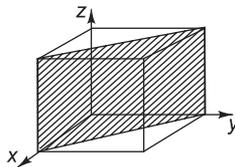
Fig. 1.45

1.9 The unit cell edges a, b and c of an orthorhombic crystal are 0.05 nm, 0.04 nm and 0.03 nm, respectively. Of a family of parallel equidistant planes, the one that is closest to the origin of the unit cell makes intercepts on the a, b and c edges at 0.025 nm, 0.02 nm and 0.01 nm, respectively. Find the Miller indices of the set of parallel planes.

(A.U, B.E/B.Tech, Jan. 2006)

Solution Given Data

$$a = 0.05 \text{ nm} = 0.05 \times 10^{-9} \text{ m}$$
$$b = 0.04 \text{ nm} = 0.04 \times 10^{-9} \text{ m}$$
$$c = 0.03 \text{ nm} = 0.03 \times 10^{-9} \text{ m}$$

Intercepts of the plane are

$$pa = 0.025 \text{ nm} = 0.025 \times 10^{-9} \text{ m}$$

$qb = 0.02$ nm $= 0.02 \times 10^{-9}$ m

$rc = 0.01$ nm $= 0.01 \times 10^{-9}$ m   Miller indices = ?

From the given data, we get

$p = 0.5$

$q = 0.5$

$r = 0.33$

Therefore, we have

$$\frac{1}{p} : \frac{1}{q} : \frac{1}{r} = \frac{1}{0.5} : \frac{1}{0.5} : \frac{1}{0.33}$$

$$\frac{1}{p} : \frac{1}{q} : \frac{1}{r} = 2 : 2 : 3$$

Thus, Miller indices of the given set of parallel planes are **(2, 2, 3)**.

1.10 Iron has BCC structure with atomic radius 0.123 Å. Find the lattice constant and also the volume of the unit cell.

(A.U, B.E/B.Tech, May/June 2005)

Solution   Given Data

$r = 0.123$ Å $= 0.123 \times 10^{-10}$ m $a = ?$; $V = ?$

In a bcc structure, we have

$$a = \frac{4}{\sqrt{3}} r \tag{1}$$

Substitution of the given value for r into Eq. (1) gives

$$a = \frac{4 \times 0.123 \times 10^{-10}}{\sqrt{3}}$$

i.e., $a = 0.284 \times 10^{-10}$ m

The unit cell volume of a BCC structure is

$$V = a^3 \tag{2}$$

Substituting the value for 'a' into Eq. (2), we get

$$V = \mathbf{0.023 \times 10^{-30} \ m^3}$$

1.11 Magnesium has an HCP structure. The radius of magnesium atom is 0.1605 nm. Calculate the volume of the unit cell of magnesium.

(A.U, B.E/B.Tech, Jan. 2006)

Solution Given Data

$r = 0.1605$ nm $= 0.1605 \times 10^{-9}$ m $V = ?$

In an HCP structure, the unit cell volume is

$$V = 3\sqrt{2}\, a^3 \tag{1}$$

where $a = 2r$ is the lattice constant. Therefore, we have

$$V = 3\sqrt{2}\, (2r)^3$$

$$V = 24\sqrt{2}\, r^3 \tag{2}$$

Substituting the given value for 'r' into Eq. (2), we get

$$V = 24\sqrt{2} \times (0.1605\ 10^{-9})^3$$

$$\mathbf{V = 0.140 \times 10^{-27}\ m^3}$$

1.12 Determine lattice constant of *Fcc* lead crystal of radius 1.746 Å.

[A.U, B.E/B. Tech, Jan. 2011]

Solution Given Data

$r = 1.746$ Å $= 1.746 \times 10^{-10}$ m

$a = ?$

$$r = \frac{a}{2\sqrt{2}}$$

$$a = r2\sqrt{2}$$

$$= 1.746 \times 10^{-10} \times 2 \times \sqrt{2}$$

$$a = 4.937 \times 10^{-10}\ m$$

$$\mathbf{= 4.937\ \text{Å}}$$

1.13 The lattice constant of a cubic lattice is 4.12 Å. Find the lattice spacings between (111), (112) and (123) lattice plane.

[A.U, B.E/B. Tech, Jan. 2011]

Solution   Given Data

$$a = 4.12 \text{ Å} = 4.12 \times 10^{-10} \text{ m}$$
$$d_{(111)} = ?$$
$$d_{(112)} = ?$$
$$d_{(123)} = ?$$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$d_{(111)} = \frac{4.12 \times 10^{-10}}{\sqrt{1+1+1}}$$

$$= \frac{4.12 \times 10^{-10}}{\sqrt{3}}$$

$$= 2.3786 \text{ Å}$$

$$d_{(112)} = \frac{4.12 \times 10^{-10}}{\sqrt{1+1+4}}$$

$$= 1.6819 \text{ Å}$$

$$d_{(123)} = \frac{4.12 \times 10^{-10}}{\sqrt{1+4+9}}$$

1.14   Copper has FCC structure and its atomic radius is 1.273 Å. Find the lattice parameter and the density of copper. Avagadro's number 6.023×10^{26} atoms/kilomole and atomic weight of copper is 63.5 gm.

(A.U, B.E/B. Tech, Jan. 2011)

Solution   Given Data

$$r = 1.273 \times \text{Å} = 1.273 \times 10^{-10} \text{ m}$$
$$M = 63.5 \text{ gms} = 63.5 \ 10^{-3} \text{ kg}$$
$$\underline{N_A = 6.023 \times 10^{23} \text{ m}^{-1}}$$
$$a = ? \ \rho = ?$$

$$\rho = \frac{nM}{N_A \, a^3}$$

Morever in an FCC structure, we have

$$a = 2\sqrt{2} \, r$$

$$\rho = \frac{n\,M}{N_A\,(2\sqrt{2})^3}$$

$$\rho = \frac{4 \times 63.5 \times 10^{-3}}{6.023 \times 10^{23} \times (2\sqrt{2})^3 \times (1.273 \times 10^{-10})^3}$$

ρ = **9034 kg/m³**

1.15 The interplanar distance of (110) planes in a BCC crystal is 2.03 Å. What is the lattice parameter of the crystal?

(A.U, B.E/B. Tech, Jan. 2011)

Solution Given Data

$d_{(110)} = 2.03 \text{ Å} = 2.03 \times 10^{-10}$ m
$a = ?$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$d_{(110)} \times \sqrt{h^2 + k^2 + l^2} = a$$

$$a = 2.03 \times 10^{-10} \times \sqrt{1^2 + 1^2 + 0^2}$$

$$a = \mathbf{2.8 \times 10^{-10}}\ \mathbf{m}$$

1.16 The density of BCC iron is 7860 kg/m³ and its atomic weight is 55.85. Calculate its atomic radius.

(Avagardo's number = 6.023×10^{26}/kmol) *(A.U, B.E/B.Tech, Jan. 2011)*

Solution Given Data

$\rho = 7860$ kg/m³
$M = 55.85 \times 10^{-3}$ kg
$N_a = 6.023 \times 10^{26}$/kmole = 6.023×10^{23} mol^{-1}
$r = ?$

In a BCC structure,

$$n = 2,\ \alpha = \frac{4}{\sqrt{3}}\,r$$

$\therefore$

$$\rho = \frac{nM}{N_A \, a^3}$$

$$\rho = \frac{nM}{N_A \left(\dfrac{4}{\sqrt{3}} r\right)^3}$$

$$r^3 = \frac{nM (\sqrt{3})^3}{N_A (4)^3 \, \rho}$$

$$r^3 = \frac{2 \times 55.85 \times 10^{-3} \times (\sqrt{3})^3}{6.023 \times 10^{23} \times (4)^3 \times 7860}$$

$$r^3 = 7.669 \times 10^{-31}$$

$$\boldsymbol{r = 9.15 \times 10^{-11} \text{ m.}}$$

QUESTIONS AND ANSWERS

1.1 Define crystal.
A crystal is defined as a solid in which atoms or molecules are regularly arranged. That is, the crystal is said to possess a long-range order.

1.2 Define single crystal. Give a few examples.
Single crystal is a solid in which atoms or molecules are regularly arranged throughout its entire volume. Examples are ruby, quartz, GaAs.

1.3 Define polycrystal. Give a few examples.
Polycrystal is a solid made up of a large number of single crystals with different orientations. Examples are Cu, Al, steel, brass.

1.4 What is a noncrystalline solid? Give a few examples.
It is a solid with no regular arrangement of atoms or molecules. Examples are window glass, polyethylene, PVC, Teflon, Bakelite.

1.5 Define space lattice.
Space lattice is defined as an infinite three-dimensional array of points in which every point has an identical environment as any other point in the array.

1.6 Define basis.
The basis is a unit assembly of atoms or molecules identical in composition and associated with every lattice point in a lattice.

1.7 What is meant by crystal structure?
Crystal structure is the basis repeated with correct periodicity in all directions. That is, Lattice + Basis = Crystal structure.

1.8 Define unit cell.
Unit cell is defined as the smallest geometric figure, the repetition of which gives the actual crystal structure.

1.9 What are lattice parameters?
The primitives and interfacial angles are called lattice parameters.

1.10 What are primitives?
The intercepts of the unit cell on the crystallographic axes are called primitives.

1.11 What are interfacial angles?
The angles between the crystallographic axes are called interfacial angles.

1.12 Define primitive cell.
The unit cell formed by the primitives is called the primitive cell. It contains only one lattice point.

1.13 Name the seven crystal systems.
The crystals systems are cubic, tetrgonal, orthorhombic, monoclinic, triclinic, rhombohedral and hexagonal crystal systems.

1.14 Mention the lattice parameters of a cubic crystal system. Give a few examples for cubic crystal systems.
$a = b = c$; $\alpha = \beta = \gamma = 90°$. Examples are NaCl, Po.

1.15 Mention the lattice parameters of a tetragonal crystal system. Give a few examples for tetragonal crystal systems.
$a = b \neq c$; $\alpha = \beta = g = 90°$. Examples are $NiSO_4$, SnO_2.

1.16 Mention the lattice parameters of an orthorhombic crystal system. Give a few examples for orthorhombic crystal systems.
$a \neq b \neq c$; $\alpha = \beta = \gamma = 90°$. Examples are KNO_3, $BaSO_4$.

1.17 Mention the lattice parameters of a monoclinic crystal system. Give a few examples for monoclinic crystal systems.

$a \neq b \neq c$; $\alpha \neq \beta \neq \gamma \neq 90°$. Examples are Na_2SO_3, $FeSO_4$.

1.18 Mention the lattice parameters of a triclinic crystal system. Give an example for monoclinic crystal systems.

$a \neq b \neq c$; $\alpha \neq \beta \neq \gamma \neq 90°$. Example: $CuSO_4$.

1.19 Mention the lattice parameters of a rhombohedralcrystal system. Give a few examples for monoclinic crystal systems.

$a = b = c$; $\alpha = \beta = \gamma = 90°$; $a, b, g < 120°$; Example: $CaSO_4$.

1.20 Mention the lattice parameters of a hexagonal crystal system. Give a few examples for hexagonal crystal systems.

$a = b \neq c$; $\alpha = \beta = 90°$, $\gamma = 120°$. Examples are SiO_2, AgI.

1.21 What are Bravais lattices?

There are only 14 ways of arranging lattice points in space so that each point has identical environment. The 14 types of arrangements are called Bravais lattices.

1.22 Define simple cubic structure.

It refers to a cubic structure in which there is one lattice point at each corner of the unit cell.

1.23 Define body-centred cubic structure.

It refers to a cubic structure in which there is one lattice point at each corner of the unit cell and there is a lattice point at the centre of the unit cell.

1.24 Define face-centred cubic structure.

It refers to a cubic structure in which there is one lattice point at each corner of the unit cell. Moreover, there is one lattice point at the centre of each face of the unit cell.

1.25 Define base-centred cubic structure.

It refers to a cubic structure in which there is one lattice point at each corner of the unit cell and there is one lattice point at the centre of the two bases of the unit cell.

1.26 What are lattice planes?

The crystal lattice is considered to be made up of a set of parallel equidistant planes passing through the lattice points. These planes are called lattice planes.

1.27 What are Miller indices?

A plane in a crystal is designated by three integers ($h\,k\,l$) called Miller indices. They are associated with the intercepts of the plane on the crystallographic axes of the unit cell.

1.28 Draw the crystal planes with Miller indices (1 0 0) and (0 1 0).
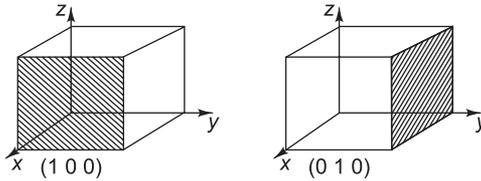
Fig. 1.46

1.29 Draw the crystal planes with Miller indices (0 0 1) and (1 0 1).
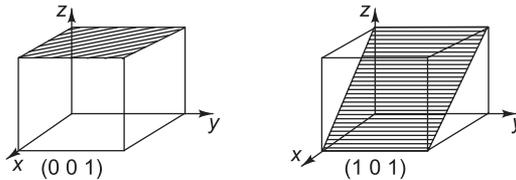
Fig. 1.47

1.30 Draw the crystal planes with Miller indices (1 1 0) and (1 1 1).
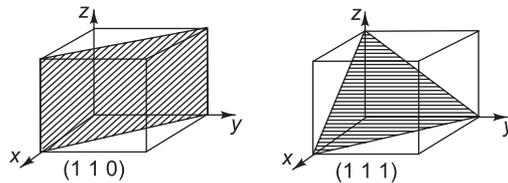
Fig. 1.48

1.31 Draw the crystal planes with Miller indices (1 1 2) and (2 0 0).
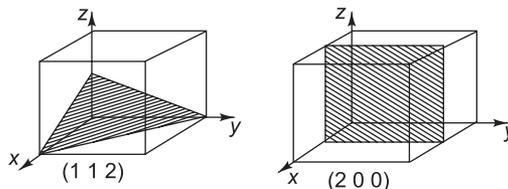
Fig. 1.49

1.32 Draw the crystal plane with Miller indices ($\bar{1}\,0\,0$).
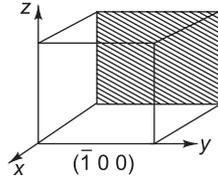
Fig. 1.50

1.33 Find the Miller indices of a lattice plane with intercepts 5*a* and 5*b* on the *x* and *y* axes. The plane is parallel to the *z* axis. Given *a*, *b*, *c* are the primitive vectors of the lattice.

For the given plane, the intercepts are *pa*, *qb*, *rc* = 5*a*, 5*b*

i.e.,
$$\frac{1}{p}:\frac{1}{q}:\frac{1}{r}=\frac{1}{5}:\frac{1}{5}:\frac{1}{\infty}$$

or
$$\frac{1}{p}:\frac{1}{q}:\frac{1}{r}=\frac{1}{5}:\frac{1}{5}:0$$

Multiplying the reciprocals by their LCM, we get the Miller indices as (1 1 0).

1.34 Give the expression for interplanar spacing in a cubic crystal.

The distance between adjacent planes of a set of parallel planes of Miller indices ($h\,k\,l$) in a cubic crystal is given by

$$d=\frac{a}{\sqrt{h^2+k^2+l^2}}$$

where *a* is the cube edge.

1.35 Give the expression for the angle between the normals to the planes ($h_1\,k_1\,l_1$) and ($h_2\,k_2\,l_2$) in a cubic crystal system.

The angle between the normals to the two planes ($h_1\,k_1\,l_1$) and ($h_2\,k_2\,l_2$) in a cubic system is

$$\cos\theta=\frac{h_1\,h_2+k_1\,k_2+l_1\,l_2}{\sqrt{h_1^2+k_1^2+l_1^2}\,\sqrt{h_2^2+k_2^2+l_2^2}}$$

1.36 Define atomic radius.

It is defined as half the distance between nearest neighbouring atoms in a crystal.

1.37   Define coordination number.
Coordination number is the number of equidistant neighbours of an atom in a crystal.

1.38   Define atomic packing factor.
Atomic packing factor is defined as the ratio of volume occupied by atoms in the unit cell to the volume of the unit cell.

1.39   What is meant by nearest neighbour distance?
It is the distance between the centres of two neighbouring atoms.

1.40   Give the number of atoms in a unit cell of a simple cubic structure.

$$n = \frac{1}{8} \times 8 \quad \text{or} \quad n = 1$$

1.41   Give the number of atoms in a unit cell of a body-centred cubic structure.

$$n = \text{Contribution of corner atoms}$$
$$+ \text{Contribution of body-centred atom}$$

$$n = \left(\frac{1}{8} \times 8\right) + 1n = \left(\frac{1}{8} \times 8\right) + 1$$

i.e., $n = 2$

1.42   Give the number of atoms in a unit cell of a face-centred cubic structure.

$$n = \text{Contribution of corner atoms}$$
$$+ \text{Contribution of face-centred atoms}$$

$$n = \left(\frac{1}{8} \times 8\right) + 1$$

i.e., $n = 4$

1.43   Give the number of atoms in a unit cell of an HCP structure.

$$n = \text{Contribution of corner atoms}$$
$$+ \text{Contribution of base-centred atoms}$$
$$+ \text{Contribution of inner atoms}$$

$$n = \left(\frac{1}{6} \times 12\right) + \left(\frac{1}{2} \times 2\right) + 3$$

i.e., $n = 6$

1.44   Give the coordination number of a simple cubic structure.

$$N_C = 6$$

1.45   Give the coordination number of a body-centred cubic structure.

$$N_C = 8$$

1.46   Give the coordination number of a face-centred cubic structure.

$$N_C = 12$$

1.47   Give the coordination number of an HCP structure.

$$N_C = 12$$

1.48   Give the expression for the atomic radius of a simple cubic structure.

In an SC structure, the atomic radius r and the cube edge a are related by

$$r = \frac{a}{2}$$

1.49   Give the expression for the atomic radius of a body-centred cubic structure.

$$r = \frac{\sqrt{3}\,a}{4}$$

1.50   Give the expression for the atomic radius of a face-centred cubic structure.

$$r = \frac{a}{2\sqrt{2}}$$

1.51   Give the expression for the atomic radius of an HCP structure.

$$r = \frac{a}{2}$$

1.52   Give the atomic packing factor of a simple cubic structure.

$$PF = \frac{\pi}{6} = 0.52$$

1.53   Give the atomic packing factor of a body-centred cubic structure.

$$PF = \frac{\sqrt{3}\pi}{8} = 0.68$$

1.54   Give the atomic packing factor of a face-centred cubic structure.

$$PF = \frac{\pi}{3\sqrt{2}} = 0.74$$

1.55   Give the atomic packing factor of an HCP structure.

$$PF = \frac{\pi}{3\sqrt{2}} = 0.74$$

1.56   What is crystal imperfection?
Any deviation in a natural crystal from the regular and the periodic arrangement of atoms is known as **crystal imperfection**.

1.57   Mention the four categories of crystal imperfection.
Point defect; Line defect; Surface and grain boundary defect; Volume defect.

1.58   What is point defect?
Zero-dimensional defects are called point defects.

1.59   What is line defect?
One-dimensional defects are called line defects.

1.60   What is surface defect?
Two-dimensional defects are called surface defects.

1.61   What is volume defect?
Three-dimensional defects are called volume defects.

1.62   What is lattice vacancy?
When the atoms are arranged in a regular and periodical manner, the crystal is known as a perfect crystal. When an atom is missing in a perfect crystal it is said to be a lattice vacancy.

1.63   What is Schottky defect?
If an atom is transferred from the lattice site to the surface of the crystal, it is known as **Schottky defect**.

1.64   What is Frenkel defect?
If an atom is transferred from the lattice site to the interstitial position, the defect is known as **Frenkel defect**.

1.65   What is edge dislocation?
It refers to the defect in a crystal in which an extra half plane from the top of the crystal to its middle is introduced.

1.66   What is screw dislocation?
It refers to the displacement of the atoms in one part of the crystal relative to the rest of the crystal.

1.67   What is grain boundary?
These are the surface imperfections which separate crystals of different orientations in a polycrystalline aggregate.

1.68   What is tilt boundary?
If the angle of dislocation in a crystal is less than 10°, then the defect is said to be **tilt boundary**.

1.69   What is twin boundary?
The crystal may be distorted along a plane in such a way that the mirror image of one undistorted portion looks like the other one. This type of defect is known as **twin boundary**.

1.70   What is stacking fault?
It refers to the discrepancy in the packing sequence of layers of atoms in a crystal.

✓ EXERCISE PROBLEMS

1.1   Copper has an FCC structure and its atomic radius is 1.273 Å. Find the lattice parameter and density of copper.
$$[a = 3.6 \times 10^{-10} \ m \ \rho = 9039 \ kg/m^3]$$

1.2   A crystal of BCC structure has atomic radius 1.2 Å. Find the volume of its unit cell. $\qquad [a^3 = 2.218 \times 10^{-29} \ m^3]$

1.3   Calculate the interplanar distance for (3 2 1) plane in simple cubic lattice with interatomic spacing equal to 4.12 Å.
$$[d = 1.1011 \times 10^{-10} \ m]$$

1.4   Sketch (1 1 2) and (1 2 3) plane for given Miller indices.

1.5 Iron of atomic weight 55.85 g solidifies into BCC structure and has a density 7860 kgm^{-3}. Calculate the radius of an atom.

[r = 1.242 Å]

1.6 Iron has BCC structure with atomic radius of 0.362 Å. Find the Lattice constant and the Volume of the unit cell.

[V = 0.58 × 10^{-30} m^3]

1.7 Calculate the number of atoms per unit cell of a metal having the lattice parameters of 1.2 Å and the density 5.23 gkm^3. Atomic weight of the metal is 41.61 and Avagardro's number is 6.023 × 10^{23}/mol.

[n = 1]

1.8 Copper crystallises into an FCC structures. Its atomic radius is 2.43 Å. Calculate the inter planar spacing for (1 0 1) and (1 2 1) planes in the copper crystal.

[d$_{(101)}$ = 4.86 Å, d$_{(121)}$ = 2.80 Å]

1.9 Copper Crystallising into an FCC structure has an atomic radius of 3.210 Å and an atomic weight of 26.2. Determine the density of copper. Avogadro's number is 6.023 × 10^{23}/mol.

[ρ = 2325 kg/m^3]

1.10 Magnesium has an HCP structure. The radius of magnesium atom is 1.023 mm. Calculate the volume of the unit cell of magnesium. *[V = 36.33 × 10^{-27} m^3]*

QUESTION BANK

PART-A

1.1 A material having different properties in different directions is known as _____ .

1.2 Give two examples for the metals having hexagonal close packed structure.

1.3 The Miller indices are the same for _____ planes.

1.4 Among the crystal structures, which one has the highest packing fraction?

1.5 How many ways are there in arranging lattice points in space so that each point has identical environment?

1.6 The number of atoms in the unit cell of an HCP structure is _____ .

1.7 Give the coordination number of a simple cubic structure.

1.8 What is the coordination number of a body-centered cubic structure?

1.9 Give the coordination number of a face-centered cubic structure.

1.10 The coordination number of a hexagonal closely packed structure is _____ .

1.11 Give the expression for the atomic radius of a simple cubic structure.

1.12 What is the expression for the atomic radius of a BCC structure?

1.13 State the expression for the atomic radius of an FCC structure.

1.14 Write the expression for the atomic radius of an HCP structure.

1.15 Give the atomic packing factor of a simple cubic structure.

1.16 The atomic packing factor of a BCC structure is _____ .

1.17 Give the atomic packing factor of an FCC structure.

1.18 Write the atomic packing factor of an HCP structure.

1.19 The number of atoms in the unit cell of a simple cubic structure is _____ .

1.20 What is point defect?

1.21 Give two examples of single crystals.

1.22 What is surface defect?

1.23 List out two examples for polycrystalline solids.

1.24 Whether stacking fault sequence refers to the discrepancy in the packing sequence of layers of atoms in a crystal?

1.25 What is the angle of dislocation in a crystal if the defect in it is a tilt boundary?

1.26 _____ and _____ are called the lattice parameters.

1.27 Lattice + _____ = Crystal structure.

1.28 What is point defect?

1.29 Give two examples for noncrystalline solids.

1.30 The microscopic property of a cubic crystalline material that is correlated to the macroscopic property of material density is _____ .

1.31 How do you represent the normal to the plane with Miller indices (hkl)?

1.32 The planes that determine the physical and chemical properties of solids are those with _____ Miller index numbers. (higher, lower, any value)

1.33 Coordination number is the number of _____ of an atom in a crystal.

1.34 How many lattice points does a primitive cell have?

1.35 The edge length of the unit cell is called _____ .

1.36 Draw the crystal plane with Miller indices (100) and (100).

1.37 What is volume defect?

1.38 Bismuth has $a = b = c = 4.74$ Å and angles $a = b = g = 60°$. What is its crystal structure?

1.39 Give the Miller indices of a plane making an intercept $3a$ on the x axis and parallel to the y and z axes.

1.40 Most of the alkali metals crystallise into which structure?

1.41 Transition and rare earth metals crystallise in _____ (FCC, BCC, HCP) structure.

1.42 The elements of _____ group in the periodic table crystallise in HCP structures.

1.43 How many atoms per unit cell are there in a BCC structure?

1.44 The number of atoms per unit cell in an FCC structure is

_____ .

1.45 Give the number of atoms per unit cell in an SC structure.

1.46 Polonium exhibits simple cubic structure at _____ temperatures. (High, low, all)

1.47 Give two examples for an FCC crystal structure.

1.48 Define crystal.

1.49 What is single crystal? Give a few examples.

1.50 Define polycrystals. Give a few examples.

1.51 What is noncrystalline solid? Give a few examples.

1.52 Define space lattice.

1.53 What is basis in a crystal lattice ?

1.54 What is meant by crystal structure?

1.55 Define unit cell.

1.56 What are lattice parameters?

1.57 What are primitives?

1.58 How do you define interfacial angles?

1.59 Define primitive cell.

1.60 Name the seven crystal systems.

1.61 Mention the lattice parameters of a cubic system. Give a few examples for cubic crystal systems.

1.62 What are Bravais lattices?

1.63 How can you define a simple cubic structure?

1.64 Define FCC structure.

1.65 What is BCC structure?

1.66 How are atoms arranged in base-centered cubic structures?

1.67 What are lattice planes?

1.68 How will you define Miller indices?

1.69 Draw the crystal planes with Miller indices (100) and (010).

1.70 Draw the crystal planes with Miller indices (001) and (101).

1.71 Draw the crystal planes with Miller indices (110) and (111).

1.72 Draw the crystal planes with Miller indices (112) and (200).

1.73 Define atomic packing factor and coordination number.

1.74 How will you define atomic radius and nearest neighbour distance?

1.75 What is crystal imperfection?

1.76 Mention the four categories of crystal imperfection.

1.77 What is Schottky defect?

1.78 How does Frenkel defect appear in a crystal structure?

1.79 Show edge dislocation and screw dislocation in a crystal structure.

1.80 What is grain boundary?

1.81 Define tilt boundary in crystal structure.

1.82 What is twin boundary?

1.83 How will you define stacking fault?

1.84 Find the Miller indices for a plane which intersects at a, $b/2$, $3c$ on x, y, z axes respectively in a simple cubic crystal.

1.85 Calculate the density of copper, (FCC structure) if its atomic radius = 1.278 Å.

Given atomic weight of copper = 63.54.

1.86 State the relationship between cube edge and interplanar distance.

1.87 Give the stacking sequence of atoms in FCC and HCP structures.

1.88 Mention any two applications of crystal imperfections.

1.89 What are dislocations?

1.90 Determine the spacing between (100) planes, (110) planes and (111) planes in an NaCl crystal having lattice constant $a = 5.64$ Å.

1.91 Show that for a simple cubic system $d_{100} : d_{110} : d_{111}$ is equal to $\sqrt{6} : \sqrt{3} : \sqrt{2}$.

1.92 Draw a unit cell for the given data, $a = 2$ cm, $b = 2$ cm, $c = 3$ cm and $a = \beta = 90°$ and $g = 120°$.

1.93 What is Burgers vector?

1.94 Calculate the nearest neighbouring distance in BCC and FCC crystals.

1.95 The unit cell of aluminium is FCC with lattice constant $a = 0.405$ nm. How many unit cells are there in an aluminium foil of 0.005 cm thick and area 25 cm square.

1.96 Molybdenum has BCC structure and density of $10.2 \times 10^3 \, kg/m^3$. Calculate its atomic radius. The atomic weight of molybdenum is 95.94 and Avogadro's number is $6.023 \times 10^{26} \, mole^{-1}$.

1.97 Zinc has HCP structure. The height of the unit cell is 4.935 Å. Find how many atoms are there in the unit cell and what is the volume of the unit cell?

PART-B

1.1 How will you determine the Miller indices of a given plane? What are the distinct features of Miller indices?

1.2 Calculate the packing density for simple cubic and face centered cubic crystals.

1.3 Determine the coordination number and packing density for a hexagonal closely packed structure. Show that HCP structure demands on axial ratio of 1.633.

1.4 Calculate coordination number and packing factor for BCC crystal structure. Show that in a simple cubic lattice, the separation between the successive lattice planes (100), (110) and (111) are in the ratio of $1 : 0.71 : 0.58$.

1.5 What are crystal imperfections? Classify them in the order of their geometry.

1.6 Explain briefly the following: edge dislocation and screw dislocation.

1.7 Explain the following: Stacking fault, twin boundary and grain boundary.

1.8 Derive the packing factor for BCC, SC and FCC crystal structures. Name some materials having these structures.

1.9 Define atomic radius and determine the same for SC, BCC, FCC and HCP structures. Give two examples for each.

1.10 Calculate the number of atoms per unit cell, coordination number, atomic radius and packing fraction in a simple cubic unit cell.

1.11 Derive the number of atoms per unit cell, coordination number, atomic radius and packing fraction in a BCC unit cell.

1.12 Calculate the number of atoms per unit cell, coordination number, atomic radius and packing fraction in an FCC unit cell.

1.13 Calculate the number of atoms per unit cell, coordination number, atomic radius and packing fraction in an HCP unit cell.

1.14 What are Miller indices? Derive an expression for the interplanar spacing for (hkl) planes of a cubic structure.

1.15 Name the seven crystal systems. Give the relationship.

1.16 What is packing fraction? Prove that the packing fraction of HCP is 0.74.

1.17 Prove that packing fraction of FCC and HCP are the same.

1.18 Explain surface defects in crystalline materials.

1.19 Explain the special features of three types of lattices in a cubic crystal.

1.20 Metallic iron changes from BCC to FCC form at 910°C . At this temperature, the atomic radius of iron atom in the two

structures are 0.1258 and 0.129 nm respectively. Calculate the volume change in percentage during this structural change.

1.21 How many atoms per millimeter square surface area are there in (100) plane, (110) plane and (111) plane for copper which has FCC structure and a lattice constant $a = 3.61 \times 10^{-10}$ m.

1.22 Find the radius of the largest sphere that can fit in the largest interstitial void of a BCC lattice. Also, show that the maximum radius of the sphere that can fit into void at the body center of FCC structure coordinated by facial atom is 0.414r.

1.23 Describe the Bravais lattices.

1.24 Describe Czocharlski and Bridgeman techniques of crystal growth.

UNIT 2

PROPERTIES OF MATTER

2.1 INTRODUCTION

It involves the study of physical, mechanical, electrical and magnetic properties of engineering materials.

2.2 ELASTICITY

Elasticity is the property of a body by virtue of which it tends to regain its original shape and size, on the removal of deforming forces.

Based on its elastic properties, a material body is classified as
- Perfectly elastic body
- Perfectly plastic body

Perfectly Elastic Body   It is a material body which regains its original shape and size, when the deforming forces are removed.

Perfectly Plastic Body   It is a material body which retains its altered shape and size, when the deforming forces are removed.

2.3 BASIC CONCEPTS IN ELASTICITY

2.3.1 Stress

It is the restoring force developed inside the deformed body per unit area which brings back the body to its original state, when the deforming forces are removed.

Its unit is Newton/m^2

Depending on the direction of deforming (or restoring) forces, stress can be classified as
- Normal stress
- Tangential stress or shearing stress

Normal Stress   It is defined as the restoring force per unit area which acts perpendicular to the surface of the body.

Tangential Stress   It is defined as the restoring force per unit area which acts along the surface of the body.

2.3.2 Strain

It is defined as the ratio of the change in dimension to the original dimension of a stressed body.

Based on the direction of the deforming (or restoring) forces, strain can be classified as

- Longitudinal strain or Tensile strain
- Shearing strain
- Volumetric strain

Longitudinal Strain   Let the deforming force act parallel to length of the body (i.e. longitudinally). Then, by definition

$$\text{Longitudinal strain} = \frac{\text{Change in length of the body}}{\text{Original length of the body}}$$

Shearing Strain   Let the body be subjected to a shearing stress (Fig. 2.1). Then by definition,

$$\text{Shearing strain} = \phi \text{ (in radian)}$$
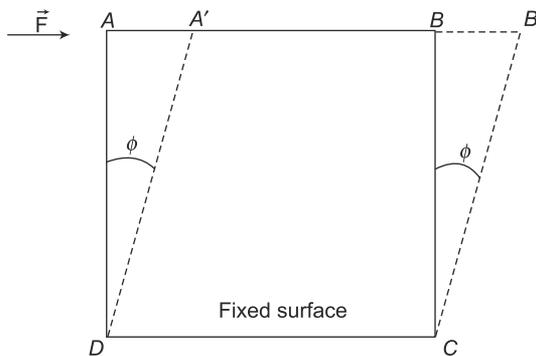
Fig. 2.1 Shearing Strain

In a sheared body, only the shape changes which its size remains unchanged, when deforming forces act on it.

Volumetric Strain   Let the deforming force act perpendicular to the surface of the body.

By definition,

$$\text{Volumetric strain} = \frac{\text{Change in volume of the body}}{\text{Original volume of the body}}$$

2.3.3 Elastic Limit

It is the maximum stress up to which the body can recover its original state (shape and size) after removal of the deforming forces.

2.3.4 Hooke's Law

Within the elastic limit, stress (developed in a body) is directly proportional to strain (experienced by the body).

i.e.   Stress $\propto$ Strain

or $\dfrac{\text{Stress}}{\text{Strain}} = \text{const} = E$

where E is proportionality constant called the **modulus of elasticity**. It depends on the material and on the nature of stress.

That is, modulus of elasticity is defined as the ratio of strain developed in the body to the stress applied.

2.3.5 Yield Point

At and beyond the yield point (which is beyond the elastic limit), strain rapidly increases even for small increases in stress.

2.3.6 Breaking Stress

It is the stress exceeding the elastic limit which results in breaking the body. Breaking stress depends on the material of the body but it does not depend on the dimensions of the body.

Breaking force $\equiv$ Breaking stress $\times$ Area

(i.e. it depends on the dimensions of the body)

2.4   CLASSIFICATION OF ELASTIC MODULUS

Elastic Modulus

Young's Modulus	Rigidity Modulus	Bulk Modulus
It corresponds to longitudinal strain	It corresponds to shearing strain	It corresponds to volumetric strain
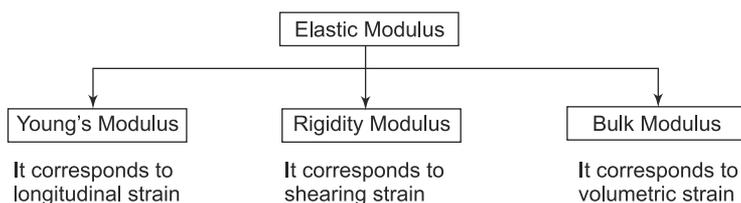
2.4.1 Young's Modulus

It is defined as the ratio between the longitudinal stress to the longitudinal strain, within elastic limits.

$$\text{Young's modulus } (Y) = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}} \text{ N/m}^2$$

Let us consider a wire of length L with an area of cross section A. Let one end of the wire be fixed and the longitudinal force F is applied normal to the cross-sectional area. Let '1' be change in length due to applied force, Fig. (2.2).
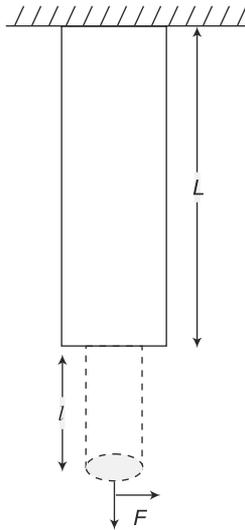
Fig. 2.2 Young's Modulus

Young's Modulus Y is

$$Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$$

$$Y = \frac{F/A}{l/L} = \frac{FL}{Al}$$

i.e.
$$Y = \frac{FL}{Al} \tag{2.1}$$

Note

Beyond the elastic limit,

$$Y = \frac{L}{A} \cdot \frac{dF}{dL}$$

Interpretation of Young's Modulus

Let $L = 1$ m (i.e. unit length), $l = 1$ m and $A = 1$ m^2

Then, Eq. (2.1) gives $Y = F$

Thus, Young's modulus may be defined as the force required to double the length of the wire of unit cross-sectional area.

2.4.2 Rigidity Modulus

It is defined as the ratio between the tangential stress to the shearing strain, within elastic limits.

$$\text{Rigidity modulus } (\eta) = \frac{\text{Tangential Stress}}{\text{Shearing Strain}} \text{ N/m}^2$$

Let us consider a rectangular block fixed at its lower face HDCG with an area of cross section A. A force F is applied tangentially on its upper face *AEKB* and it is deformed to *A1B1CDE1GHK1*, i.e. the line joining the two faces are shifted by an angle 'ϕ' (Fig. 2.3).
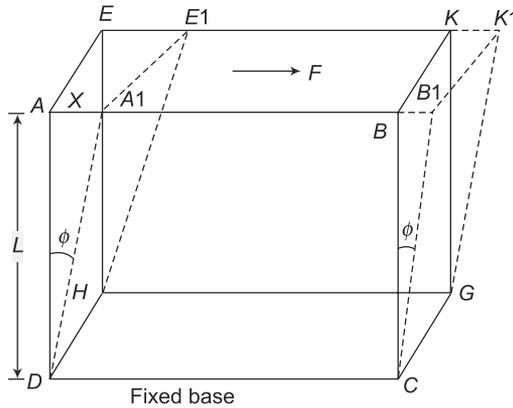
Fig. 2.3 Rigidity Modulus

Rigidity modulus of the material is

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$\eta = \frac{F/A}{\phi} \tag{2.2}$$

As ϕ is small within the elastic limit, we have $\tan \phi = \phi$. Hence, we get

$$\eta = \frac{FL}{Ax} \tag{2.3}$$

Interpretation of Rigidity Modulus

Let $A = 1 \ m^2$ (unit area) and $\phi = 1$ radian. Then, Eq. (2.2) gives

$$\eta = F$$

Thus, rigidity modulus may be defined as the shearing stress per unit shear.

Note

Above the elastic limit,

$$\eta = \frac{1}{A} \cdot \frac{dF}{d\phi}$$

2.4.3 Bulk Modulus

It is defined as the ratio between the volumetric stress or bulk stress to the volumetric strain or bulk strain, within elastic limits.

$$\text{Bulk modulus } (K) = \frac{\text{Volumetric stress}}{\text{Volumetric strain}} \ \text{N/m}^2$$

Let us consider a body of volume v with an area of cross section A. Let three equal forces act on the body in mutually perpendicular direction and the change in volume be v due to action of forces, (Fig. 2.4).
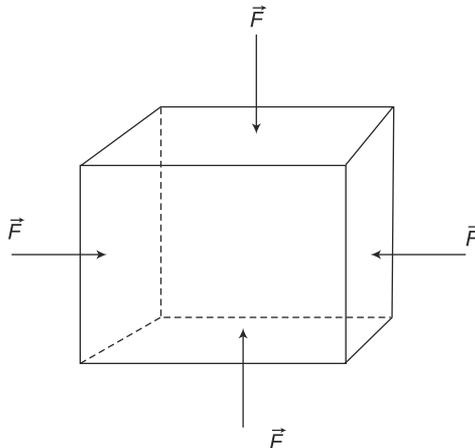
Fig. 2.4 Bulk Modulus

Bulk modulus of the material is

$$K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

$$K = \frac{F/A}{v/V}$$

where A is the cross-sectional area, v is the change in volume and V is the original volume of the body.

$$K = \frac{PV}{v} \qquad (2.4)$$

where $P = F/A$ is the pressure (i.e. force per unit area).

Reciprocal of K, i.e. $1/K$ is called the compressibility.

Note

Above elastic limit,

$$K = -V\frac{dP}{dV}$$

Negative sign indicates that V is inversely proportional to P.

2.5 Poisson's Ratio

It is defined as the ratio between the lateral strain per unit stress to the longitudinal strain per unit stress, within elastic limits.

Poisson's *ratio* is defined as

$$\sigma = \frac{\text{Lateral strain per unit stress }(\beta)}{\text{Longitudinal strain per unit stress }(\alpha)} = \text{const}$$

$\sigma = $ const (for the given material)

Let a tensile stress be applied on the wire in the y-direction. As a result, the wire elongates in the y-direction and shortens in the x-direction (Fig. 2.5).

That is, $\quad L \rightarrow L + l$

$\qquad\quad D \rightarrow D - d \qquad$ (within elastic limit)

$$\sigma = \frac{-(D-d)L}{lD} \qquad (2.5)$$

The negative sign indicates that the longitudinal and lateral strains are taking place in opposite direction, i.e. one strain increases and another decreases.
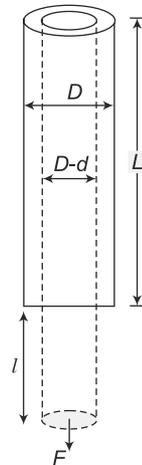
Fig. 2.5 Poisson's Ratio

Poisson's ratio σ is an elastic constant but not an elastic modulus. That is, σ does not correlate the stress and strain.

Note

Above elastic limit, $\sigma = -\dfrac{L}{D}\dfrac{dD}{dL}$

2.5.1 Relation between Elastic Modulii

The modulii of elasticity Y, η, K and the elastic constant σ of a body are interrelated. When two of them are known, others can be determined using these correlations. That is,

$$\alpha = \frac{1}{Y} \qquad (2.6)$$

$$\alpha + \beta = \frac{1}{2\eta} \qquad (2.7)$$

$$\alpha - 2\beta = \frac{1}{3K} \qquad (2.8)$$

$$Y = \frac{9K\eta}{3K + \eta} \qquad (2.9)$$

$$\sigma = \frac{3K - 2\eta}{6K + 2\eta} \qquad (2.10)$$

$$\sigma = \frac{Y}{2\eta} - 1 \qquad (2.11)$$

where α is the longitudinal strain, and β is the lateral strain.

Using Eqs. (2.6), (2.7), (2.8) and (2.11), we may have

$$Y = 3K(1 - 2\sigma) \qquad (2.12)$$
$$Y = 2\eta + (1 + \sigma) \qquad (2.13)$$

2.5.2 Limiting Values of σ

From Eqs. (2.12) and (2.13), we have

$$3K(1 - 2\sigma) = 2\eta(1 + \sigma)$$

As K and η are positive quantities, the factors $(1 - 2\sigma)$ and $(1 + \sigma)$ should have the same sign. That is, if $(1 - 2\sigma)$ is positive then $(1 + \sigma)$ is also positive and vice versa. Such a situation is possible, if

$$-1 < \sigma < \tfrac{1}{2}$$

But, σ cannot have negative values as it would mean simultaneously elongation or shrinkage in lateral and longitudinal directions. Since no material behaves in this way, we have

$$0 < \sigma < \tfrac{1}{2}$$

2.6   STRESS–STRAIN DIAGRAM

Let a wire of uniform cross-sectional area be loaded (Fig. 2.6). The load is increased step by step and the corresponding strain is measured. The graphical representation of the results (called the stress–strain diagram) is given in Fig. 2.7.
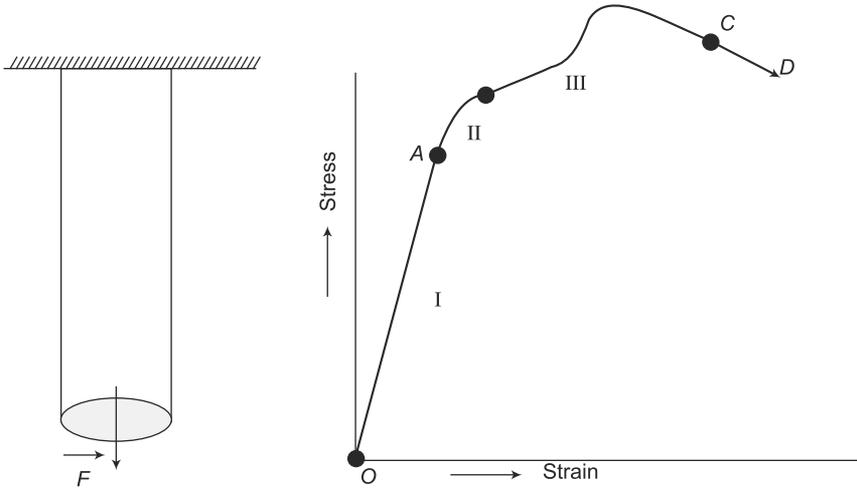
Fig. 2.6   **Fig. 2.7**   Loaded Wire and Stress–Strain Diagram

Segment I   Up to the point A, the material obeys the Hooke's law. That is, strain is proportional to stress. Upon removal of deforming forces, wire regains its original length. The point 'A' refers to the limit of proportionality.

Segment II   It may be divided into lower and upper regions. In the lower region (a small range beyond 'A'), elongation of the wire is greater than that allowed by Hooke's law (or greater than proportionality constant). However, upon the removal of the load, the wire regains its original length.

In the upper region (relatively far off from point A), the wire does not regain its original length upon removal of the load. That is, a

permanent increase in length is produced. The minimum stress at which the wire refuses to regain its original length is called the elastic limit.

Segment III A small increase in the load produces elongation of the wire. That is, the wire behaves like a viscous fluid. It happens at the point B which is called the yield point.

At the point C, the wire thins down, so that its cross section does not remain uniform and it breaks down. The corresponding stress is called the breaking stress.

The breaking stress depends on the nature of wire but it does not depends on its dimensions.

But, Breaking force ($\equiv$ Breaking stress × Area) depends on the dimensions of the wire.

At the point *D*, the wire actually breaks down.

2.7 FACTORS AFFECTING ELASTICITY

1. Stress
2. Annealing
3. Temperature
4. Impurity
5. Nature of crystals

Effect of Stress Repeated cycles of stresses lead to the loss of elasticity of the body even within the elastic limit.

Effect of Annealing Annealing (i.e. heating the material to very high temperature and cooling it slowly) weakens the elastic property.

Effect of Temperature With an increase in temperature, the elastic property is weakened.

Effect of Impurities Elastic property of the material may be strengthened or weakened depending on the impurities added.
Example: K + Au → Elasticity of Au gets strengthened
C + molten Fe → Elasticity of Fe gets weakened

Effect of Nature of Crystals Elastic property depends on the nature of the crystal. For single crystals, elasticity is more. For polycrystals, elasticity is less.

2.8   BENDING OF BEAMS

1. Beam

It is a bar of uniform cross section whose length is much greater than its thickness.

2. Longitudinal Filament

A rectangular beam may be considered to be made of a large number of layers placed one above the other (Fig. 2.8). Further, each layer may be considered to be made of a large number of longitudinal filaments which are lying parallel to the length of the beam.

Filaments in a layer

Layers in a beam
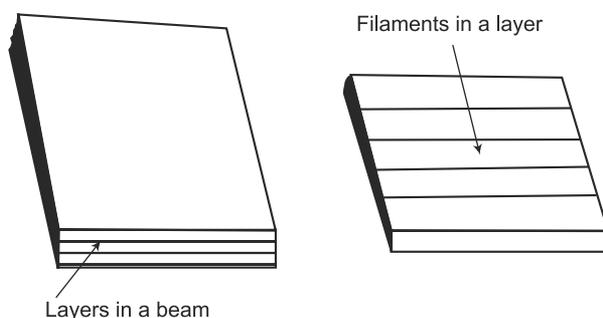
Fig. 2.8 Beams

3. Neutral Surface

Let a beam be subjected to equal and opposite couples at its ends. It leads to the bending of the beam into a circular arc (Fig. 2.9).

Neutral filament
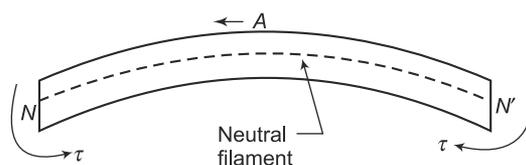
Fig. 2.9 Bending of a Beam

Due to bending, filaments on the convex side are lengthened while those on the concave side are shortened. The filaments of a layer NN' in between the convex and the concave sides are neither shortened

nor lengthened and their lengths remain constant. This layer NN' is called neutral layer or neutral surface.

4. Plane of Bending

It is the plane in which the beam bends. It is the same as the plane of applied couple.
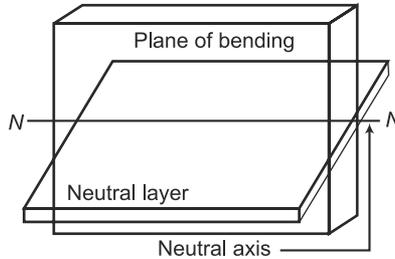
Fig. 2.10 Plane of Bending, Neutral Layer and Neutral Axis

5. Neutral Axis

It is line of intersection of neutral layer with the plane of bending.

6. Bending Moment

Let the beam be bent under the action of equal and opposite couples τ applied at its ends (Fig. 2.11). Let PQ be a reference (chosen) plane. Layer above NN' are subjected to pulling forces resulting in elongation. Layers below NN' are subjected to pushing forces resulting in contraction. As a result, a restoring couple will be produced to compensate the resultant of the pulling and pushing forces. The magnitude of this restoring couple is called the bending moment.
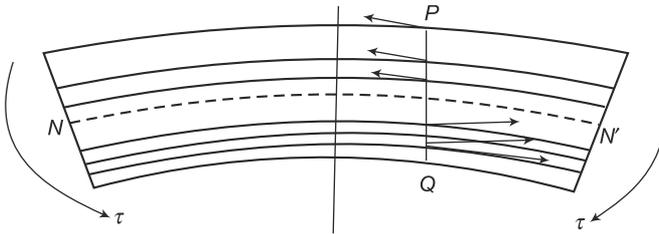
Fig. 2.11 Bending of a Beam

The bending moment is defined as the sum of moments of all the couples arising in a bent beam and trying to resist its deformation caused by the external couple.

2.8.1 Expression for Bending Moment

Consider a small portion of a bent beam (Fig. 2.12). Let R be the radius of curvature of the neutral axis NN'. Let NN' subtend an angle ϕ at its centre of curvature O. Consider a layer (or filament) KL at a distance Z above the neutral axis. Due to bending of the beam, the length of the filament KL increases from arc NN' to arc KL.
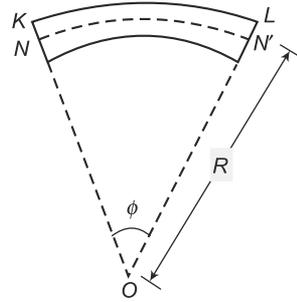
Fig. 2.12 Bending Moment

Change in length = arc KL – arc NN'

From the triangle ΔOKL, we get

$$\text{arc } KL = (R + Z)\phi$$

From the triangle $\Delta ONN'$, we get

$$\text{arc } NN' = R\phi$$

Therefore, increase in length of the filament KL is

$$(R + Z)\,\phi - R\phi = Z\phi$$

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{Z\phi}{R\phi} = \frac{Z}{R}$$

i.e. strain in the filament is directly proportional to its distance from the neutral axis.

Let Y be the Young's modulus of elasticity of the beam. Let 'a' be the cross-sectional area of the filament. Young's modulus is defined as

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

Hence,   Longitudinal stress = $Y \times$ Longitudinal strain

i.e.   Longitudinal stress = $\dfrac{YZ}{R}$

On the other hand, we have

$$\text{Force} = \text{Stress} \times \text{Cross-sectional area} = \frac{YZa}{R}$$

Moment of this force about a line through the neutral filament NN' is

$$M_{NN'} = \frac{YZa}{R} Z = \frac{Ya}{R} Z^2$$

Therefore, internal bending moment (i.e. sum of the moments of all the forces of extension and compression acting over the whole cross section of the beam) is

$$\tau = \Sigma \frac{YaZ^2}{R} = \frac{Y\Sigma aZ^2}{R}$$

i.e.
$$\tau = \frac{Y \cdot I_g}{R} \qquad (2.14)$$

where $I_g = \Sigma aZ^2$ is the geometrical moment of inertia of the cross section about an axis through its centroid and it is perpendicular to the plane of bending.

Let A be the cross-sectional area of the beam. Let k be the radius of gyration of the cross section about the axis passing through the centroid and it is perpendicular to the plane of bending.

Then, we have $\qquad I_g = Ak^2$

Hence, Eq. (2.14) may be rewritten as

$$\tau = \frac{YAk^2}{R} \qquad (2.15)$$

Case 1 (Beam of Rectangular Cross Section) Let b be the breath of the beam. Let d be the thickness of the beam. Then, we have

$$I_g = \frac{bd^3}{12}$$

Case 2 (Beam of Circular Cross Section) Let r be the cross-sectional radius of the beam. Then, we have

$$I_g = \frac{\pi r^4}{4}$$

2.9 DEPRESSION OF A CANTILEVER

Cantilever

It is a beam fixed horizontally at one end and loaded at the other end.

Consider a cantilever of length l loaded with a mass m at its free end (Fig. 2.13). In this case, AB is the neutral axis in the undepressed state and AB' is the neutral axis in the depressed state.
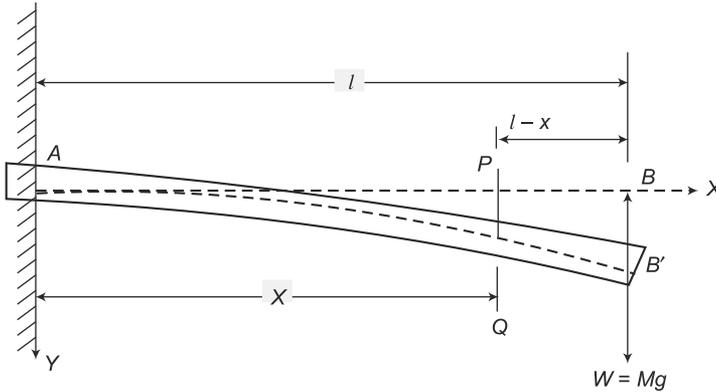
Fig. 2.13 Cantilever Depression

Let us consider (Fig. 2.13) a point on the transverse cross section PQ of the beam with coordinates (x, y). The moment of external couple about this reference point is

$$M_e = W(l - x)$$

where $W \equiv mg$ is the force applied to the free end and g is the acceleration due to gravity.

Internal bending moment of the beam is given as

$$M_i = \frac{YI_g}{R}$$

At the equilibrium state, we have

$$M_i = M_e$$

i.e.

$$\frac{YI_g}{R} = W(l - x)$$

$$\frac{1}{R} = \frac{W(l - x)}{YI_g} \tag{2.16}$$

If y is the depression of the beam at the section PQ then the radius of curvature R at this section is given by

$$\frac{1}{R} = \frac{d^2y}{dx^2} \tag{2.17}$$

Substituting Eq. (2.17) into Eq. (2.16), we get

$$\frac{d^2y}{dx^2} = \frac{W(l-x)}{YI_g}$$

Integrating this equation, we have

$$\frac{dy}{dx} = \frac{W}{YI_g}\left(lx - \frac{x^2}{2}\right) + C_1 \tag{2.18}$$

where C_1 is the constant of integration.

Determination of C_1   For determining the constant C_1, the boundary conditions are used. That is, at the fixed end, we have

$$y = 0 \text{ and } \frac{dy}{dx} = 0 \text{ at } x = 0$$

Substituting these conditions into Eq. (2.18), we get $C_1 = 0$.

i.e.
$$\frac{dy}{dx} = \frac{W}{YI_g}\left(lx - \frac{x^2}{2}\right)$$

Integrating this equation, we get

$$y = \frac{W}{YI_g}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$$

where C_2 is the integration constant.

Substituting the boundary conditions (at the fixed end) into the above equation, we get $C_2 = 0$.

Thus, we have

$$y = \frac{W}{YI_g}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) = \frac{Wx^2}{2YI_g}\left(l - \frac{x}{3}\right) \tag{2.19}$$

Equation (2.19) gives the depression of a cantilever at a distance of x from the fixed end.

Depression at the Free End   At the free end, we have $x = l$ and $y = y$

Therefore, Eq. (2.19) gives the depression of cantilever at the free end as

$$y = \frac{Wl^3}{3YI_g} \tag{2.20}$$

Case 1 (Beam of Rectangular Cross Section)   Let b and d be the breadth and thickness of the beam, respectively. Then, the geometrical moment of inertia is

$$I_g = \frac{bd^3}{12}$$

Then Eq. (2.20) gives

$$y = \frac{4Wl^3}{Ybd^3}$$

Case 2 (Beam of Circular Cross Section)   Let r be the cross-sectional radius of the beam. Then, the geometrical moment of inertia is

$$I_g = \frac{\pi r^4}{4}$$

Then, Eq. (2.20) gives,

$$y = \frac{4Wl^3}{3Y\pi r^4} \tag{2.21}$$

Note

In the derivation of Eq. (2.20), mass of the beam is considered to be negligible.

2.9.1   Experimental Determination of Young's Modulus by Depression of Cantilever

Construction
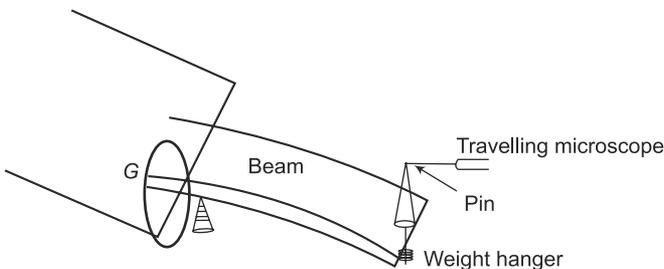
Fig. 2.14   Young's Modulus Determination

A rectangular beam is rigidly clamped on the table by a *G*-clamp (Fig. 2.14). A weight hanger of mass w is attached to the free end of the beam. A pin is fixed as the free end by means of wax. A travelling microscope measures the position of the pin.

Procedure

With no mass added to the weight hanger, the microscope is adjusted to have its horizontal cross wire coinciding with the tip of the pin. The corresponding vertical scale reading is noted. Load at the free end is increased in steps by adding slotted weights to the weight hanger and every time, vertical scale reading of the microscope is noted after achieving the coincidence of pin-tip with the horizontal cross wire of the microscope. The free end is now unloaded in steps and the corresponding vertical scale readings of the microscope are noted. The observations are tabulated in Table 2.1.

Table 2.1   Observations

Sl. No.	Load (M) kg	Microscope Reading (m)			Depres-sion y for M kg (m)	M/y kg m^{-1}
		Loading Mode	Unloading Mode	Mean		
					Mean	

Mean depression y of the beam for a given mass M is determined.

Obviously, Young's modulus of a rectangular cross-sectional beam is

$$Y = \frac{4Wl^3}{ybd^3} = \frac{4Mgl^3}{ybd^3} = \frac{4gl^3}{bd^3}\left(\frac{M}{y}\right) \tag{2.22}$$

Substitution of mean value of M/y into the above equation along with the measured values of l, b, d gives the Young's modulus of the beam.

2.10 UNIFORM BENDING OF A BEAM

It is defined as the mode of bending in which every element of the beam is bent with the same radius of curvature. In uniform bending, the beam is elevated due to applied load.
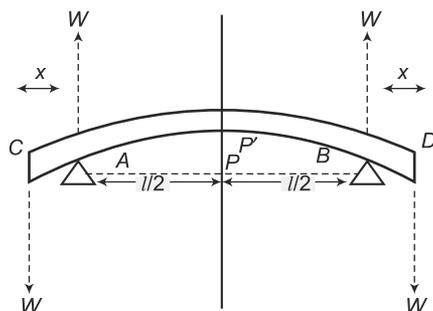
Fig. 2.15 Uniform Bending of a Beam

Beam *CD* of negligible mass is symmetrically supported by two knife edges *A* and *B* (Fig. 2.15). Let the distance between knife edges *A* and *B* be *l*. Let equal weights *W* be applied to the ends *C* and *D* of the beam. Moreover, distance $CA = BD = x$. A reaction *W* is produced at the points *A* and *B* acting vertically upwards to compensate the applied external forces. It makes the beam to bend, to form an arc of a circle of radius *R*. Let the reference point *P* of the beam be displaced to *P′*. The force *W* applied at the point *C* and the corresponding reaction *W* developed at the point *A* of the beam constitute a couple.

The moment of this couple about the point *P* is

M_e = contribution of applied at *C* force-contribution of reaction developed at *A*

i.e.
$$M_e = W(l + x) - Wl = Wx \qquad (2.23)$$

Internal bending moment is

$$M_i = \frac{YI_g}{R} \qquad (2.24)$$

At equilibrium, we have

$$M_e = M_i$$

i.e.
$$Wx = \frac{YI_g}{R} \qquad (2.25)$$

Equation (2.25) shows that radius of curvature R is a constant for the given load W. Hence, the bending is called uniform bending.
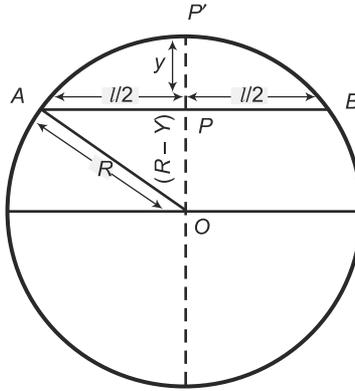
Fig. 2.16 Constant Radius of Curvature

Consider the arc of a circle formed by the bent beam (Fig. 2.16).
Consider the triangle ΔOAP, we have

$$OA^2 = AP^2 + PO^2$$

i.e.
$$R^2 = \left(\frac{l}{2}\right)^2 + (R-y)^2$$

$$2Ry - y^2 = \frac{l^2}{4} \tag{2.26}$$

As the elevation y is small compared to the radius of curvature R (i.e. $y^2 \ll 2Ry$), Eq. (2.26) simplified as

$$2Ry = \frac{l^2}{4}$$

i.e.
$$R = \frac{l^2}{8y} \tag{2.27}$$

Substituting Eq. (2.27) into Eq. (2.25), we get

$$Wx = \frac{8YI_g y}{l^2}$$

Elevation y at the middle of the beam, i.e. at the point P' is

$$y = \frac{Wxl^2}{8YI_g} \tag{2.28}$$

Therefore Eq. (2.28) may be rewritten for the Young's modulus as

$$Y = \frac{Wxl^2}{8yI_g} \tag{2.29}$$

Equation (2.29) may be used for determining the Young's modulus of the beam material.

2.10.1 Experimental Determination of Young's Modulus of the Material by Uniform Bending

Construction
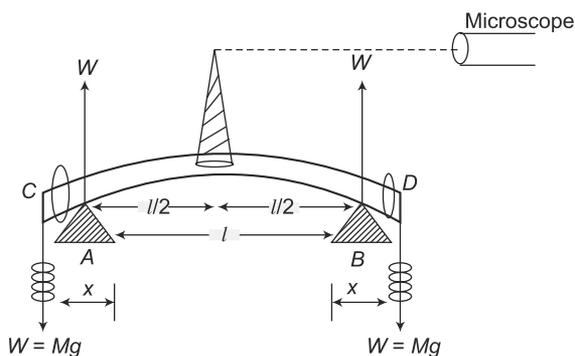
Fig. 2.17 Young's Modulus Determination by Uniform Bending

The given beam is symmetrically supported by two knife edges *A* and *B* (Fig. 2.17). A pin is vertically placed (by means of wax) at the centre of the beam. Two weight hangers of equal weight *w* are suspended at the ends of the beam by means of loops of strings.

Procedure

Initially, the hangers are several times loaded and unloaded with equal weights to bring the beam to the elastic mood. With no added slotted weight in the weight hanger, the travelling microscope is focused to have the pin tip coincide with the horizontal cross wire of the telescope. The corresponding vertical scale reading of the microscope is note. Now, loads are added in steps to the weight hanger and every time, vertical scale readings of the microscope are noted after achieving the coincidence of the pin-tip with the horizontal cross wire of the microscope. The experiment is repeated with the removal of loads in the same steps from the weight hanger. The readings are

tabulated (Table 2.2). Elevation of the beam for a particular added mass is calculated.

Table 2.2 Observations

Sl. No.	Load (M) kg	Reading of Microscope (m)			Depression y for M kg (m)
		Loading Mode	*Unloading Mode*	*Mean*	
				Mean	

The breadth *b* and the thickness *d* of the beam are measured distance *x* between the hangers and their neighbouring knife edges is measured. The distance *l* between the knife edges is measured. The mean value of elevation *y* of the beam is calculated. Then, the Young's Modulus of the material of the beam is determined by

$$Y = \frac{3Mgxl^2}{2ybd^3} \tag{2.30}$$

2.11 NON-UNIFORM BENDING OF A BEAM

It is defined as the mode of bending in which radius of curvature is not the same for all the elements in the beam. In non-uniform bending, the beam is depressed due to applied load.
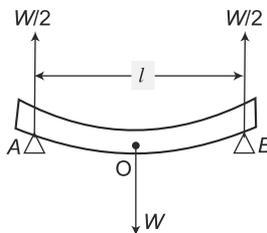
Fig. 2.18 Non-uniform Bending

Beam *CD* of negligible mass is symmetrically supported by two knife edges *A* and *B* (Fig. 2.18). Let *l* be the distance between the knife edges. The beam is loaded at the middle point *O* with a weight *W*. As a result, a reaction *W/2* is produced at the points *A* and *B* acting vertically upwards to compensate the applied external force. The beam bends down with the maximum depression at the loaded point *O*. The bent beam may be considered to be made of two inverted cantilevers, each of length *l/2* fixed at *O* and carrying a load *W/2* at *A* and *B*. That is, depression of the loaded beam at *O* is the same as produced in cantilever of length *l/2* loaded by a weight *W/2* at the free end. Thus, we may now consider the following case:
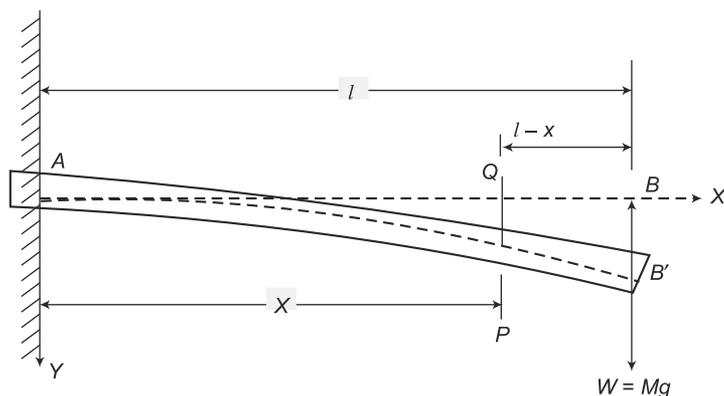
Fig. 2.19 Bending of Beam

External bending moment about *P* is

$$M_e = \frac{W}{2}\left(\frac{l}{2} - x\right) \qquad (2.31)$$

Internal bending moment is

$$M_i = \frac{YI_g}{R} \qquad (2.32)$$

Let the coordinates of the point *P* be (x, y) with the point *O* as the origin of the coordinate system. Then, the radius of curvature *R* at the point *P* is

$$\frac{1}{R} = \frac{d^2y}{dx^2} \qquad (2.33)$$

At equilibrium, we have

$$M_i = M_e$$

From Eqs. (2.31) and (2.32), we get

$$YI_g \frac{d^2y}{dx_2} = \frac{W}{2}\left(\frac{l}{2} - x\right)$$

Integration of this equation gives

$$\frac{dy}{dx} = \frac{W}{2YI_g}\left(\frac{lx}{2} - \frac{x^2}{2}\right) + C_1 \qquad (2.34)$$

where C_1 is the integration constant.

Determination of C_1 For determining the constant C_1, the boundary conditions are used. That is, at the fixed end, we have

$$y = 0 \text{ and } \frac{dy}{dx} = 0 \text{ at } x = 0$$

Substituting these conditions into Eq. (2.34), we get $C_1 = 0$.

i.e. $$\frac{dy}{dx} = \frac{W}{2YI_g}\left(\frac{lx}{2} - \frac{x^2}{2}\right)$$

Integrating this equation, we get

$$y = \frac{W}{2YI_g}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$$

where C_1 is the integration constant.

Determination of C_2 Substituting the boundary conditions (at the end O) into the above equation we get $C_2 = 0$

Thus, we have

i.e. $$y = \frac{W}{2YI_g}\left(\frac{lx^2}{4} - \frac{x^3}{6}\right) \qquad (2.35)$$

Equation (2.35) gives the depression of the cantilever of length $l/2$ at a distance x from the fixed end.

Depression at the Free End At the end (i.e. at the point A), we have

$$x = l/2 \text{ and } y = y$$

Therefore, Eq. (2.35) gives the depression at the free end as

$$y = \frac{Wl^3}{48YI_g} \qquad (2.36)$$

Equation (2.36) gives the depression of the non-uniformly bent beam at its middle point.

Case 1 (Beam of Rectangular Cross Section) Let b and d be the breadth and thickness of the beam respectively.

Then, the geometrical moment of inertiar is $I_g = \dfrac{bd^3}{12}$

Thus, Eq. (2.36) gives

$$y = \frac{Wl^3}{4Ybd^3} \tag{2.37}$$

Case 2 (Beam of Circular Cross Section) Let r be the cross-sectional radius of the beam. Then, the geometrical moment of inertia is $I_g = \pi r^4/4$.

Therefore, Eq. (2.36) may be rewritten as

$$y = \frac{Wl^3}{12Y\pi r^4} \tag{2.38}$$

Note

In the derivation of Eq. (2.38), the mass of the beam is considered to be negligible.

2.11.1 Experimental Determination of Young's Modulus by Non-Uniform Bending of Beam

Construction
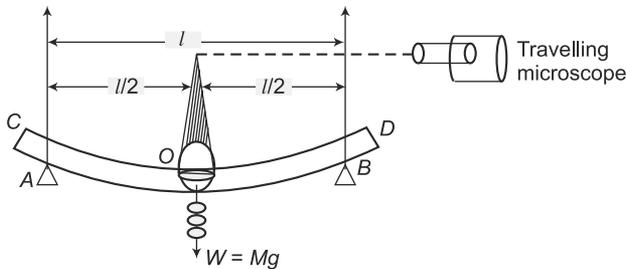
Fig. 2.20 Non-uniform Bending of Beam

The given rectangular beam is supported by two knife edges A and B with equal lengths of the beam projecting beyond the knife edges (Fig. 2.20). A pin is vertically placed at the middle of the beam by means of wax. A weight hanger is attached to the mid-point of the beam through a metallic loop.

Procedure

The travelling microscope is adjusted to have its horizontal cross wire coincided with the pin-tip. The vertical scale reading of microscope is noted. Loads are added to the hanger in steps and every time, vertical scale reading of microscope is noted. The experiment is repeated while unloading the hanger is same steps. The readings are tabulated (Table 2.3).

Table 2.3   Observations

Sl. No.	Load (M) kg	Microscope Reading (m)			Depression y for M kg (m)
		Loading Mode	Unloading Mode	Mean	
				Mean	

The distance l between the knife edges, breadth b and thickness d of the beam are measured. Then, Young's modulus of the material of the beam is determined by

$$Y = \frac{Mgl^3}{4ybd^3} \tag{2.39}$$

2.12   SHAPED GIRDERS
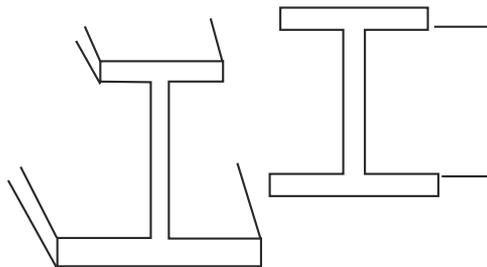
Fig. 2.21   I-shaped Girders

A girder is defined as a beam used to withstand heavy loads with minimum depression produced in it. Girders with upper and lower sections broadened and the middle section tapered are called the I-shaped girders (as they resemble the English alphabet I). In general, a girder supported at its ends say, by the opposite walls of a room) bends under its own weight and also under the internal loads. It is desirable to minimize the depression of the beam. Depression at the middle point of the rectangular beam given by

$$y = \frac{Wl^3}{4Ybd^3} \tag{2.40}$$

Analysis of the above equation gives the conditions for minimizing the depression as
1. Decrease in the load applied
2. Decrease in the length of the girder
3. Use of material (e.g. steel) of greater Young's modulus
4. Increase in the breadth of the beam
5. Increase in the thickness of the beam

In a bent beam, as it is known, the filaments (or layers) above the neutral surface are elongated and those below the neutral surface are shortened. The neutral axis does not experience any change in length. That is, topmost and lowermost surfaces of the beam are subjected to maximum stresses while the stresses diminish to zero at the neutral surfaces. In order to withstand such a pattern of stress distribution inside the beam, maximum amount of material is located at the topmost and lowermost surfaces while a minimum amount of material is located in the middle portion of the beam. A girder of rectangular cross section and a similar I-shaped girder will have the same strength. There is economy of material in the I-shaped girder without any loss of efficiency. In this type of girder, the excess material not bearing any stress is removed.

SUMMARY

▸ Elasticity is the property of the body by virtue of which it tends to regain its original shape and size, on the removal of deforming forces.

▸ A perfectly elastic body is a material body which regains its original shape and size, when the deforming forces are removed.

▶▶ A perfectly plastic body is a material body which retains its altered shape and size, when the deforming forces are removed.

▶▶ Stress is defined as the restoring force developed inside the deformed body per unit area which brings back the body to its original state, when the deforming forces are removed.

▶▶ Depending on the direction of deforming forces, stress can be classified as normal stress and tangential or shearing stress.

▶▶ Strain is defined as the ratio of the change of the change in dimension to the original dimension of a stressed body.

▶▶ Depending on the direction of deforming or restoring forces, strain can be classified as longitudinal or tensile strain, shearing strain and volumetric strain.

▶▶ Elastic limit is the maximum stress up to which the body can recover its original state after removal of deforming forces.

▶▶ Hooke's law states that within elastic limit, stress developed in a body is directly proportional to strain experienced by the body.

▶▶ Modulus of elasticity is defined as the ratio of strain developed in the body to the stress applied.

▶▶ Breaking stress is the stress exceeding the elastic limit which results in breaking the body.

▶▶ Elastic modulus can be classified as Young's modulus, rigidity modulus and bulk modulus.

▶▶ Young's modulus is defined as the ratio between the longitudinal stress to the longitudinal strain, within elastic limits.

▶▶ Bulk modulus is defined as the ratio between the volumetric stress or bulk stress to the volumetric strain or bulk strain, within elastic limits.

▶▶ Rigidity modulus is defined as the ratio between the tangential stress to the shearing strain, within elastic limits.

▶▶ Poisson's ratio is defined as the ratio between the lateral strain per unit stress to the longitudinal strain per unit stress, within elastic limits.

▶▶ Stress, annealing, temperature, impurity and nature of crystals are the factors that affect elasticity.

▶▶ Beam is bar of uniform cross section whose length is much greater than its thickness.

▶▶ Neutral axis is a line of intersection of neutral layer with the plane of bending.

▶▶ Bending moment is defined as the sum of moments of all the couples arising in a bent beam and trying to resist its deformation caused by the external couple.

▶▶ Cantilever is a beam fixed horizontally at one end and loaded at the other end.

▶▶ Uniform bending of a beam is defined as the mode of bending in which every element of the beam is bent with the same radius of curvature.

▶▶ In uniform bending, the beam is elevated due to applied load.

▶▶ Non-uniform bending of a beam is defined as the mode of bending in which radius of curvature is not the same for all the elements in the beam.

▶▶ In non-uniform bending, the beam is depressed due to applied load.

▶▶ Girders with upper and lower sections broadened and the middle section tapered are called I-shaped girders.

▶▶ As the girders resemble the English alphabet I, they are named I-shaped girders.

✓ SOLVED PROBLEMS

2.1 A copper wire of 3 m length and of 0.6 mm diameter is suspended from a rigid support. An elongation of 4.3 mm is observed when the wire is loaded with a mass of 5 kg. Determine the Young's modulus of copper.

Solution   Given Data

$l = 3$ m

$d = 0.6$ mm $= 0.6 \times 10^{-3}$ m

$m = 5$ kg

$\Delta l = 4.3$ mm $= 4.3 \times 10^{-3}$ m

$Y = ?$

Young's modulus is defined as

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

Therefore, we have

$$Y = \frac{\text{Force/Cross-sectional area}}{\text{Change in length/Original length}}$$

i.e. $\qquad Y = \dfrac{Mg \,/\, \pi r^2}{\Delta l \,/\, l}$ $\hfill$ (1)

where $g = 9.81$ m/s^2 is the acceleration due to gravity and r is the radius of the wire. Substituting the given data into Eq. (1), we get

$$Y = \frac{5 \times 9.81 \times 3}{\pi \times (0.3 \times 10^{-3}) \times 4.3 \times 10^{-3}}$$

$$Y = \mathbf{121 \times 10^9 \ N/m^2}$$

2.2 A steel wire of 2 m length and of 0.5 mm diameter is stretched by 5 cm. Calculate the work done in this process if the Young's modulus of steel is 2×10^{11} N/m^2.

Solution Given Data

$$l = 2 \ \text{m}$$

$$d = 0.5 \ \text{mm} = 0.5 \times 10^{-3} \ \text{m}$$

$$\Delta t = 5 \ \text{cm} = 5 \times 10^{-2} \ \text{m}$$

$$\underline{Y = 2 \times 10^{11} \ \text{N/m}^2}$$

$$W = ?$$

Work done in stretching a wire of unit volume is given as

$$W' = \frac{\text{Stress} \times \text{Strain}}{2}$$

On the other hand,

$$\text{Stress} = \text{Young's Modulus} \times \text{Strain}$$

Therefore, we have

$$W' = \frac{\text{Young's modulus} \times (\text{strain})^2}{2}$$

Then, work done in stretching the given wire is

$$W = W' \times \text{volume of the wire}$$

i.e. $\qquad W = \dfrac{Y \times (\Delta l \,/\, l)^2 \times \pi r^2 l}{2}$

$$W = \frac{Y(\Delta l)^2 \pi r^2}{2l}$$

$$W = \frac{2\times10''\times(5\times10^{-2})\times3.14\times(0.25\times10^{-3})^2}{2\times2}$$

W = 24.5 joules

2.3 Calculate the Young's modulus in the cantilever depression method used. The length of the cantilever beam is 2 m which is suspended with a load of 150 *g*. The depression is found to be 3 m. The thickness of the beam is 5 mm and breadth of the beam is 3 cm.

Solution   Given Data

$l = 150 \times 10^{-3}$ kg
$M = 150 \times 10^{-3}$ kg
$b = 3 \times 15^2$ m
$d = 5 \times 10^{-3}$ m
$y = 3 \times 10^{-2}$ m

$$Y = \frac{4gl^3}{bd^3}\left(\frac{M}{y}\right)$$

$$= \frac{4\times9.8\times(150\times10^{-3})^3}{3\times10^{-2}\times(5\times10^{-3})^3}\left(\frac{150\times10^{-3}}{3\times10^{-2}}\right)$$

$$= \mathbf{0.0176 \times 10^{10}\ N/m^2}$$

2.4   A beam of circular cross section has a radius of 1 cm and a length of 1 m. The beam is supported by two knife edges placed symmetrically with respect to the middle of the beam. The distance between the knife edge is 0.8 m. When the middle of the beam is loaded a by mass of 75 g, a depression of 1.5×10^{-3} is observed in the middle of the beam. Determine the Young's modulus of the beam material.

Solution   Given Data

$l = 0.8$ m
$y = 1.5\times10^{-3}$ m
$l = 0.8$ m
$r = 1\ cm = 1\times10^{-2}$ m
$M = 75g = 75\times10^{-3}$ kg
$Y = ?$

For non-uniform bending, the Young's modulus of the beam material is given as

$$Y = \frac{Mgl^3}{48 y I_g} \qquad (1)$$

where I_g is the geometrical moment of inertia of the beam and g is the acceleration due to gravity.

For a beam of circular cross section, we have

$$I_g = \frac{\pi r^4}{4} \qquad (2)$$

From Eqs. (1) and (2), we have

$$Y = \frac{Mgl^3}{12 y \pi r^4} \qquad (3)$$

Substituting the given data into Eq. (3), we get

$$Y = \frac{75 \times 10^{-3} \times 9.81 \times (0.8)^3}{12 \times 1.5 \times 10^{-3} \times 3.14 \times (1 \times 10^{-2})^4}$$

$$Y = 6.67 \times 10^8 \text{ N/m}^2$$

2.5 A rectangular beam of 1 m length, 2 cm breadth and of 0.25 cm thickness is supported by two knife edges placed symmetrically with respect to the middle of the bar. The distance between the knife edges is 0.8 m. The ends of the bar are loaded with equal masses of 50 g each. The depression of the beam at its middle point is 1.5×10^{-3} m. Determine the Young's modulus of the beam material.

Solution Given Data

$$l = 0.8 \text{ m}$$
$$y = 1.5 \times 10^{-3} \text{ m}$$
$$x = 0.1 \text{ m}$$
$$M = 50 \text{ g} = 50 \times 10^{-3} \text{ kg}$$
$$b = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$
$$\underline{d = 0.25 \text{ cm} = 0.25 \times 10^{-2} \text{ m}}$$
$$Y = ?$$

For uniform bending, the Young's modulus of the beam material is given as

$$Y = \frac{Mgxl^2}{8yI_g} \tag{1}$$

where I_g is the geometrical moment of inertia of the beam and g is the acceleration due to gravity.

For a rectangular beam, we have

$$I_g = \frac{bd^3}{12} \tag{2}$$

where b is the breadth of the beam and d is the thickness of the beam.

From Eqs. (1) and (2), we have

$$Y = \frac{3Mgxl^2}{2ybd^3} \tag{3}$$

Substituting the given data into Eq. (3), we get

$$Y = \frac{3 \times 50 \times 10^{-3} \times 9.81 \times 0.1 \times (0.8)^2}{2 \times 1.5 \times 10^{-3} \times 2 \times 10^{-2} \times (0.25 \times 10^{-2})^3}$$

$$Y = 10.05 \times 10^{10} \text{ N/m}^2$$

QUESTIONS AND ANSWERS

2.1 What is meant by elasticity of a body?
Elasticity is the property of the body by virtue of which it tends to regain its original shape and size, on the removal of deforming forces.

2.2 What is meant by a perfectly elastic body?
A perfectly elastic body is a material body which regains its original shape and size, when the deforming forces are removed.

2.3 What is meant by a perfectly plastic body?
A perfectly plastic body is a material body which retains its altered shape and size, when the deforming forces are removed.

2.4 Define stress and give its unit of measurement.
Stress is defined as the restoring force developed inside the deformed body per unit area which brings back the body to its original state, when the deforming forces are removed. Its unit is N/m^2.

2.5 Define strain.

Strain is defined as the ratio of the change in dimension to the original dimension of a stressed body.

2.6 State Hooke's law.

Hooke's law states that within elastic limit, stress developed in a body is directly proportional to strain experienced by the body.

Stress ∝ strain

Stress/strain = Constant = E = Modulus of elasticity.

2.7 Define the modulus of elasticity.

Modulus of elasticity is defined as the ratio of strain developed in the body to the stress applied.

Stress/Strain = Modulus of elasticity.

2.8 Define Young's modulus.

It is defined as the ratio between the longitudinal stress to the longitudinal strain, within elastic limits.

$$\text{Young's modulus } (Y) = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}} \; \text{N/m}^2$$

2.9 Define rigidity modulus.

It is defined as the ratio between the tangential stress to the shearing strain, within elastic limits.

$$\text{Rigidity modulus } (\eta) = \frac{\text{Tangential Stress}}{\text{Shearing Strain}} \; \text{N/m}^2$$

2.10 Define bulk modulus.

It is defined as the ratio between the volumetric stress or bulk stress to the volumetric strain or bulk strain, within elastic limits.

$$\text{Bulk modulus } (K) = \frac{\text{Volumetric stress}}{\text{Volumetric strain}} \; \text{N/m}^2$$

2.11 What is meant by Poisson's ratio?

Poisson's ratio is defined as the ratio between the lateral strain per unit stress to the longitudinal strain per unit stress, within elastic limits.

$$\text{Poisson's ratio} = \frac{\text{Lageral strain per unit stress}}{\text{Longitudinal strain per unit stress}} = \text{constant}$$

2.12 Define the bending moment of a beam.

Bending moment is defined as the sum of moments of all the couples arising in a bent bean and trying to resist its deformation caused by the external couple.

2.13 What is meant by a cantilever?

Cantilever is a beam fixed horizontally at one end and loaded at the other end.

2.14 What is meant by uniform bending of a beam?

Uniform bending of a beam is defined as the mode of bending in which every element of the beam is bent with the same radius of curvature. In uniform bending, the beam is elevated due to applied load.

2.15 What is meant by non-uniform bending of a beam?

Non-uniform bending of a beam is defined as the mode of bending in which radius of curvature is not the same for all the elements in the beam. In non-uniform bending, the beam is depressed due to applied load.

2.16 What is an I-shaped girder?

Girders with upper and lower sections broadened and the middle section tapered are called I-shaped girders. As the girders resemble the English alphabet I, they are named I-shaped girders.

EXERCISE PROBLEMS

2.1 A load of 3 kg produces an elongation of 3 mm in a wire of 3 m length and of 1 mm diameter. Determine the Young's modulus of the wire material.

$$[Y = 37.45 \times 10^9 \ N/m^2]$$

2.2 A wire of 2 m length and of 1 mm diameter is subjected to a load of 5 kg. The Young's modulus of the wire material is 130 GPa. Determine the elongation produced in the wire.

$$[\Delta l = 0.96 \times 10^{-3} \ m]$$

2.3 Determine the force required to double the length of a wire of 1 sq. cm cross-sectional. The Young's modulus of the wire material is 75 GPa. $[F = 75 \times 10^5 \ N]$

2.4 A cantilever of 0.8 m length has a depression of 10 mm at its free end. Determine the depression at a distance of 0.5 m from the fixed end. $[y = 4.64 \times 10^{-3} \ m]$

2.5 A rod of 1.26 cm diameter is non-uniformly bent by applying a load of 1 kg at its middle point. The depression at the middle point is 0.03 cm and the distance between the knife edges is 70 cm. Determine the Young's modulus of the rod material. $[Y = 18.88 \times 10^{10} \ N/m^2]$

✓ QUESTION BANK

PART-A

2.1 What is meant by elasticity of a body?
2.2 What is meant by a perfectly elastic body?
2.3 What is meant by a perfectly plastic body?
2.4 Define stress and give its unit of measurement.
2.5 Define normal stress.
2.6 Define tangential stress.
2.7 Define strain.
2.8 Define longitudinal strain.
2.9 Define shearing strain.
2.10 Define volumetric strain.
2.11 What is meant by elastic limit?
2.12 What is meant by yield point?
2.13 What is meant by breaking stress?
2.14 State Hooke's law.
2.15 Define the modulus of elasticity.
2.16 Mention the factors affecting the elasticity of a body.
2.17 Define Young's modulus.
2.18 Define rigidity modulus.
2.19 Define bulk modulus.
2.20 What is meant by Poisson's ratio?
2.21 Give the relation between Young's modulus, rigidity modulus and bulk modulus of a body.
2.22 Define beam.
2.23 Define bending moment of a beam.
2.24 What is meant by the longitudinal filament of a beam?

2.25 What is meant by the neutral axis of a beam?
2.26 What is meant by the plane of bending of a beam?
2.27 What is meant by the neutral surface of a beam?
2.28 What is meant by a cantilever?
2.29 What is meant by uniform bending of a beam?
2.30 What is meant by non-uniform bending of a beam?
2.31 What is meant by a girder?
2.32 What is an I-shaped girder?
2.33 State the advantage of an I-shaped girder.

(PART-B)

2.1 Explain stress-strain diagram. Discuss the factors affecting the elasticity of a material.
2.2 What is meant by bending of beams? Derive the expression for bending moment for rectangular and circular cross sections.
2.3 What is meant by a cantilever? Derive the expression for the depression produced due to a load hanging *t* the end of cantilever beam. Describe an experiment to determine the Young's modulus of the cantilever material using this expression.
2.4 Describe with necessary theory, the method to determine the Young's modulus of the material by uniform bending.
2.5 Explain with necessary theory, the determination of Young's modulus of elasticity of the material of a beam, supported at its ends and loaded in the material.
2.6 Describe an experiment to determine the Young's modulus of a beam by uniform bending.
2.7 Describe an experiment to determine the Young's modulus of a beam by non-uniform bending.

UNIT 4
QUANTUM PHYSICS AND APPLICATIONS

4.1   QUANTUM PHYSICS

In the 1920s, Schrodinger, Heisenberg et al., developed quantum physics to overcome the limitations of the Bohr theory of atom. The fundamental difference between Newtonian physics and quantum physics lies in their subject matter of study. In the study of motion of a particle, Newtonian physics assumes that a particle's position, mass, velocity and acceleration have definite and measurable values at any instant. On the other hand, quantum physics assigns probabilistic values to these quantities. In fact, Newtonian physics is an approximation of quantum physics. Newtonian physics is applicable to the macroscopic universe while quantum physics is applicable to the microscopic universe. In this chapter, we will consider some prominent features of quantum physics.

4.2   BLACK-BODY RADIATION

A body, depending upon its temperature, may emit radiation of particular characteristics. For example, the hot filament of an incandescent lamp emits white light. Similarly, a less hot material glows red. The results of such observations of a hot body are shown in Fig. 4.1.

As seen, the central wavelength of radiation shifts to infrared as the temperature of the body is decreased. Even at room temperature, the body emits radiation, but it is in the far infrared region.

Consider a closed blackened cavity with a narrow hole at a point and a projection just opposite to the hole (Fig. 4.2). Radiation entering into the cavity will be trapped inside the cavity. That is, the cavity neither reflects nor transmits radiations. Hence, it appears black and it is called a black body. The radiations trapped in the cavity and in
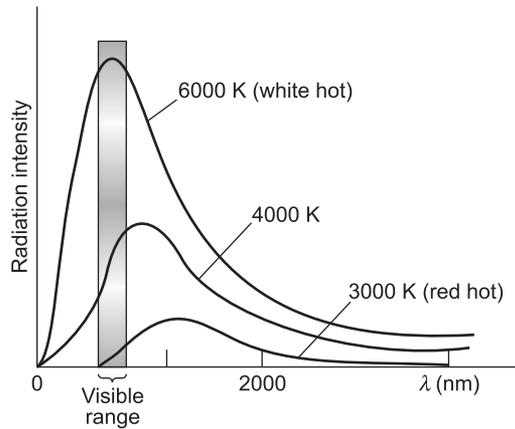
Fig. 4.1 Observations of a Hot Body

thermal equilibrium with the walls of the cavity are called black-body radiations. **A perfectly black body is defined as the body which can absorb radiations of all wavelengths.** At high temperatures, the black body emits radiations of all wavelengths. According to Kirchoff's law, a substance absorbing any radiation from a black body will emit the same radiation. Therefore, the total radiations from the point O

Fig. 4.2 Black Body

(Fig. 4.2) must be identical to the radiations from the walls of the black body.
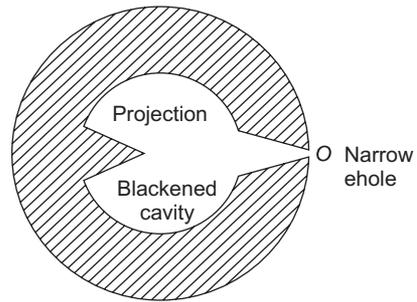
4.2.1 Planck's Quantum Theory of Black-Body Radiation

To explain the experimentally observed energy distribution in the spectrum of a black body and to overcome the failure of classical mechanics in this aspect, Planck derived the radiation law using the following assumptions:

1. The energy changes of a radiation are quantised. That is, they can have only discrete values.
2. The energy changes of radiation are integral multiples of an elementary energy unit called quantum.

3. The elementary energy quantum E should have finite values greater than zero.
4. The elementary energy quantum E is proportional to the frequency v of radiation. That is,

$$E \propto v$$

or $$E = hv \qquad (4.1)$$

where h is the Planck's constant.

Electromagnetic radiation consists of discrete energy particles called photons. Each photon carries an energy quantum hv. The electromagnetic radiations trapped in a cavity and in thermal equilibrium with the walls of the cavity are called black-body radiations. In equilibrium conditions, black-body radiations are considered to be the photon gas. Moreover, the photons are assumed to have the following characteristics:

- The photons do not interact with each other and they interact only with the walls of the cavity.
- The photons are indistinguishable and any number of photons may have the same energy.

Thus, photons are considered as bosons obeying Bose–Einstein statistics. Applying Bose–Einstein statistics to the photon gas, we get the number of photons having energies in between E and $E + dE$ as

$$dn = \frac{g(E)dE}{\exp(E/kT)-1} \qquad (4.2)$$

where k is Boltzmann's constant, T is the absolute temperature, and $g(E)\, dE$ is the number of energy states in the energy range of E and $E + dE$

As $E = hv$ for photons, $g(E)\, dE$ may be replaced by $g(v)\, dv$ which gives the number of modes of oscillations in the frequency range v and $v + dv$. For determining $g(v)dv$, a spherical shell of radii hv/c and $h(v + dv)/c$ is constructed. Here, c is the velocity of light in vacuum.

Volume of the spherical shell is

$$V' = \frac{4}{3}\pi \frac{h^3}{c^3}(v+dv)^3 - \frac{4}{3}\pi \frac{h^3}{c^3}v^3 \qquad (4.3)$$

Since dv is small, the higher order terms of dv in Eq. (4.3) are neglected.

Therefore, we have

$$V' = \frac{4}{3}\pi\frac{h^3}{c^3}(v^3 + 3v^2 dv - v^3)$$

i.e., $\qquad V' = \frac{4\pi h^3}{c^3}v^2 dv \qquad\qquad (4.4)$

The volume of phase space is $V = h^3$ and there are two states of polarisation of the radiation. The number of states in the frequency range v and $v + dv$ are given as

$$g(v)dv = \frac{8\pi V}{c^3}v^2 dv \qquad\qquad (4.5)$$

Substitution of Eq. (4.5) into Eq. (4.2) gives

$$dn = \frac{8\pi V}{c^3}\cdot\frac{v^2 dv}{\exp(hv/kT)-1} \qquad\qquad (4.6)$$

Energy of dn photons in the frequency range v and $v + dv$ is equal to $hvdn$. Hence, energy density is given by

$$E' = \frac{hv\cdot dn}{V} \qquad\qquad (4.7)$$

The energy density distribution for black-body radiation is then given by

$$E(v)dv = \frac{hvdn}{V}$$

i.e., $\qquad E(v) = \frac{hv}{V}\cdot\frac{dn}{dv} \qquad\qquad (4.8)$

Substituting Eq. (4.6) into Eq. (4.8), we get

$$E(v) = \frac{8\pi V}{c^3}\cdot\frac{1}{V}\frac{hv^3 dv}{\exp(hv/kT)-1}\cdot\frac{1}{dv}$$

i.e., $\qquad E(v) = \frac{8\pi hv^3}{c^3}\cdot\frac{1}{\exp(hv/kT)-1} \qquad\qquad (4.9)$

Equation (4.9) represents the **Planck's law for black-body radiation**. It gives

$$E(v)dv = \frac{8\pi hv^3}{c^3}\cdot\frac{dv}{\exp(hv/kT)-1} \qquad\qquad (4.10)$$

Often, Planck's law is written in terms of wavelength.

If λ is the wavelength, we have $v = c/\lambda$, we have $dv = -cd\lambda/\lambda^2$. Neglecting the negative sign (due to physical considerations), we may now rewrite Eq. (4.10) as

$$\text{i.e.,} \qquad E\,(\lambda)\,d\lambda = \frac{8\pi h}{\lambda^3}\cdot\frac{c}{\lambda^2}\cdot\frac{d\lambda}{\exp(hc/\lambda kT)-1}$$

$$E\,(\lambda)\,d\lambda = \frac{8\pi hc}{\lambda^5}\cdot\frac{d\lambda}{\exp(hc/\lambda kT)-1} \tag{4.11}$$

$$\text{or} \qquad E\,(\lambda) = \frac{8\pi hc}{\lambda^5}\cdot\frac{1}{\exp(hc/\lambda kT)-1} \tag{4.12}$$

Equation (4.12) represents the energy density corresponding to wavelength λ of the black-body. Equation (4.12) also represents the **Planck's law for black-body radiation**.

Planck's radiation law can be used to derive the other important experimentally determined laws such as Rayleigh–Jeans law, Wien's displacement law, etc.

4.2.2 Rayleigh–Jeans Law

In the limit of long wavelengths and high temperature, we have $hc/\lambda kT \ll 1$. Then, the term $\exp(hc/\lambda kT)$ can be expanded to

$$\exp\,(hc/\lambda kT) = 1+\frac{hc}{\lambda kT}+\left(\frac{hc}{\lambda kT}\right)^2\cdot\frac{1}{2!}+\cdots$$

$$\exp\,(hc/\lambda kT) \approx 1+\frac{hc}{\lambda kT} \tag{4.13}$$

Taking into account, Eq. (4.12) may be rewritten as

$$E(\lambda) = \frac{8\pi kT}{\lambda^4} \tag{4.14}$$

Equation (4.14) represents the **Rayleigh–Jeans law**.

This is good approximation to Eq. (4.12) only at moderate and long infrared wavelengths. In that region, $E(\lambda) \sim 1/\lambda^4$.

4.2.3 Wien's Displacement Law

It concerns the maximum in the radiation curves of Fig. 4.1.

Wien's displacement laws holds for the limit of short wavelengths and low temperature. In such cases, we have exp $(hc/\lambda kT)$ >>1.

Hence Eq. (4.12) reduces to

$$E(\lambda) = \frac{8\pi hc}{\lambda^5}\exp(-hc/\lambda kT) \tag{4.15}$$

Applying the first mathematical condition for maximum i.e., $\partial E/\partial\lambda = 0$ to Eq. (4.15), we get

$$\left(\frac{hc}{kT} - 5\lambda\right)\exp(-hc/\lambda kT) = 0$$

$$\therefore \qquad \frac{hc}{kT} - 5\lambda = 0$$

i.e., $\qquad \lambda T = \dfrac{hc}{5k} \equiv \text{const.} \tag{4.16}$

representing the Wien's displacement law. Thus, according to Wien's displacement law, if the maximum of the curve occurs at λ_{max} for a temperature T, then at a temperature T' the maximum will occur at λ'_{max} given by

$$\lambda_{max} \cdot T = \lambda'_{max} \cdot T'^{-1} = 2.9 \times 10^{-3} \text{ m.K} \tag{4.17}$$

4.3   COMPTON EFFECT

In 1923, Compton examined the behaviour of X-rays scattered from a block of graphite. He measured the wavelength of the scattered X-rays at different scattering angles. Many of the scattered X-rays were found to have lower frequencies. This is called Compton scattering.

When high frequency monochromatic radiations such as γ rays, X-rays, etc., fall on a substance, the scattered radiations contain two components. One component is of the same frequency and the other component is of lower frequency than that of the incident radiation. In addition to this, an electron is liberated from the atom of the substance (i.e., recoil electron). This phenomenon is called the Compton effect.

The scattered radiation of the same frequency as that of incident radiation is called **unmodified radiation**. The scattered radiation of lower frequency than that of incident radiation is called **modified radiation**.

4.3.1 Theory of Compton Effect

The phenomenon of light scattering by a metal is considered as a collision between the photon and the electron in the metal (Fig. 4.3).
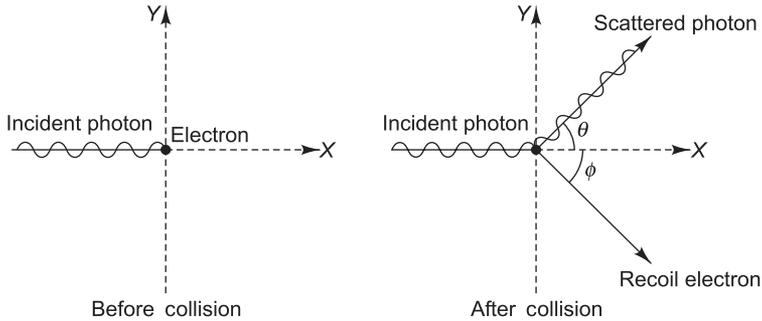
Fig. 4.3 Illustration of Compton Effect

The electron is assumed to be free and at rest before its collision with the incident photon. After collision, the relativistic mass of the electron is taken into account. This is justified because a small part of the energy of the incident radiation gives sufficient energy to the recoil electron.

Consider a photon of energy hv colliding with an electron at rest. Even when the electron is bound to the nucleus of the atom, it receives sufficient energy from the incident photon. So, it can be treated as practically free. During the collision process, a portion of the energy is imparted to the electron. As a result, the electron gains kinetic energy to recoil. Let ϕ be the recoil angle of the electron and θ be the scattering angle of the photon.

Before the collision, let

Energy of the incident photon $= hv$

Momentum of the incident photon $= hv/c$

Rest mass of the electron $= m_0$

Rest mass energy of the electron $= m_0c^2$

Momentum of the rest electron $= 0$

After the collision, let

Energy of the scattered photon $= hv'$

Momentum of the scattered photon $= hv'/c$

Velocity of the recoil electron $= v$

Relativistic mass of the electron is $m = \dfrac{m_0}{(1 - v^2/c^2)^{1/2}}$

Mass of the recoil electron $= m$

Relativistic energy of the recoil electron $= mc^2$

Momentum of the recoil electron $= mv$

Applying the principle of conservation of energy, we get

$$h\nu + m_0c^2 = h\nu' + mc^2 \tag{4.18}$$

Applying the principle of conservation of momentum along the direction of the incident photon, we have

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \tag{4.19}$$

Applying the principle of conservation of momentum in a direction perpendicular to the direction of the incident photon, we have

$$0 + 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \tag{4.20}$$

From Eq. (4.19), we have

$$h\nu = h\nu' \cos\theta + mvc\cos\phi$$

or $\qquad mvc\cos\phi = h\nu - h\nu'\cos\theta \tag{4.21}$

From Eq. (4.20), we have

$$mvc\sin\phi = h\nu'\sin\theta \tag{4.22}$$

From Eqs. (4.21) and (4.22), we get

$$m^2v^2c^2 = (h\nu - h\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2$$

$$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\,\nu'\cos\theta + h^2\nu'^2\sin^2\theta$$

$$m^2v^2c^2 = h^2(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta) \tag{4.23}$$

From Eq. (4.18), we have

$$mc^2 = h(\nu - \nu') + m_0c^2$$

Upon squaring, we get

$$m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2hm_0c^2(\nu - \nu') + m_0^2\,c^4 \tag{4.24}$$

Subtracting Eq. (4.23) from Eq. (4.24), we have

$$m^2c^2(c^2 - v^2) = 2h^2vv'(\cos\theta - 1) + 2h(v - v')m_0c^2 + m_0^2c^4$$

$$m^2c^2(c^2 - v^2) = -2h^2vv'(1 - \cos\theta) + 2h(v - v')m_0c^2 + m_0^2c^4$$

Using the expression for the relativistic mass of electron, the above equation is rewritten as

$$\frac{m_0^2}{\left(1 - \dfrac{v^2}{c^2}\right)}c^2(c^2 - v^2) = -2h^2vv'(1 - \cos\theta) + 2h(v - v')m_0c^2 + m_0^2c^4$$

$$m_0^2c^4 = -2h^2vv'(1 - \cos\theta) + 2h(v - v')m_0c^2 + m_0^2c^4$$

$$2h^2vv'(1 - \cos\theta) = 2h(v - v')m_0c^2$$

$$\frac{v - v'}{vv'} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

i.e., $$\qquad \frac{1}{v'} - \frac{1}{v} = \frac{h}{m_0c^2}(1 - \cos\theta) \qquad\qquad (4.25)$$

Equation (4.25) shows that $v' < v$ as the constants h, m_0, c have positive values and the maximum value of $\cos\theta$ is equal to 1.

Equation (4.25) may be rewritten as

$$\frac{c}{v'} - \frac{c}{v} = \frac{h}{m_0c}(1 - \cos\theta)$$

or $$\qquad \lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta)$$

i.e. $$\qquad \lambda' - \lambda = \frac{h}{m_0c}2\sin^2(\theta/2) \qquad\qquad (4.26)$$

Conclusions   From Eq. (4.26), it is clear that
- Change in wavelength is independent of the wavelength of the incident photon. It depends only on the angle of scattering of the photon.
- When $\theta = 0$, we have $\lambda' - \lambda = 0$. That is, no scattering takes place along the direction of the incident photon.

- When $\theta = \pi/2$, we have $\lambda' - \lambda = \dfrac{h}{m_0 c}$

i.e., $\lambda' - \lambda = \dfrac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} = 0.024 \,\text{Å}$

This difference in wavelength is called **Compton shift** and it is a constant.

- When $\theta = \pi$, we have

$$\lambda' - \lambda = \dfrac{2h}{m_0 \, c}$$

i.e., $\lambda' - \lambda = 0.048 \,\text{Å}$

Thus, when θ varies between 0 and π, the wavelength of the scattered photon varies between λ and $\lambda + 2h/(m_0 c)$ provided the incident photon is of a very small wavelength.

4.3.2 Experimental Verification of Compton Effect

Figure 4.4 shows the schematic experimental arrangement for the verification of Compton effect. The monochromatic X-rays from the X-ray tube A are collimated by the slit S_1. The emerging parallel beam of X-ray is allowed to the carbon block B which acts as a scatterer. The scattered X-rays are collimated by the slit S_2. The parallel beem of scattered X-rays fall on the crystal C of the Bragg's spectrometer D. Using Bragg's law, the wavelength of the scattered radiation is determined. The scattering angle is measured by the Bragg's spectrometer.
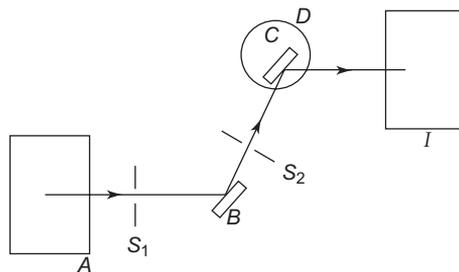
A–X-ray tube; S_1, S_2–Slits; B–Carbon scatterer;
C–Spectrometer crystal; D–Bragg's spectrometer;
I–Ionisation chamber

Fig. 4.4 Experimental Study of Compton Effect

The X-rays reflected by the crystal C of the Bragg's spectometer pass through an ionisation chamber I to get its intensity measured. The scattering angle can be varied by rotating the scatterer B. For different values of scattering angles, the intensity and wavelength of the scattered X-rays are measured. The results are graphically presented (Fig. 4.5).
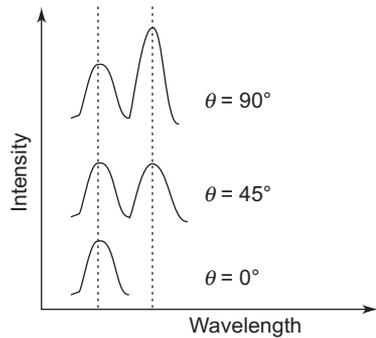
Fig. 4.5 Intensity of Scattered Radiation

We observe that for each value of $\theta \neq 0$, there are distinct intensity peaks at two wavelength: one of which corresponds to the incident radiation of wavelength λ and the other has a higher wavelength λ'. The peak corresponding to λ' is called modified peak.

As seen in Fig. 4.5, with increase in θ, $\lambda' - \lambda$ increases and the shift in wavelength $\Delta\lambda$ increases in accordance with the results obtained by Compton. Using different X-ray sources, it may also be found that the Compton shift for a given scattering angle does not depend on the wavelength of the incident X-rays. Thus, the Compton effect is experimentally verified.

4.4   WAVE NATURE OF MATTER

A wave is spread out over a relatively large region of space and it cannot be said to be located at some definite (particular) point. But, a particle is located at some definite point of space. Hence, it appears difficult to accept the conflicting idea that a particle has dual nature (i.e., both corpuscular and wave nature). However, the acceptance of dual nature of radiation is necessary because some experiments (photoelectric effect, geometrical optics) show that radiation is a stream of particles while the other experiments (interference and diffraction) show that radiation is a wave motion. On a macroscopic level, wave and corpuscular descriptions are distinctly different. But, on a microscopic level, this boundary is considerably smeared and the motion of a microscopic object is simultaneously of wave and corpuscular nature.

In 1924, the French physicist Louis de Broglie put forward a bold hypothesis. That is, matter also exhibits dual characteristics.

Particles of matter such as electrons and protons also exhibit wave properties. This suggestion is based on the following facts:

(i) The entire universe consists of matter and radiation only.

(ii) Nature is symmetrical in many respects.

4.4.1 de Broglie's Matter Waves

The waves associated with the moving particles of matter (electrons, protons, etc.) are known as **matter waves** or **pilot waves** or **de Broglie's waves**.

The wavelength of a matter wave is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \tag{4.27}$$

where h is Planck's constant; m is mass of the particle; v is velocity of the particle; p is momentum of the particle.

4.4.2 Derivation of de Broglie's Equation

Let us consider the photons (i.e., corpuscles of light). Planck's theory of radiation, considering the light as a wave, gives the energy of a photon as

$$E = h\nu = \frac{hc}{\lambda} \tag{4.28}$$

where, c is velocity of light in vacuum, ν is frequency of light radiation, λ is wavelength of light radiation, and h–Planck's constant.

Einstein's mass–energy relation, considering the light as a stream of particles, gives the energy of a photon as

$$E = mc^2 \tag{4.29}$$

From Eqs (4.28) and (4.29), we get

$$\lambda = \frac{h}{mc} = \frac{h}{p} \tag{4.30}$$

where $p \equiv mc$ is the momentum of the photon.

de Broglie carried over this idea to the material particles. Thus, if a particle having a mass m travels with a velocity v then its momentum is mv. Hence, the wavelength associated with this particle is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \tag{4.31}$$

4.4.3 Properties of Matter Waves

1. The lighter the particle, the greater is the wavelength associated with it.
2. The smaller the velocity of the particle, the greater is the wavelength.
3. They are not electromagnetic waves.
4. They are generated by the motion of microparticles.
5. They do not depend on the charge of the particles.
6. The velocity of matter waves is greater than that of light.

Proof Phase velocity of matter waves is

$$u = v\lambda \tag{4.32}$$

By photon analogy, energy of a particle is

$$E = hv \tag{4.33}$$

By Einstein's theory,

$$E = mc^2 \tag{4.34}$$

From Eqs (4.32) and (4.34), we have

$$v = \frac{mc^2}{h} \tag{4.35}$$

de Broglie's equation gives

$$\lambda = \frac{h}{mv} \tag{4.36}$$

Substituting Eqs (4.35), (4.36) into Eq. (4.32), we get

$$u = \frac{c^2}{v} \tag{4.37}$$

Since the velocity of a particle cannot exceed the velocity of light in vacuum, the phase velocity of matter waves is greater than the velocity of light.

7. Matter waves are pilot waves in the sense that their only function is to pilot or guide the material particle.
8. Matter wave is not a physical phenomenon. It is rather a symbolic representation of what we know about the particles.
9. The group velocity w of a de Broglie wave is equal to the velocity of the moving particle.

      i.e., $w = v$

4.5   G P Thomson Experiment

G P Thomson performed this experiment to demonstrate that the electrons behave as waves.

Construction

By means of an induction coil in a discharge tube *HC*, a beam of cathode rays is produced. The electrons are passing through a fine hole *H*, incident on a thin gold foil *G* of thickness around 10^{-8} m. The emergent beam of electrons falls on a photographic plate *P*. By means of a fluorescent screen *S*, visual examination of the pattern obtained is viewed. A very high vacuum is maintained in the recording part *GP* of the apparatus while air is allowed to leak into the discharge tube section through a needle valve (Fig. 4.6).
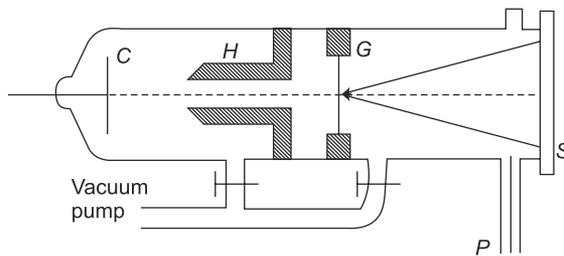
Fig. 4.6 Schematic Representation of GP Thomson Experiment

Working

A beam of electrons of known velocity passes through the thin gold foil *G* and falls on the photographic plate. Then the photographic plate is developed. A symmetrical pattern consisting of concentric rings about a central spot is obtained as shown in Fig. 4.7. This pattern is identical to the pattern produced by X-rays in the powdered crystal method. Now by using a magnetic field, the cathode

Fig. 4.7 Output Pattern

rays in the discharge tube are defected. The pattern obtained on the screen *S* is found to shift correspondingly. Thus, we confirm that the pattern obtained was due to diffracted electrons and not due to secondary X-rays which was generated by the electrons going through the foil. Now the film *G* is removed. The pattern disappears, revealing that the presence of the film is essential.
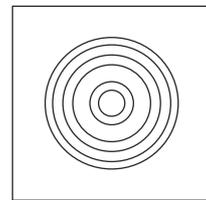
Conclusion

This experiment clearly reveals that an electron beam behaves as waves because the diffraction pattern can be produced only by waves. If the electron S behaved as corpuscles, the electrons passing through the foil would have been scattered through a wide angle.

4.5.1 Calculation of λ from Radii of the Rings

Let XY be the incident beam passing through the film at Y. YZ is the path of the beam after Bragg reflection at Y. The reflected beam falls at Z on the photographic plate at a distance D from the central point O. Let the distance from film to plate, i.e. YO be L (Fig. 4.8).

Now, $\angle ZYO = 2\theta$

θ is the glancing angle given by Bragg relation

$$n_\lambda = 2d \sin \theta$$

$$\frac{D}{L} = \tan 2\theta = 2\theta$$

$$[\because \theta \text{ is small}]$$

$$\theta = \frac{D}{2L}$$

But $2d \sin \theta = 2d\theta = n\lambda$

$$\theta = \frac{n\lambda}{2d}$$

$$\frac{n\lambda}{2d} = \frac{D}{2L}$$

$$\lambda = \frac{Dd}{nL}$$
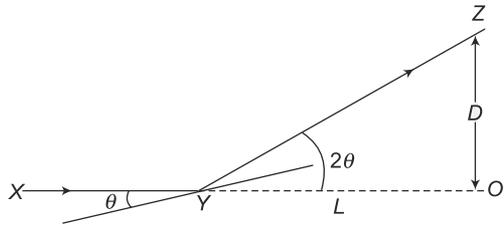
Fig. 4.8 Calculation of λ

From the above, wavelength is calculated which agrees with the value calculated by the equation $\lambda = \dfrac{h}{mv}$. This provides ultimate conformation for the wave nature of the electron.

4.6 Heisenberg's Uncertainty Principle

It is a direct consequence of the dual nature of matter. In classical mechanics, a particle occupies a definite position in space at any instant of time and has a definite momentum. Hence, it is possible to determine

(preciously and simultaneously) the position and momentum of the particle. But, the peculiar properties of microparticles (wave-corpuscular duality) impose several restrictions on the possibility of deterministic description of particles states.

4.6.1 Uncertainty Principle

In any simultaneous and accurate determination of the values of both members of a particular pair of physical variables that describe the behaviour of an atomic system, the product of uncertainties or errors in the knowledge of two variables is equal to or greater than Planck's constant h. That is,

$$\Delta x \cdot \Delta p_x \geq h \qquad (4.38)$$

where Δx is the uncertainty in determining the position of the particle; Δp_x is the uncertainty in determining the momentum of the particle.

Similarly, Heisenberg's uncertainty principle gives

$$\Delta E \cdot \Delta t \geq h \qquad (4.39)$$

$$\Delta J \cdot \Delta \theta \geq h \qquad (4.40)$$

where ΔE is the uncertainty in determining the energy; Δt is the uncertainty in determining the time; ΔJ is the uncertainty in determining the an angular momentum; $\Delta \theta$ is the uncertainty in determining the angle.

Thus, according to the uncertainty principle, it is not possible to determine accurately and simultaneously the values of the position and momentum of a microparticle at any time. The uncertainty principle sets limits on accuracy measurement in the case of atomic particles only.

4.6.2 Experimental Illustrations of Uncertainty Principle

Determination of the Position of a Particle by a Microscope

Consider an electron whose position is to be determined using a microscope. The resolving power of the microscope is given by

$$\Delta x = \frac{\lambda}{2 \sin \theta} \qquad (4.41)$$

where λ is the wavelength of the light used; θ is the semivertical angle of the cone of light; Δx is the uncertainty in determining the position of the particle.

To identify the electron, atleast one photon must strike the electron and scatter inside the microscope.

Let $p = \dfrac{h}{\lambda}$ be the momentum of the incident photon (i.e., before striking the electron). And, let the scattered photon enter the field of view of the microscope. Its x-component of momentum p_x may be between $p \sin \theta$ and $-p \sin \theta$. Therefore, the uncertainty in the x-component of momentum is

$$\Delta p_x = p \sin \theta -(-p \sin \theta)$$
$$= 2\,p \sin \theta$$

$$\Delta p_x = 2\frac{h}{\lambda}\sin\theta \qquad (4.42)$$

Fig. 4.9 Schematic Representation of Determining the Position of a Particle

Using Eqs. (4.41) and (4.42), we get

$$\Delta x \cdot \Delta p_x = \frac{\lambda}{2\sin\theta}\cdot\frac{2h}{\lambda}\sin\theta$$

$$\Delta x \cdot \Delta p_x = h \qquad (4.43)$$

That is, the product of uncertainties in the position and momentum is of the order of Planck's constant.

4.6.3   Diffraction by a Single Slit

Suppose a narrow beam of electrons pass through a single narrow slit and produce a diffraction pattern (Fig. 4.10).
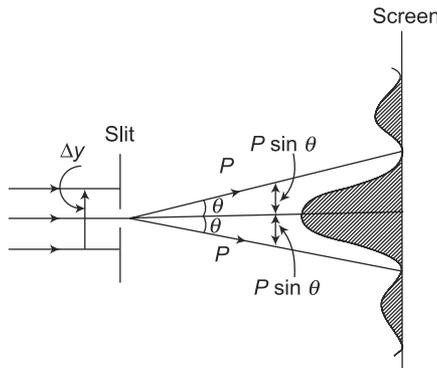
Fig. 4.10 Diffraction by a Single Slit

The equation for the diffraction pattern due to a single slit is

$$d \sin \theta \equiv n\lambda \tag{4.44}$$

where d is the width of the slit; θ is the glancing angle; n is the order of the spectrum; λ is the wavelength of light used.

In this case, the width of the slit is equal to be uncertainty in the measurement of the position Δy of the electron. That is, the electrons have passed through the slit, but, we cannot say definitely at what place of the slit. Therefore, for first minimum of the diffraction pattern (when $n = 1$), we have

$$\Delta y \sin \theta = \lambda \tag{4.45}$$

where θ is the angle of diffraction corresponding to the first-order minimum.

From Eq. (4.45), we get

$$\Delta y = \frac{\lambda}{\sin \theta} \tag{4.46}$$

Initially, the electrons are moving along the x-axis and hence they have no component of momentum along y-axis. After diffraction at the slit, the electrons are deviated from their initial path and have a component $p \sin \theta$. Therefore, they y-component of the momentum may lie between $p \sin \theta$ and $-p \sin \theta$. That is, the uncertainty in y-component of momentum is

$$\Delta p_y = p \sin \theta - (-p \sin \theta)$$
$$= 2 p \sin \theta$$

$$\Delta p_y = 2 \frac{h}{\lambda} \sin \theta \tag{4.47}$$

From Eqs. (4.46) and (4.47), we have

$$\Delta y \cdot \Delta p_y = \frac{\lambda}{\sin \theta} \cdot \frac{2h}{\lambda} \sin \theta = 2h$$

$$\Delta y \cdot \Delta p_y \approx h \tag{4.48}$$

Thus, the product of uncertainties in position and momentum is of the order of Planck's constant.

4.7   SCHRODINGER'S WAVE EQUATION

Developing de Broglie's idea about the dual nature of microparticles and using Hamilton's principle in classical mechanics, Schrodinger in 1926 derived the famous equation which serves as the fundamental equation in quantum physics. Schrodinger's wave equation has two different forms, namely, Schrodinger's time-independent wave equation and Schrodinger's time-dependent wave equation.

4.7.1   Schrodinger's Time-independent Wave Equation

Consider a stationary wave associated with a moving microparticle of mass m and of velocity v. Let Ψ be the wave function of the particle along the x, y, z coordinate axes at any time t. Let u be the wave velocity.

The general wave equation is

$$\nabla^2\Psi = \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} \tag{4.49}$$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ is the Laplacian operator.

The solution of Eq. (4.49) may be written as

$$\Psi = \Psi_0 e^{-i\omega t} \tag{4.50}$$

where Ψ_0 is the amplitude of the wave function at the point (x, y, z); $\omega = 2\pi v$ is the angular frequency of the wave; v is the frequency of the wave.

Differentiating Eq. (4.50) with respect to t, we get

$$\frac{\partial\Psi}{\partial t} = -i\omega\,\Psi_0 e^{-i\omega t} = -i\omega\,\Psi$$

$$\frac{\partial^2\Psi}{\partial t^2} = -\omega^2\Psi \tag{4.51}$$

Substituting Eq. (4.51) into Eq. (4.49), we get

$$\nabla^2\Psi = -\frac{1}{u^2}\omega^2\Psi$$

$$\nabla^2\Psi = -\frac{1}{u^2}(2\pi v)^2\,\Psi$$

i.e., $\qquad \nabla^2 \Psi = -\dfrac{4\pi^2 v^2}{u^2}\Psi$ $\qquad\qquad$ (4.52)

On the other hand, the wave velocity is defined as $u = v\lambda$. Here, λ is the wavelength. Therefore, Eq. (4.52) is rewritten as

$$\nabla^2\Psi = -\dfrac{4\pi^2 v^2}{v^2\lambda^2}\Psi$$

i.e., $\qquad \nabla^2\Psi = -\dfrac{4\pi^2}{\lambda^2}\Psi$ $\qquad\qquad$ (4.53)

Moreover, the total energy of the particle is the sum of its kinetic and potential energies. That is,

$$E = \dfrac{1}{2}mv^2 + V$$

or $\qquad E = \dfrac{p^2}{2m} + V$ $\qquad\qquad$ (4.54)

where $p = mv$ is the momentum of the particle and V is the potential energy of the particle.

From Eq. (4.54), we get the momentum of the particle as

$$p = \left[2m(E-V)\right]^{1/2}$$

Then, using de Broglie's hypothesis, we get

$$\lambda \equiv \dfrac{h}{p} = \dfrac{h}{\left[2m(E-V)\right]^{1/2}}$$ $\qquad\qquad$ (4.55)

Substitution of Eq. (4.55) into Eq. (4.54) gives

$$\nabla^2\Psi = -\dfrac{4\pi^2}{h^2}2m(E-V)\Psi$$

or $\qquad \nabla^2\Psi = -\dfrac{2m}{\hbar^2}(E-V)\Psi$ $\qquad\qquad$ (4.56)

where $\hbar = h/2\pi$.

Equation (4.56) may be rewritten as

$$\nabla^2\Psi + \dfrac{2m}{\hbar^2}(E-V)\Psi = 0$$ $\qquad\qquad$ (4.57)

Equation (4.57) is **Schrodinger's time-independent wave equation in three dimensions**. It is applicable to the particles having time-independent potential energy.

We may now examine Eq. (4.57) in the limits of zero potential energy of the particle and one dimensional motion of the particle.

Free Particle Case Let the particle be free. That is, its potential energy is zero. Then, the Schrodinger's time-independent wave equation is modified as

$$\nabla^2 \Psi + \frac{2m}{\hbar^2} E\Psi = 0 \tag{4.58}$$

One-Dimensional Case Let the particle move along only one coordinate axis, say, the x axis. Then, the wave function Ψ depends only on one coordinate x. In this case, Eq. (4.31) is modified as

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\Psi = 0 \tag{4.59}$$

Equation (4.59) is **Schrodinger's time-independent wave equation in one dimension**.

4.7.2 Schrodinger's Time-dependent Wave Equation

Let the potential energy of a moving microparticle depend on time. For such a particle, Schrodinger's wave equation will have another form which is called Schrodinger's time-dependent wave equation.

Differentiating the wave function given by Eq. (4.50) with respect to t, we get

$$\frac{\partial \Psi}{\partial t} = -i\omega \Psi_0 e^{-i\omega t} = -i\omega \Psi$$

or $$\frac{\partial \Psi}{\partial t} = -i2\pi v \Psi$$

$$\frac{\partial \Psi}{\partial t} = -i2\pi \frac{hv}{h}\Psi$$

i.e., $$\frac{\partial \Psi}{\partial t} = -i\frac{E\Psi}{\hbar}$$

Thus, we have $E\Psi = -\frac{\hbar}{i}\frac{\partial \Psi}{\partial t}$

or
$$E\Psi = i\hbar \frac{\partial \Psi}{\partial t} \tag{4.60}$$

Substituting Eq. (4.60) into Eq. (4.57), we get

$$\nabla^2 \Psi + \frac{2m}{\hbar^2} i\hbar \frac{\partial \Psi}{\partial t} - \frac{2m}{\hbar^2} V\Psi = 0$$

$$\frac{\hbar^2}{2m} \nabla^2 \Psi + i\hbar \frac{\partial \Psi}{\partial t} - V\Psi = 0$$

or
$$-\frac{\hbar^2}{2m}\nabla^2 \Psi + V\Psi = i\hbar \frac{\partial \Psi}{\partial t} \tag{4.61}$$

On the other hand, the Hamiltonian operator $\hat{H}$ and the energy operator $\hat{E}$ are given as

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V$$

$$\hat{E} = i\hbar \frac{\partial}{\partial t}$$

Then, Eq. (4.61) can be rewritten as

$$\hat{H}\Psi = \hat{E}\Psi \tag{4.62}$$

Equation (4.62) is **Schrodinger's time-dependent wave equation in three dimensions**.

One-Dimensional Case Now, we may examine Eq. (4.61) in the limit of one-dimensional motion of the particle. Let the particle move only along one coordinate axis, say, the x axis. Then, the wave function depends only on one coordinate x. In this case, Eq. (4.62) is modified as

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi = i\hbar \frac{\partial \Psi}{\partial t} \tag{4.63}$$

Equation (4.63) is **Schrodinger's time-dependent wave equation in one dimension**.

4.7.3 Significance of Wave Function

Schrodinger assumed that the wave function Ψ is the amplitude of some sort of wave associated with the system. It soon became clear

that this interpretation was wrong. The correct physical interpretation of wave function was given by Max Born. He postulated that $|\Psi|^2$ gives the probability density for finding the particles at given locations in space.

The significant features of wave function are as follows:

- The wave function Ψ has no physical interpretation. But, the square of its absolute magnitude $|\Psi|^2$ (or $\Psi\Psi^*$ if Ψ is a complex quantity with Ψ^* being its complex conjugate) evaluated at a particular point, at a particular time, is proportional to the probability of experimentally finding the particle there at that time.

- It measures the variations of the matter waves. It statistically relates the corpuscular and wave nature of matter.

- It is a complex quantity. Hence, it cannot be measured.

- It identifies the state of a particle in an atomic structure.

- It is a measure of the probability of finding the particle at a particular position. It cannot predict the exact location of the particle.

- It is a complex quantity, while the probability is a real and positive quantity. Hence, a term called the **position probability density** $p(r, t)$ is introduced. It is defined as the product of the wave function and its complex conjugate.

  That is,

  $$p(r,t) = \left|\Psi(r,\ t)\right|^2 = \Psi\,\Psi^* \tag{4.64}$$

- The probability of finding the particle within a volume element dt is given by

  $$p = \int \left|\Psi\right|^2 d\tau \tag{4.65}$$

- If a particle is definitely present, then the corresponding probability value is one. That is,

  $$p = \int_{-\infty}^{+\infty} \left|\Psi\right|^2 d\tau = 1 \tag{4.66}$$

  Equation (4.66) is called the **normalisation condition**.

- The wave function Ψ has no physical meaning, while the probability density p has a physical meaning.

4.7.4 Particle in One-Dimensional Box

This study determines the allowed energies and the position probability functions of a free particle confined to a box. Consider a particle of mass m confined to a one-dimensional box of width L and of infinite height (Fig. 4.11). Let the particle move only in one direction X along a straight line.
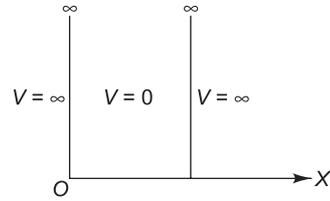
Fig. 4.11 Particle in a One-Dimensional Box

It is assumed that the walls of the box are rigid and elastic. Moreover, the particle is considered to be free. Rigid walls ensure that the particle cannot penetrate the walls and it is confined to the box. Elasticity of the walls ensures elastic collisions of the particle with the walls.

As the particle is free, its potential energy is constant (i.e., $V = 0$ ≡ const.). Hence, Schrodinger's one-dimensional time-independent wave equation can be used to study the state of the given particle. Thus, we have

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\Psi = 0 \tag{4.67}$$

As the particle is free, Eq. (4.67) is simplified as

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}E\Psi = 0 \tag{4.68}$$

The wave number k can be defined as

$$\frac{2mE}{\hbar^2} = k^2 \tag{4.69}$$

Then, Eq. (4.68) may be rewritten as

$$\frac{d^2\Psi}{dx^2} + k^2\Psi = 0 \tag{4.70}$$

Equation (4.70) is analogous to the equation of simple harmonic motion. Then, solution of Eq. (4.70) has the form

$$\Psi = A \sin kx + B \cos kx \tag{4.71}$$

where A and B are constants determined through the boundary conditions.

As the particle is confined to the box, it does not exist outside the box and also at the walls. Obviously, the wave function at the walls is zero. Therefore, the boundary conditions are

$$\Psi = 0, \text{ at } x = 0, \tag{4.72}$$

$$\Psi = 0, \text{ at } x = L, \tag{4.73}$$

Substitution of Eq. (4.72) into Eq. (4.71) gives

$$B = 0$$

Similarly, substituting Eq. (4.73) into Eq. (4.71), we get $A \sin kL = 0$

i.e., $kL = n\pi$, where $n = 1, 2, 3, ...$

or $k = \dfrac{n\pi}{L}$ (4.74)

Taking into account Eqs (4.71) and (4.74), the wave function is rewritten as

$$\Psi = A \sin \frac{n\pi}{L} x \tag{4.75}$$

As the particle is confined to the given box, the normalisation condition gives

$$\int_0^L |\Psi|^2 \, dx = 1$$

i.e., $\displaystyle\int_0^L A^2 \sin^2\left(\frac{n\pi}{L} x\right) dx = 1$

or $\dfrac{1}{2}\displaystyle\int_0^L \left(1 - \cos\left(\frac{2\pi n}{L} x\right)\right) dx = 1$

$$\frac{A^2}{2}\left[x - \frac{L}{2\pi n}\sin\left(\frac{2\pi n}{L} x\right)\right]_0^L = 1$$

or $A = \sqrt{\dfrac{2}{L}}$ (4.76)

Taking into account Eqs (4.75) and (4.76), we get the allowed wave functions as

$$\Psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \tag{4.77}$$

From Eqs (4.69) and (4.74), we get

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

i.e., $E = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$

or $E_n = \dfrac{n^2h^2}{8mL^2}$ (4.78)

As follows from Eq. (4.78), the particle can have only discrete values of energy. That is, energy of the particle is quantised. Moreover, the particle cannot have zero energy.

The values of energy E_n, given by Eq. (4.78), for which Schrodinger's time-independent wave equation can be solved are called **eigen energy values**, and the corresponding wave functions Y_n, given by Eq. (4.77), are called **eigen wave functions**. Eigen wave functions and the corresponding probability densities of the particle in a one-dimensional box are shown in Fig. 4.12 and Fig. 4.13, respectively.
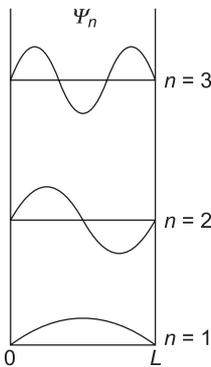
Fig. 4.12   Eigen Wave Function
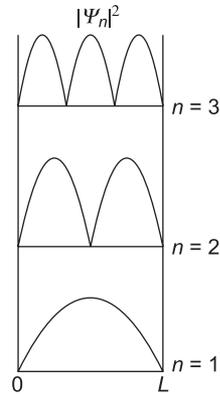
Fig. 4.13   Probability Density

Using Eq.(4.78), the lowest possible energy of the given particle can be calculated. It is called the ground-state energy which corresponds to the lowest possible eigen value with $n = 1$. Then,

Equation (4.78) gives the ground-state energy of the particle as

$$E_1 = \frac{h^2}{8\,mL^2}$$ (4.79)

On the other hand, momentum of the particle is given as

$$p_n \equiv \pm\sqrt{2mE_n} = \pm\frac{nh}{2L} \tag{4.80}$$

From Eq. (4.65), it follows that the particle can have only discrete values of momentum. That is, momentum of the particle is quantised. As the particle is moving back and forth, its average momentum for any value of n is given as

$$\bar{p}_n = \frac{\left(+\dfrac{nh}{2L}\right) + \left(-\dfrac{nh}{2L}\right)}{2} = 0 \tag{4.81}$$

4.8   BASICS OF A MICROSCOPE

A microscope is a device used to view a magnified image of a small object. In general, microscopes are classified as simple and compound microscope. A **simple microscope** is made of a single biconvex magnifying lens held in a simple frame. A **compound microscope** is made of two lenses or a system of lenses for better magnification. Depending on the applications, many other microscopes such as phase contrast microscope, ultraviolet microscope, metallurgical microscope, electron microscope, etc., have been designed. Microscopes are characterised by their magnifying power and resolving power.

Magnifying Power   The magnifying power M of a microscope is defined as the ratio between the angle subtended by the final image at the eye β to the angle subtended by the object at the eye α, placed at the near point.

$$M = \frac{\beta}{\alpha} \tag{4.82}$$

Resolving Power   It is the ability of a microscope to form distinct and separable images of the two point objects which are close to each other. The resolving power is given as

$$\text{Resolving power} = \frac{2NA}{\lambda} \tag{4.83}$$

where, NA is the numerical aperture of the objective of the microscope and λ is the wavelength of light through vacuum.

Therefore, the resolving power of a microscope can be increased by decreasing the value of λ. Thus, by using ultraviolet light and quartz lenses, the resolving power can be increased.

4.8.1 Electron Microscope

The properties of materials depend on structure. The structure of a material is determined by composition, heat treatment and processing. It is necessary to find both composition and microstructure with highest resolution for the study of material behaviour and for designing new materials. An electron microscope provides both physical and chemical analysis. The electron microscope is characterised by its superior resolution, resulting from very small wavelengths as compared to other forms of radiation like X-rays and neutrons.

The resolving power of an optical instrument is inversely proportional to the wavelengths of the radiation used for illuminating the object under study. A microscope can easily distinguish one object from another if the minimum distance between the two points is

$$d = \frac{\lambda}{n\sin\theta} \tag{4.84}$$

where λ is the wavelength of radiation to illuminate the objects and $n \sin\theta$ is the numerical aperture of the objective lens, n is the refractive index of the object space and θ is the semivertical angle of the cone of rays entering the objective lens.

Higher magnification as well as resolving power can be obtained by utilising waves of shorter wavelengths. Electron microscopes use electron waves whose wavelength is given by

$$\lambda = \frac{12.27}{\sqrt{V}}\,\text{Å} \tag{4.85}$$

For V = 1000 volts, Eq. (4.85) gives λ = 0.1225 Å which is extremely small. Electron microscopes giving magnifications more than 200000X are available. The highest magnification obtained with the best optical microscope is 2000X.

Principle The working of an electron microscope is based upon the following facts:
   (i) Electrons exhibit wave properties like light rays but have much shorter wavelength.

(ii) Electrons can be focussed by electric and magnetic fields just as light rays can be focussed with the help of lenses.

4.8.2 Transmission Electron Microscope (TEM)

Principle A beam of electrons is transmitted through the specimen. The electron beam coming out produces a magnified image on the fluorescent screen or on the photographic film. In order to get large magnification, magnetic lenses of short focal lengths are used.

Construction A transmission electron microscope is shown in Fig. 4.14.

The basic components of a TEM are electron gun, electromagnetic condensing lens, object holder, electromagnetic objective and projective lenses and photographic plate or fluorescent screen.

Fig. 4.14 Transmission Electron Microscope

Highly accelerated electron beams are produced by the electron gun. An electromagnetic condensing beam is used to condense the beam of electrons. The object holder holds a very thin sample of the specimen. Electromagnetive objective and projective lenses project the image on the screen. The whole arrangement is kept in a high vacuum metal frame.

The accelerated electron beam from the electron gun is made to pass through the condensing lens. The condensing lens consists of a current-carrying coil that produces a magnetic field (acts as lenses) to focus the electron beam on the specimen in the holder. The focal length of the magnetic lens can be varied by varying the current through the coils. Thus the focusing can be varied by changing the current through the coil and velocity of electrons can be varied by accelerating potential difference. When the beam is incident on the sample, some part of it is transmitted and is focused by objective lens. The high angle diffracted electrons are blocked by the aperture and thereby increase the contrast. Then the beam is passed through the projector lens and is projected on the fluorescent screen.

Uses

1. It is used in the investigation of atomic structures and structure of crystals.
2. It is used in biology, medicine, physics, chemistry metallurgy, etc.
3. It is used in the study of structure of textile fibres, purification of lubricating oils, composition of paper and paints, surfaces of metals and plastics.
4. It is used to study disease-causing viruses.

Merits

1. It has high resolution.
2. It has high resolving power of 1 A to 2 A.
3. Specimen of size up to 0.2 nm can be examined.
4. High contrast image is obtained due to transmitted and diffracted beams.
5. 1,00,000 times greater than that of the size of the object magnification can be obtained.

Demerits

1. Three-dimensional image of the object cannot be obtained.
2. Not suitable for thick objects.
3. Object must be very thin.
4. During sample preparation, structural change may take place.
5. The electrons interact with the sample; and can cause damage to the biological samples.

4.8.3 Scanning Electron Microscope (SEM)

The advantage of SEM is that large specimens can be examined in three dimensions. It makes the investigation of surfaces much easier as the specimen surface can be scanned without the labour involved in preparing replicas. This is for surface observation. We can get magnifications up to 300000X. SEM studies yield the following information about the specimen:

  (i) *Topography* The surface features of an object and its texture
 (ii) *Morphology* The shape, size and arrangement of particles making up the object on the surface of the sample
(iii) *Composition* The constituting elements and compounds of the sample and their relative ratios

(iv) *Crystallographic Information* The arrangement of atoms in the specimen and their degree of order

When accelerated primary electrons strike the specimen, they interact and emit a wide spectrum of electromagnetic waves ranging from optical wavelength to characteristic X-rays. These waves are used to investigate its physical and chemical properties.
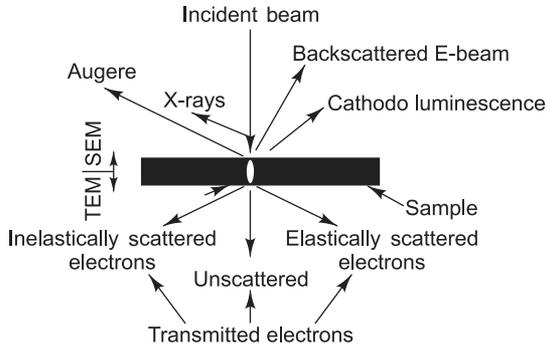
Fig. 4.15 Interaction of Electron Beam with a Specimen

Construction The scanning electron components are shown in Fig. 4.16. It consists of an electron gun to produce accelerated beam of electrons. Electromagnetic lenses are used to condense the beam of electrons. A scanning coil is placed after the second condensing lens. The collector collects the secondary electron. In the collector, a scintillator is placed and is connected to a photomultiplier and finally the signal is sent to CRO.

Working When the electron beam interacts with the specimen, the back scattered electrons are emitted when electrons are bounced by the atoms and being reflected back nearly 180°. These electrons can be used to differentiate different atomic elements and its intensity is more for heavier elements.

Secondary electrons are also produced when the electron beam interacts with specimen atoms and removes one of its electron with extra energy. These reactions occur near the surface of the specimen, and they are used in topographical study of the specimen.

Higher energy electron drops into the place of exposed secondary electrons and that causes an energy surplus in the atom, which releases another low energy electron called Augur electron, which gives compositional information about the specimen.

During the same process along with Augur electrons, X-rays are also emitted which provide further information on the composition of the specimen.

All the above said electrons are detected by the electron detector and are converted into light and fed to a photomultiplier through a light pipe along with X-rays which modulate the brightness of the CRT output device.
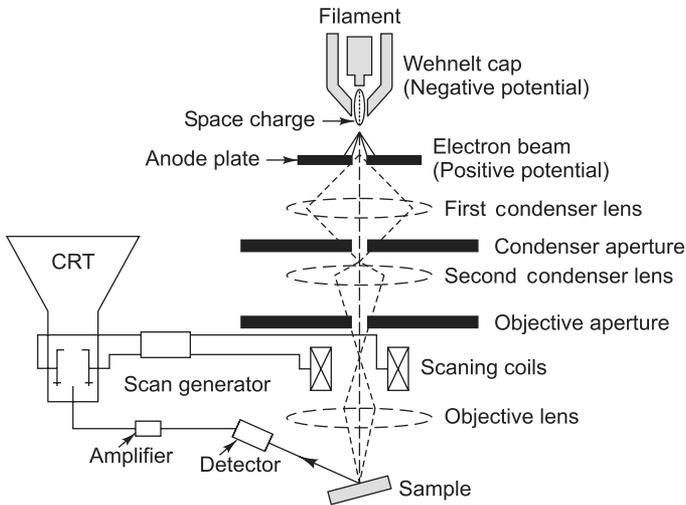
Fig. 4.16 Scanning Electron Microscope

Merits
1. Three-dimensional image of the object is obtained.
2. Object image has large depth of focus.
3. It can be used to examine specimen of large thickness.
4. Image can be viewed directly on the screen.
5. 3,00,000 times greater than that of the size of the object magnification can be obtained.

Demerits
The resolution of the image is limited to 10–20 nm, hence it is very poor.

Uses
1. It is used in various fields such as physics, chemistry, biology, engineering, industries, etc., for a variety of applications.
2. Three-dimensional view of the structure of the specimen can be obtained.

4.8.4 Differences between Optical and Electron Microscopes

S No.	Optical Microscope	Electron Microscope
1	Absorption of light is the principle used	Transmission or diffraction of electrons is the principle used
2	Incandescent lamp is the source	Electron gun is the source
3	Magnification is 2000 times as that of the size of the object	Magnification is 100000 times in TEM and 300000 times in SEM as that of the size of the object
4	0.00002 is the resolving power	1 A is the resolving power
5	Poor resolution	Better resolution
6	Optical lenses are used	Magnetic lenses are used
7	Operation does not require vacuum	Operation requires vacuum
8	We obtain a two-dimensional image of the object	We obtain a three-dimensional image of the object in TEM and SEM
9	Low cost	High cost
10	Compact in size	Larger in size

SUMMARY

➤ Max Planck proposed quantum hypothesis to overcome the failure of classical concepts to explain the experimentally observed behaviour of the black-body radiation.

➤ Planck's quantum theory on black-body radiation supported the particle nature of radiation. The radiation energy emitted from a hot black body is in the form of discrete packets or quanta called photons.

➤ Rayleigh–Jeans law states that the energy density radiated by a black body in the limit of long wavelength and high temperature is proportional to its temperature and is inversely proportional to λ^4.

➤ Wein's displacement law holds for the limit of short wavelength and low temperature.

➤ When a high frequency monochromatic radiation falls on a substance, scattered radiations contain two components. One component is of the same frequency and the other component is of lower frequency than that of the incident radiation. In

addition to this, an electron is liberated from the atom of the substance. This phenomenon is called the Compton effect.

▶▶ Change in wavelength is independent of the wavelength of the incident photon. It depends only on the scattering of photon.

▶▶ The waves associated with the moving matter are de-Broglie waves or matter waves.

▶▶ The wavelength of matter waves $\lambda = h/mv$.

▶▶ The concept of matter waves was created by the dual nature of light (both particle and wave). This reveals that the particles of matter undergo diffraction and interference.

▶▶ Schrodinger's wave equation is a matter wave equation and is based on wave nature of particles.

▶▶ Time-dependent and time-independent are two different forms of Schrodinger's wave equation.

▶▶ The wave function is defined as the probability amplitude of a particle to find its location and measures the variation of matter waves.

▶▶ A free particle can have any energy and its energy spectrum is continuous.

▶▶ A microscope is characterised by its magnifying power and resolving power.

▶▶ Resolving power is the ability to show two closer objects as separate ones.

▶▶ Magnifying power is the ability to show the magnified image of an object.

▶▶ As electrons posses very small wavelength, an electron microscope has more magnifying power and resolving power when compared with an optical microscope.

SOLVED PROBLEMS

4.1 The surface temperature of the sun is 6000 K and its radius is 7×10^8 m. Assuming that it radiates as a black body, find the energy it radiates at the middle of the visible range, of about 5550 nm per second.

Solution Given Data

$T = 6000$ K

$r = 7 \times 10^8$ m

$$\lambda = 550 \times 10^{-9}\,\text{m}$$

Radiant energy of a black body per square metre per second is given by Planck's law as

$$E(\lambda) = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{\exp(hc/\lambda kT) - 1}$$

Hence, energy radiated by the sun per second is

$$E' = \text{Area of the sun} \times E$$

$$E' = 4 \times 3.14\, r^2\, E$$

Substituting the values, we get

$$E' = 7.852 \times 10^{24}\,\text{W}$$

4.2 A classical particle of mass $m = 10^{-6}$ kg moves with a velocity of $v = 10^{-2}$ m/s. Find the de Broglie wavelength of the particle.

Solution The de Broglie's equation gives

$$\lambda = \frac{h}{mv} = 6.6 \times 10^{-26}\,\text{m}$$

4.3 Calculate the de Broglie wavelength of an electron accelerated by a potential of $V = 2$ kV.

Solution Energy gained by the electron due to the potential,

$$E = eV \tag{1}$$

where e is the charge of electron and V is the potential.

$$\text{Kinetic energy of the electron} = \frac{1}{2}mv^2 \tag{2}$$

where v is the velocity of electron and m is the mass of the electron.

From Eqs (1) and (2), we get

$$eV = \frac{1}{2}mv^2$$

$$\therefore \qquad v = \sqrt{\frac{2eV}{m}} \tag{3}$$

Momentum of electron $p = mv = \sqrt{\dfrac{2eV}{m}} \times m$

$\therefore \qquad P = \sqrt{2meV}$ \hfill (4)

Then, de Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

\hfill (5)

Substituting the numerical values $h = 6.6256 \times 10^{-34}$ J/s; $e = 1.602 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg, we get,

$$\lambda = \frac{12.26}{\sqrt{V}} \, \overset{\circ}{A}$$

i.e., $\qquad \lambda = \mathbf{2.73 \times 10^{-11}}$ **m**

4.4 An electron has a speed of 600 m/s with an accuracy of 0.005%. Calculate the uncertainty with which we can locate the position of the electron.

Solution Given Data

$\qquad v = 600$ m/s; $m = 9.1 \times 10^{-31}$ kg

$\qquad h = 6.63 \times 10^{-34}$ J/s

The momentum of an electron $= mv$

$$= 9.1 \times 10^{-31} \times 600$$

$$\Delta p_x = \left(\frac{0.005}{100}\right) mv$$

$$= 5 \times 10^{-5} \times 9.1 \times 10^{-31} \times 600$$

$$\Delta x = \frac{h}{\Delta p_x} = \frac{6.63 \times 10^{-34}}{5 \times 10^{-5} \times 9.1 \times 10^{-31} \times 600}$$

$$\Delta x = \mathbf{0.02254} \text{ m}$$

4.5 An electron is confined to a box of length 10^{-9} m. Calculate the minimum uncertainty in its velocity.

Solution   Given Data

$m = 9.1 \times 10^{-31}$ kg

$h = 6.63 \times 10^{-34}$ J/s

According to the uncertainty principle,

$$\Delta x \cdot \Delta p_x = h$$

Therefore, if Δx is maximum, then Δp_x must be minimum

i.e., $(\Delta x)_{max} \cdot (\Delta p_x)_{min} = h$

According to the problem, $(\Delta x)_{max} = 10^{-9}$ m

$$\therefore \qquad (\Delta p_x)_{min} = \frac{h}{(\Delta x)_{max}} = \frac{6.6 \times 10^{-34}}{10^{-9}}$$

$$= 6.6 \times 10^{-26}$$

Again $(\Delta p_x)_{min} = m \, (\Delta v_x)_{min}$

$\therefore \qquad (\Delta v_x)_{min} = 7.3 \times 10^4$ **m/s.**

4.6   Find the lowest energy of a neutron confined to a nucleus of size 10^{-14} m. Given, mass of neutron is 1.67×10^{-27} kg and Planck's constant is 6.62×10^{-34} J/s.

Solution   Given Data

$L = 10^{-14}$ m

$m = 1.67 \times 10^{-27}$ kg

$h = 6.62 \times 10^{-34}$ J/s

$E_1 = ?$

The lowest energy of a neutron confined to the nucleus is given as

$$E_1 = \frac{h^2}{8mL^2} \qquad (1)$$

Substituting the values of h, m and L into Eq. (1), we get

$$E_1 = \frac{(6.62 \times 10^{-34})^2}{8 \times 1.67 \times 10^{-27} \times (10^{-14})^2}$$

i.e., $E_1 = 3.28 \times 10^{-13}$ **J**

4.7   A ground-state energy particle is confined to a one-dimensional box of width 20Å. Determine the probability of finding the particle in an interval of 1Å from the centre of the box.

Solution   Given Data

$$L = 20 \text{ Å} = 20 \times 10^{-10} \text{ m}$$

$$\Delta x = 1\text{Å} = 1 \times 10^{-10} \text{ m}$$

$$|\Psi_1^2|.\Delta x = ?$$

The probability of binding the particle in an internal of *Dx* from the centre of the box is given as

$$p \equiv \left|\psi_1^{\,2}\right|\Delta x = \left[\sqrt{\frac{2}{L}}\sin\left(\frac{\pi(L/2)}{L}\right)\right]^2 \times \Delta x$$

$$p = \frac{2}{L}\sin^2\left(\frac{\pi}{2}\right)\Delta x$$

i.e., $p = \dfrac{2}{L}\Delta x$ (1)

Substituting the values of *L* and Δ*x* into Eq. (1), we get

$$P = \frac{2 \times 1 \times 10^{-10}}{20 \times 10^{-10}}$$

i.e.,   **P = 0.1**

4.8   An electron is confined to an infinitely deep potential well of width 3.5 Å. Determine the lowest two quantum energies that the electron can have.

Solution   Given Data

$$L = 3.5 \text{ Å} = 3.5 \times 10^{-10} \text{ m}$$

$$E_1 = ?; E_2 = ?$$

Application of the Schrodinger's wave equation to the electron in a one-dimensional box gives

$$E_n = \frac{n^2 h^2}{8mL^2}$$ (1)

where *n* = 1, 2, 3, ...; *h* = 6.62 × 10^{-34} J/s; *m* = 9.1 × 10^{-31} kg.

Equation (1) gives

$$E_1 = \frac{1^2 \cdot h^2}{8mL^2} \tag{2}$$

$$E_2 = \frac{2^2 \cdot h^2}{8mL^2} \tag{3}$$

Substituting the given data into Eq. (2), we get

$$E_1 = \frac{(6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (3.5 \times 10^{-10})^2}$$

$$E_1 = 4.914 \times 10^{-19} \text{ J} = \frac{4.914 \times 10^{-19}}{1.6 \times 10^{-19}} \text{eV}$$

i.e., **E_1 = 3.07 eV**

Substituting the given data into Eq. (3), we get

$$E_2 = \frac{4 \times (6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (3.5 \times 10^{-10})^2}$$

i.e., $E_2 = 4 \times 4.914 \times 10^{-19}$ J

$$E_2 = 19.66 \times 10^{-19} \text{ J}$$

or **E_2 = 12.28 eV**

4.9 Determine the energy difference between the first and second quantum states for a free electron in a cubical solid of 1 m³ volume.

Solution Given Data

$L = 1$ m

$E_2 - E_1 = ?$

Application of the Schrodinger's wave equation to the electron in a three-dimensional box gives

$$E_n = \frac{(n_x^2 + n_y^2 + n_z^2)h^2}{8mL^2} \tag{1}$$

where n_x, n_y, n_z = 1, 2, 3, ... are the quantum numbers;

$h = 6.62 \times 10^{-34}$ J/s; $m = 9.1 \times 10^{-31}$ kg.

For the first quantum state, we have

$$n_x = 1, \; n_y = 1, \; n_z = 1$$

Hence, Eq. (1) gives

$$E_1 = \frac{3h^2}{8mL^2} \tag{2}$$

For the second quantum state, we have

$$n_x = 1, \; n_y = 1, \; n_z = 2$$

Therefore, Eq. (1) gives

$$E_2 = \frac{6h^2}{8mL^2} \tag{3}$$

From Eqs (2) and (3), we get

$$E_2 - E_1 = \frac{3h^2}{8mL^2} \tag{4}$$

Substituting the given data into Eq. (4), we get

$$E_2 - E_1 = \frac{3 \times (6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 1^2}$$

$$E_2 - E_1 = 1.806 \times 10^{-37} \, \text{J} = \frac{1.806 \times 10^{-37}}{1.6 \times 10^{-19}} \, \text{eV}$$

i.e., $\quad E_2 - E_1 = 1.129 \times 10^{-18} \, \text{eV}$

4.10 An electron is confined to an infinitely deep potential well of width 4 Å. Determine the momentum of the ground-state electron.

Solution Given Data

$$L = 4 \, \text{Å} = 4 \times 10^{-10} \, \text{m}$$

$$p_1 = ?$$

Application of Schrodinger's wave equation to the electron in a one-dimensional box gives

$$p_n = \pm \frac{nh}{2L} \tag{1}$$

For the ground-state electron (i.e., $n = 1$), Eq. (1) is modified as

$$p_1 = \pm \frac{h}{2L} \tag{2}$$

Substituting the given data into Eq. (2), we get

$$p_1 = \pm \frac{6.62 \times 10^{-34}}{2 \times 4 \times 10^{-10}}$$

i.e., $p_1 = \pm\, 0.828 \times 10^{-24}$ **kg.m.s**

The positive and negative signs in the momentum of the electron correspond to the motion of the electron in the positive and in the negative directions along the coordinate axis.

4.11 Calculate the energy of an electron moving in one dimension in an infinitely high potential box of width 0.3 nm, if the mass of the electron is 9.11×10^{-31} kg and Plank's constant is 6.25×10^{-34} J/s

(A.U, B.E/B.Tech, Jan. 2011)

Solution   Given Data

$L = 0.3$ nm $= 0.3 \times 10^{-9}$ m

$m = 9.11 \times 10^{-31}$ kg

$h = 6.25 \times 10^{-34}$ J/s

$E = ?$

$$E = \frac{n^2 h^2}{8\, mL^3}$$

$$= \frac{(6.25 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (0.3 \times 10^{-9})^2}$$

$$E = 5.955 \times 10^{-19} \text{ J/s}$$

$$E = \textbf{3.72 eV}$$

4.12 Calculate de Broglie wavelength of an electron accelerated to a potential of 100 volts.   *(A.U, B.E/B. Tech, Jan. 2011)*

Solution   Given Data

$V = 100$ volts

$\lambda = ?$

Energy gained by the electron due to the potential

$$E = eV \tag{1}$$

Kinetic energy of the electron $= \dfrac{1}{2} m v^2 \tag{2}$

$$eV = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2\,eV}{m}}$$

Momentum of electron, $P = mv = \sqrt{\dfrac{2\,eV}{m}} \times m$

$$p = \sqrt{2\,meV}$$

∴ De Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\,meV}}$$

∴ $$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.602 \times 10^{-19} \times 100}}$$

$$\lambda = \mathbf{1.226\mathring{A}}$$

4.13 Find the lowest energy of an electron confined in a box of 0.2 nm length.　　　　(A.U, B.E./B.Tech, Jan. 2011)

Solution　　Given Data

$$L = 0.2 \text{ nm} = 0.2 \times 10^{-9} \text{ m}$$

$$E = ?$$

$$En = \frac{n^2\,h^2}{8\,mL^2}$$

where $n = 1$, $h = 6.62 \times 10^{-34}$ J/s, $m = 9.1 \times 10^{-31}$ kg

$$E = \frac{(6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.2 \times 10^{-9})^2}$$

$$E = 1.507 \times 10^{-19} \text{ J}$$

$$E = \frac{1.507 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$E = \mathbf{0.94 \text{ eV}}$$

QUESTIONS AND ANSWERS

4.1   Define a perfectly black body.

A perfectly black body is defined as the body which can absorb radiations of all wavelengths.

4.2   Define Compton effect.

When high-frequency monochromatic radiations such as X-rays fall on a substance, the scattered radiations contain two components. One component is of the same frequency and the other component is of a lower frequency than that of the incident radiation. In addition to this, an electron is liberated from the atom of the substance (i.e., recoil electron). This phenomenon is called the Compton effect.

4.3   What is Compton shift?

The difference between wavelengths of unmodified and modified radiations in Compton effect is called the Compton shift.

4.4   State the de Broglie's hypothesis.

Matter exhibits dual characteristics, i.e., wave and particle nature. The waves associated with the moving particles of matter are known as matter waves or pilot waves or de Broglie's waves. The wavelength of a matter wave is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

where h is Planck's constant, m is mass of the particle, v is velocity of the particle, and p is momentum of the particle.

4.5   Give the expression for de Broglie's wavelength.

The wavelength of a matter wave is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

where h is Planck's constant, m is mass of the particle, v is velocity of the particle, and p is momentum of the particle.

4.6 State the Heisenberg's uncertainty principle.

In any simultaneous and accurate determination of the values of both the members of a particular pair of physical variables, which describe the behaviour of an atomic system, the product of uncertainties or errors in the knowledge of the two variables is equal to or greater than Planck's constant h. For example,

$$\Delta x \cdot \Delta p_x \geq h$$

where Δx is the uncertainty in determining the position of the particle, and Δp_x is the uncertainty in determining the momentum of the particle.

4.7 Formulate Schrodinger's time-independent wave equation in one dimension and three dimensions.

$$\nabla^2 \Psi + \frac{2m}{\hbar^2}(E - V)\Psi = 0 \quad \text{(Three-dimensional equation)}$$

$$\frac{d^2 \Psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\Psi = 0 \quad \text{(One-dimensional equation)}$$

where Ψ is the wave function, m is the mass of the particle, E is its total energy, V is its potential energy, and $\hbar = \dfrac{h}{2\pi}$, h is the Planck's constant.

4.8 Formulate Schrodinger's time-dependent wave equation in one dimension and in three dimensions.

$$\hat{H}\Psi = \hat{E}\Psi \quad \text{(Three-dimensional equation)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2 \Psi}{dx^2} + V\Psi = i\hbar \frac{\partial \Psi}{\partial t} \quad \text{(One-dimensional equation)}$$

where $\hat{H}$ is the Hamiltonian operator, $\hat{E}$ is the energy operator, Ψ is the wave function, m is the mass of the particle, V is its potential energy, and $\hbar = \dfrac{h}{2\pi}$, h is the Planck's constant.

4.9 What is the physical significance of wave function?

The square of its absolute magnitude $|\Psi|^2$ evaluated at a particular point, at a particular time, is proportional to the probability of experimentally finding the particle there at that time.

4.10 Formulate the eigen functions for a particle in a one-dimensional box.

$$\Psi_n = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}$$

where Ψ_n is the eigen function, L is the width of the box, x is the coordinate of the particle, and n is the quantum number.

4.11 Formulate the eigen values for a particle in a one-dimensional box.

$$E_n = \frac{n^2 h^2}{8mL^2}$$

where E_n is the eigen value, L is the width of the box, n is the quantum number, m is the mass of the particle, and h is the Planck's constant.

4.12 Formulate the ground-state energies of the particles in a one-dimensional box and in a three-dimensional box.

$$E_1 = \frac{h^2}{8\,mL^2};\ E_{1,1,1} = \frac{3h^2}{8mL^2}$$

where E_1 and $E_{1,1,1}$ are the ground-state energies of the particle, L is the width of the box, m is the mass of the particle, and h is the Planck's constant.

4.13 What are the characteristic parameters of a microscope?
Magnifying power and resolving power

4.14 Define the magnifying power of a microscope.
The magnifying power M of a microscope is defined as the ratio between the angle subtended by the final image at the eye β to the angle subtended by the object at the eye α, placed at the near point.

4.15 Define the resolving power of a microscope.
It is the ability of a microscope to form distinct and separable images of the two point objects which are close to each other. The resolving power is given as

$$\text{Resolving power} = \frac{2NA}{\lambda}$$

where, NA is the numerical aperture of the objective of the microscope and λ is the wavelength of light through vacuum.

✓ EXERCISE PROBLEMS

4.1 Calculate the energy of the electron in the energy level immediately after the lowest energy level, confined in a cubical box of 0.1 mm side. $[E_{112} = 225.75\ eV]$

4.2 A particle moves in a one-dimensional box of width 10Å and of infinite height. The particle is assumed to be in its ground state. Calculate the probability of finding the particle in an interval of 1 Å at the centre of the box. $[P = 0.2]$

4.3 An electron is confined to an infinitely deep potential well of width 5 Å. Determine the lowest quantum energy that the electron can have. $[E_1 = 1.51\ eV]$

4.4 In Compton scattering, the angle of scattering is 90° and the incident photon wavelength is 2×10^{-10} m. Calculate the angle at which the recoil electron appears. $[f = 44°\ 57′]$

4.5 An X-ray is found to have its wavelength doubled on being scattered through 90°. Calculate the original wavelength of X-ray. $[\lambda' = 0.0242\ Å]$

4.6 An electron is confined to a box of length 10^{-12} m. Calculate the minimum uncertainty in its velocity.

$$[(\Delta Vn)_{min} = 7.38 \times 10^8\ m/s]$$

4.7 A classical particle of mass $m = 10^{-8}$ kg moves with a velocity of $V = 10^{-4}$ m/s. Find de Broglie's wavelength of the particle.

$$[\lambda = 6.6 \times 10^{-22}\ m]$$

4.8 An electron is confined to an infinitely deep potential well of width 9 Å. Determines the momentum of the ground-state electron. $[E_1 = 0.46\ eV,\ E_2 = 1.81\ eV]$

4.9 An electron has a speed of 250 m/s with an accuracy of 0.02%. Calculate the uncertainty with which we can locate the position of the electron. $[\Delta_n = 0.0145\ m]$

4.10 A ground-State energy particle is confined to a one-dimensional box of width 32 Å. Determine the probability of finding the particle in an interval of 3 Å from the centre of the box.

$$[P = 0.1]$$

QUESTION BANK

PART-A

4.1 Give any one draw back of classical mechanics.

4.2 Newtonian physics is an approximation of _____ .

4.3 According to quantum mechanics, the particle has _____ .

4.4 Who proposed that matter also exhibits dual characteristics?

4.5 Write the expression for the wavelength of the matter waves.

4.6 Give any one property of the matter waves.

4.7 Write the equation of Bragg's diffraction law.

4.8 What are the characteristic parameters of a microscope?

4.9 What are the two different forms of the Schrodinger's wave equation?

4.10 Write the general wave equation.

4.11 What is meant by the resolving power of a microscope?

4.12 Write Schrodinger's time-dependent wave equation in one dimension.

4.13 A particle is confined to a one-dimensional box of parameter L. The Minimum energy of the electron is

(a) $-\dfrac{h}{8mL^2}$ 　　　　　 (b) 0

(c) $\dfrac{n^2h^2}{32\pi^2 mL^2}$ 　　　　 (d) $\dfrac{h^2}{8\,mL^2}$

4.14 Give the normalisation condition for the wave function.

4.15 Write the expression for the eigen energy values of a particle in a one-dimensional box.

4.16 What is the use of microscopes?

4.17 The wavelength associated with a particle at rest is
(a) 0 　　　　　　　　 (b) slightly greater than 1
(c) infinity 　　　　　　 (d) less than 1

4.18 What are the classifications of microscopes?

4.19 What is the principle used in optical microscopes?

4.20 What are the two types of electron microscopes?

4.21 Give any one use of SEM.

4.22 The X-rays produced in a SEM are used to investigate the _____ properties.

4.23 A quantum particle is confined to move in a one dimensional potential box of width L. When the particle is in the ground state, the probability of finding the particle is maximum at
  (a) $X = 0$ (b) $x = a$
  (c) $x = L/2$ (d) $x = L$

4.24 Topography is the study of _____ .

4.25 Give any one similarity between SEM and TEM.

4.26 TEM is operated at high _____ .

4.27 Write the equation for Heisenberg's uncertainty principle.

4.28 Write Schrodinger's time-independent wave equation for a free particle.

4.29 The group velocity of de Broglie wave is equal to the velocity of _____ .

4.30 The equation of motion of matter waves was derived by _____ .

4.31 _____ is used to study disease-causing viruses.

4.32 Who developed quantum mechanics?

4.33 What is the symbolic representation of the wave function?

4.34 Give any one significance of wave function.

4.35 The group velocity of de Broglie wave is equal to the _____ .

4.36 The material observed under an optical microscope is usually _____ to light.

4.37 The highest magnification obtained with the best optical microscope is _____ .

4.38 In electron microscope, the electron exhibits wave properties like _____ .

4.39 Phase velocity of matter wave is greater than the _____ .

4.40 Derive de Broglie's equation.

4.41 Write the properties of matter waves.

4.42 State Heisenberg's uncertainty principle.

4.43 Give the significance of wave function.

4.44 Write the expression for energy eigen values and the corresponding eigen wave function of the particle in a one-dimensional box.

4.45 Define the degeneracy of states.

4.46 State the principle of electron microscope.

4.47 What are the essential components of TEM?

4.48 What is an electron gun?

4.49 Write the operational principle of TEM?

4.50 Show that the de Broglie wavelength for an electron is found to be equal $\dfrac{12.26}{\sqrt{V}} \text{Å}$ where V is the accelerating potential difference.

4.51 Write any two dissimilarities of optical and electron microscopes.

4.52 What is meant by the nondegenerate states?

4.53 Define the resolving of power of a microscope.

4.54 Formulate the eigen values for a particle in a three-dimensional box.

4.55 Formulate Schrodinger's time-independent wave equation in one dimension and three dimensions.

4.56 What is de Broglie hypothesis?

4.57 What are the similarities between optical and electron microscopes.

4.58 What are the merits and demerits of TEM over optical microscopes?

4.59 What is the role of electron gun in TEM?

4.60 Give the equation for the diffraction pattern due to a single slit.

4.61 What is the use of Schrodinger's wave equation?

4.62 Define the magnifying power of a microscope.

4.63 What is the operational principle of an optical microscope?

4.64 Draw the schematic diagram of an optical microscope and also mention the parts.

4.65 What are the drawbacks of TEM?

4.66 What is the information obtained from the back-scattered electrons in a SEM?

4.67 What is meant by the position probability density?

4.68 Prove that the phase velocity of matter waves is greater than the velocity of light in vacuum.

4.69 Write the two fundamental equations of quantum physics.

4.70 An electron is confined to an infinitely deep potential well of width 4 Å. Determine the momentum of the ground state electron.

PART-B

4.1 Derive Schrodinger's time-dependent and time-independent wave equations for matter waves.

4.2 Obtain the expression for eigen functions and eigen energy values of a particle in a one-dimensional potential well of infinite height.

4.3 Explain the construction and working of an electron microscope.

4.4 (a) Write an account on the duality exhibited by light as well as particle.

(b) Explain Heisenberg's uncertainty principle.

4.5 What are matter waves? Describe an experiment in support of the existence of matter waves.

4.6 What do you understand by the dual nature of matter? Explain de Broglie's hypothesis.

4.7 Derive Schrodinger's time-dependent wave equation. Give the physical significance of 'Ψ' used in this equation.

4.8 Discuss in detail G.P. Thomson experiment along with its conclusions.

ACOUSTICS

5.1 INTRODUCTION

Acoustics is the study of generation, propagation and reception of sound waves. Sound is defined as a mechanical disturbance travelling through a material so that it sets the human eardrum into motion. Thus, sound gives rise to the sensation of hearing. Sound requires a material medium to travel and it cannot travel through vacuum.

The normal human ear responds to sound waves only in the frequency range of 20 Hz to 20 kHz. Sound waves of still higher frequency are called ultrasonic waves. Those with frequencies less than 20 Hz are called infrasonic waves. We will primarily deal with sound waves in air. Sound waves can be categorised on various criteria.

Classification 1

Depending on frequency, sound waves are classified as
- Infrasonics (frequency < 20 Hz)
- Audible sound (20 Hz < frequency < 20 kHz)
- Ultrasonics (frequency > 20 kHz)

Classification 2

Based on physiological effect on the human ear, sound waves are classified as
- Musical sound
- Noise

Musical sound produces a pleasant effect on the human ear. Noise produces a jarring and unpleasant effect on the human ear.

5.2   CHARACTERISTICS OF MUSICAL SOUND

The wave form of musical sound has a regular shape, amplitude and period (Fig. 5.1). But, the waveform of noise has irregular shape, amplitude and period (Fig. 5.2). Musical sound is characterised by

1. Pitch (associated with frequency)
2. Quality (associated with timbre)
3. Loudness (associated with intensity)
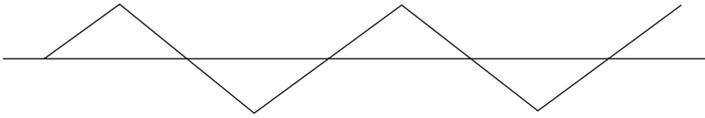
Fig. 5.1 Musical Sound
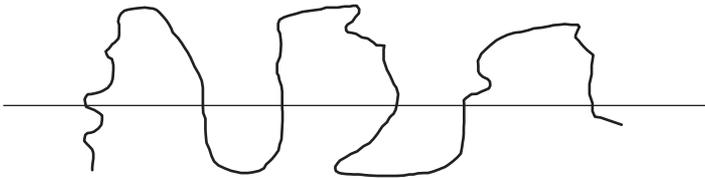
Fig. 5.2 Noise

1. Pitch

The characteristic property by virtue of which a sound appears to be grave or shrill is called pitch. It is determined by the frequency. With the increase in frequency, the pitch also increases to have a shrill sound. Similarly, with the decrease in frequency, the pitch decreases to have sound.

2. Quality

It is the property of sound which distinguishes two sounds of the same frequency and same loudness. The quality of sound depends on the overtones produced by source. In fact, by virtue of quality we are able to distinguish the sounds produced by different musical instruments.

3. Loudness

The characteristic property due to which a sound appears to be feeble or loud is called loudness. According to **Weber–Fechner law**, the

loudness L of a sound is proportional to the logarithm of its intensity I. That is,

$$L = k \log I \tag{5.1}$$

where k is the proportionality constant.

4. Intensity

It is defined as the sound energy flowing normally across unit area per second. Its unit of measurement is **watt/m^2**. Intensity of sound is empirically given by

$$I = 2(\pi v a')^2 v \rho \tag{5.2}$$

where v is the frequency of sound, a' is its amplitude, v is its velocity and ρ is the density of the medium.

Consider two sounds of intensities I_1 and I_2 and of loudness L_1 and L_2. Then, the Weber–Fechner law gives the difference in loudness as

$$\Delta L = L_1 - L_2 = k \log\left(\frac{I_1}{I_2}\right) \tag{5.3}$$

As $k = 1$ in SI units, Eq. (5.3) gives

$$\Delta L = \log\left(\frac{I_1}{I_2}\right) \text{bel} \tag{5.4}$$

The difference in loudness is expressed in the unit of **bel**. As bel is a big unit, a smaller unit (one-tenth of a bel) called **decibel (dB)** is usually employed. That is,

$$\Delta L = 10 \log\left(\frac{I_1}{I_2}\right) \text{dB} \tag{5.5}$$

Note
Frequency, timbre and intensity of a sound wave are physical quantities which can be measured. But, pitch, quality and loudness of a sound wave are physiological quantities which may be differently recognised by different persons.

5.3 ● SOUND INTENSITY MEASUREMENT

The lowest audible sound intensity (i.e., $I_0 = 10^{-12}$ watt/m^2 at 1000 Hz) to which a normal human ear responds is taken as the standard intensity. With reference to this, the intensity of a sound is measured.

The ratio of the intensity of a sound to the standard intensity is called the **intensity level or relative intensity of the sound**. Its unit of measurement is bel. Then, according to the Weber–Fechner law, the intensity level is

$$\Delta L = \log(I/I_0) \text{ bel} \tag{5.6}$$

Thus, **bel is defined as the intensity level of a sound having an intensity ten times the standard intensity.**

$$\Delta L = 10 \log(I/I_0) \text{ dB} \tag{5.7}$$

From Eq. (5.7), it follows that 1 dB corresponds to $\log (I/I_0) = 0.1$ or $I/I_0 = 1.26$. That is, 26% change in intensity corresponds to 1 dB of difference in intensity level. Table 5.1 illustrates the intensity levels of some typical sound signals.

Table 5.1   Intensity Levels of Selected Sound Signals

Sound signal	Intensity level (dB)
Motor car	30
Conversation	60–70
Busy traffic	70–80
Thunder	100–110
Painful sound	>120

5.4   SOUND LOUDNESS MEASUREMENT

Sounds of different frequencies having apparently equal loudness to a person are found to have different intensities. Taking this fact into account, another unit called **phon** is used to measure the loudness level or equivalent loudness.

Loudness level of a sound is measured with reference to a standard source producing a tone of frequency 1000 Hz. The loudness level of a sound is said to be one phon, if the intensity level of a standard tone with equal loudness is 1 dB above the standard intensity.

Sone is the unit of measurement of loudness. **Sone is defined as the loudness of 1000 Hz tone of 40 dB intensity level**. It is equal to the loudness of a sound having a loudness level of 40 phons. An empirical equation gives

$$\log L = 0.033 \, (LL\text{-}40) \tag{5.8}$$

where L is the loudness in sones and LL is the loudness level in phons.

For a frequency of 1000 Hz, the loudness level in phons is numerically equal to the intensity level in decibel. Therefore, Eqs (5.7) and (5.8) give

$$L = 445\ I^{1/3}, \text{ for a 1000 Hz tone} \qquad (5.9)$$

5.5 REVERBERATION

When produced in a hall, a pulse of sound experiences multiple reflections by various objects present in the hall. As a result, the listener receives a series of sound waves of decreasing intensities. This persistence of sound is called **reverberation**. The time required by the sound intensity to fall below the audible limit, after source is switchedoff is called **reverberation time**.

5.6 SABINE'S FORMULA FOR REVERBERATION TIME

According to Sabine, reverberation time is defined as the time taken by the sound intensity to get reduced by a factor 10^6 from its maximum steady-state value.

Sabine's formula gives the reverberation time of a hall as

$$T = \frac{0.167\ V}{A} \qquad (5.10)$$

where V is the volume of the hall and $A = \sum_i a_i S_i$ is the total absorption of sound in the hall. Here, a_i and S_i refer to the absorption coefficient and the surface area of a material in the hall.

5.7 DERIVATION OF SABINE'S FORMULA

Let a source in a hall produce sound satisfying the following assumptions:

- Constant rate of sound energy production
- Uniform distribution of sound energy
- Absence of interference of sound waves
- Absorption coefficient is independent of sound intensity
- Absence of energy loss when the sound propagates through air

Let E be the sound energy density in the hall. Consider a small element dS on a plane wall AB (Fig. 5.3). Let us determine the amount of sound energy received by the element dS. For this, two concentric semicircles of radii r and $r + dr$ are constructed.
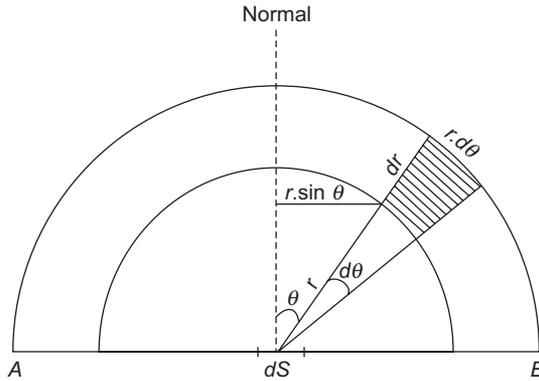
Fig. 5.3 Derivation of Sabine's Formula

Consider a small region (shaded in Fig. 5.3) lying between the radial lines making the angles of θ and $\theta + d\theta$ with the normal to dS.

$$\text{Arc of the shaded region} = (r + dr)\, d\theta$$
$$= rd\theta \text{ (since } dr \text{ and } d\theta$$
$$\text{are very small)}$$

$$\text{Radial length of the shaded region} = dr$$
$$\therefore \qquad \text{Area of the shaded region} = rdrd\theta$$

Let the shaded region be rotated about the normal through an angle $d\phi$.

Then, the distance travelled by the shaded region $= r \sin \theta\, d\phi$

Therefore, volume traced out by this area element in its rotation is

$$dV = rdrd\theta.\, r \sin \theta\, d\phi = r^2 \sin \theta\, d\theta\, dr\, d\phi$$

Sound energy contained in the volume element dV is

$$E_1 = Er^2 \sin \theta\, d\theta\, dr\, d\phi$$

Therefore, sound energy in the volume element dV per unit solid angle is

$$E_1 = \frac{Er^2 \sin \theta\, d\theta\, dr\, d\phi}{4\pi}$$

On the other hand, solid angle subtended by dS at the volume element dV is

$$d\omega = \frac{dS \cos\theta}{r^2}$$

Therefore, sound energy reaching dS from the volume element dV is

$$E_1 = \frac{Er^2 \sin\theta d\theta dr d\phi}{4\pi} \times \frac{dS \cos\theta}{r^2}$$

$$E_1 = \frac{EdS}{4\pi} \times dr \sin\theta \cos\theta d\theta d\phi$$

Total sound energy received by the element dS per second is

$$E_2 = \frac{EdS}{4\pi} \int\limits_0^C dr \int\limits_0^{\pi/2} \sin\theta \cos\theta d\theta \int\limits_0^{2\pi} d\phi$$

where C is the sound velocity.

$$E_2 = \frac{EdS}{4\pi} 2\pi \int\limits_0^C dr \int\limits_0^{\pi/2} \sin\theta \cos\theta d\theta$$

i.e., $\dfrac{EdS}{4}\int\limits_0^C dr = \dfrac{EdSC}{4\pi}$, since $\int\limits_0^{\pi/2} \sin 2\theta d\theta = 1$

Let 'a' be the absorption coefficient of the wall AB. Then, sound energy absorbed by the element dS per second is

$$E_3 = \frac{EdSCa}{4}$$

Total sound energy absorbed per second by all the materials of the wall is

$$E_4 = \frac{EC\sum adS}{4} = \frac{ECA}{4} \tag{5.11}$$

where $A = \Sigma adS$ is the total absorption of sound energy.

1. Growth of Sound Energy

Let V be the volume of the hall. Then, total sound energy in the hall is EV.

Rate of growth of sound energy is

$$\frac{d}{dt}(EV) = V\frac{dE}{dt}$$

At any instant of time, we have

Rate of growth of sound energy = Rate of energy supply by the source − Rate of energy absorption by surfaces of the wall

i.e.,

$$V\frac{dE}{dt} = P - \frac{ECA}{4} \tag{5.12}$$

where P is the power output of the source. At the steady state, we have

$$\frac{dE}{dt} = 0$$

Then, Eq. (5.12) gives the steady state energy density as

$$E_m = \frac{4P}{CA} \tag{5.13}$$

From Eq. (5.12), we get

$$\frac{dE}{dt} = \frac{P}{V} - \frac{ECA}{4V}$$

$$\frac{dE}{dt} = \frac{4P\alpha}{CA} - \alpha E$$

where $\alpha \equiv \dfrac{CA}{4V}$.

i.e.,

$$\frac{dE}{dt} + \alpha E = \frac{4P\alpha}{CA}$$

Multiplying the above equation by $e^{\alpha t}$, we get

$$\left(\frac{dE}{dt} + \alpha E\right)e^{\alpha t} = \frac{4P\alpha e^{\alpha t}}{CA}$$

i.e.,

$$\frac{d}{dt}(Ee^{\alpha t}) = \frac{4P\alpha e^{\alpha t}}{CA}$$

Integrating the above equation, we get

$$Ee^{\alpha t} = \frac{4P}{CA}e^{\alpha t} + K \tag{5.14}$$

where K is the constant of integration which can be determined through the initial conditions.

As $E = 0$ at $t = 0$, we get $K = -\dfrac{4P}{CA}$

Then, Eq. (5.14) may be rewritten as

$$E = \frac{4P}{CA}(1 - e^{-\alpha t})$$

or $\qquad E = E_m(1 - e^{-\alpha t})$ $\qquad\qquad$ (5.15)

Equation (5.15) expresses the exponential growth of sound energy with time and it is illustrated in Fig. 5.4. At $t = \infty$, Eq. (5.15) gives $E = E_m$.
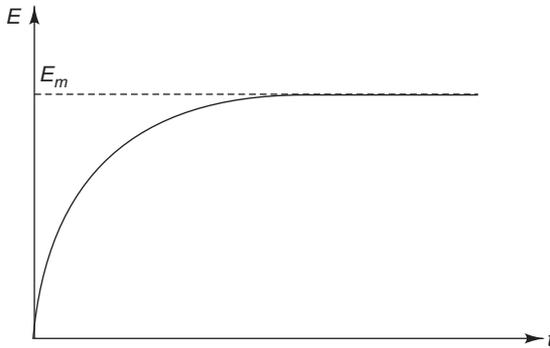
Fig. 5.4 Growth of Sound Energy

2. Decay of Sound Energy

Let the source be switched off when $E = E_m$. Thus, we have $P = 0$ at $t = 0$. Then, Eq. (5.14) gives

$$K = E_m$$

Therefore, Eq. (5.14) gives

$$E e^{\alpha t} = E_m$$

or $\qquad E = E_m e^{-\alpha t}$ $\qquad\qquad$ (5.16)

Equation (5.16) describes the exponential decay of sound energy density with time, when the source is switched off. The decay of sound energy is illustrated in Fig. 5.5.
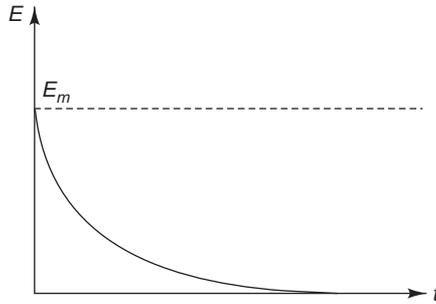
Fig. 5.5 Decay of Sound Energy

3. Reverberation Time

Sabine defines the reverberation time T as the time taken by a sound to have its intensity decreased by a factor of 10^6 from its steady state value, when the source is cut off. That is,

$$\frac{E_m}{E} = 10^6$$

Hence, From Eq. (5.16), we get

$$e^{\alpha T} = 10^6$$

Taking logarithm on both sides, we have

$$\alpha T = 6 \ln 10$$

Substitution of the expression for α into the above equation gives

$$\frac{CAT}{4V} = 6 \ln 10$$

i.e., $$T = \frac{24 \ln 10}{CA} V$$

Taking the sound velocity in air as $C = 330$ m/s, we may rewrite the above equation as

$$T = \frac{0.167V}{A} \tag{5.17}$$

or $$T = \frac{0.167V}{\sum a_i S_i} \tag{5.18}$$

where a_i and S_i refer to the absorption coefficient and the surface area of a material in the hall.

5.8 ABSORPTION COEFFICIENT

The absorption coefficient of a material depends on the frequency of the incident sound. Different materials have different absorption coefficients. Considering the interior design and the required reverberation time, appropriate absorbing materials are to be used to have an acoustically good hall. The absorption coefficient of a material may be defined in two ways.

Definition 1

The absorption coefficient of a material is defined as

$$a = \frac{I_a}{I_i} \tag{5.19}$$

where I_a is the absorbed intensity of sound and I_i is the incident intensity of sound.

Definition 2

The absorption coefficient of a surface is defined as the reciprocal of its area which absorbs the same amount of sound energy as absorbed by the unit area of an open window. The absorption coefficient is measured in **Open Window Unit (OWU)**. The absorption coefficients of some selected materials are presented in Table 5.2.

Table 5.2   Absorption Coefficients of Selected Materials

Material	Absorption coefficient (OWU)
Marble	0.01
Glass	0.02
Wood	0.06
Carpet	0.15 – 0.3
Brick wall	0.03

5.9 DETERMINATION OF ABSORPTION COEFFICIENT

The reverberation time T in a hall of volume V is measured. Let A be the total absorption in hall. A sample of material for which absorption

coefficient '*a*' is to be measured is taken. Placing the sample in the hall, now, the reverberation time T_1 is measured.

When applied to these cases, Sabine's formula gives

$$T = \frac{0.167V}{A} \tag{5.20}$$

$$T_1 = \frac{0.167V}{A + A_1} \tag{5.21}$$

where $A_1 = as$ and s is the surface area of the sample.

From Eqs (5.20) and (5.21), we get

$$\frac{1}{T_1} - \frac{I}{T} = \frac{A_1}{0.167V}$$

$$A_1 = 0.167V\left(\frac{1}{T_1} - \frac{1}{T}\right)$$

or

$$as = 0.167V\left(\frac{1}{T_1} - \frac{1}{T}\right)$$

i.e.,

$$a = \frac{0.167V}{s}\left(\frac{1}{T_1} - \frac{1}{T}\right) \tag{5.22}$$

Using Eq. (5.22), the absorption coefficient of the material can be determined.

5.10   ACOUSTICS OF BUILDINGS

Acoustics of buildings deals with the design and construction of acoustically good buildings, viz., hall, auditorium, etc. An acoustically good hall should have an optimum reverberation time, no focusing of sound, no interference of sound, no resonance effect, no echoes of sound and no extraneous noise. To ensure these qualities, the design and construction of a building should take into account the factors such as

- Reverberation time
- Loudness of sound
- Echelon effect
- Focusing and interference of sound
- Resonance
- Echo
- Creep
- Noise

1. Reverberation

It refers to the persistence of sound after switching off the source. Sabine's formula gives the reverberation time as

$$T = \frac{0.167V}{\sum_i a_i S_i} = \frac{0.167V}{A}$$

where V is the volume of hall; a_i and S_i are the absorption coefficient of a material and its surface area, respectively; and A is the total absorption.

Thus, by providing appropriate total absorption, optimum reverberation time can be ensured. Typically, the optimum value of reverberation time for music and speech are 0.5 to 1 seconds and 1 to 2 seconds, respectively. The provisions which ensure the optimum reverberation time are

- Windows and ventilators
- Sound absorbing materials on the inner surface of the building
- Curtains
- Cushions on the seats
- False ceilings

2. Loudness

To ensure uniform loudness in the hall, amplifying units and loud speakers are used. False ceilings also enhance uniform loudness. Loudness can be increased by keeping plane polished boards above the speaker and by providing racked seating arrangement.

3. Echelon Effect

When regular structures such as stairs are in the hall, the listener receives regular and successive echoes produced by these structures. This is called the echelon effect. It results in poor clarity of sound.

Remedy
- Avoiding such structures
- Covering such structures with carpets

4. Focusing and Interference

Cylindrical or spherical surface of the walls or of the ceiling lead to undesirable focusing of sound. As a result, sound intensity is not uniformly distributed in a hall.

Moreover, direct and reflected sound waves may produce interference. As a result, sound intensity is minimum in some places and it is maximum in other places. Besides, the superimposition of direct and reflected sound waves may produce a stationary wave pattern. This leads to uneven distribution of sound intensity.

Remedy

- Avoiding such structures
- Covering such structures with sound absorbing materials

5. Resonance

Hollows and crevices select their natural frequencies from the sound produced in the hall and reinforce them. It leads to a jarring effect. The magnitude of resonance is inversely proportional to the square root of the hall volume. Thus, in a large hall, the resonance effect is insignificant.

Remedy

- Covering cylindrical structures with sound absorbing materials
- Providing convex structures in the hall

6. Echo

It is the sound wave reflected and returned by a distant surface. This delayed sound wave has sufficient intensity. An echo is distinct from the direct sound wave.

7. Creep
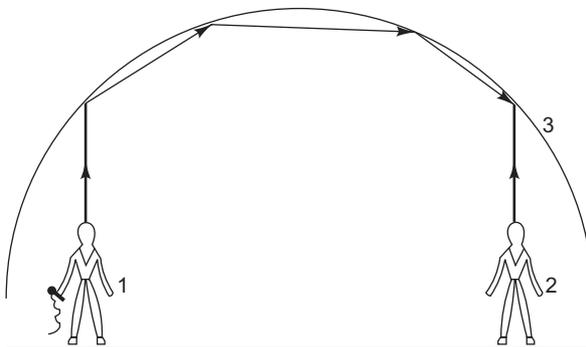
1. Speaker, 2. Listener, 3. Curved surface

Fig. 5.6   Occurrence of Creep

It occurs due to the reflection of sound along a curved surface (Fig. 5.6). Creep is produced when the sound source is located close to curved surfaces like domes.

Remedy for Echo and Creep Covering curved surfaces with sound absorbing materials

8. Noise

It adversely affects the acoustics of a hall. The noise may be extraneous (air-borne noise and structure-borne noise) or inside noise.

9. Air-borne Noise

The noise from outside reaches the hall through open windows, doors, and ventilators. This is called the air-borne noise.

Remedy

- Providing double doors and windows with sound insulating material between them
- Keeping the doors and windows closed. Then, the hall should be air-conditioned

10. Structure-borne Noise

The sound conveyed through the building structure is called structure-borne noise.

Remedy

- Providing double walls with air in between them
- Breaking the continuity in the structure by sound insulating materials

11. Inside Noise

The working machineries in a hall such as fan, air cooler, air conditioner, typewriter, etc., produce a noise called inside noise.

Remedy

- Covering the floor with carpets
- Covering the walls and ceiling with sound absorbing materials
- Placing the working machineries on absorbent pads

SUMMARY

- ▶▶ Acoustics is the study of generation, propagation and reception of sounds waves.
- ▶▶ Depending on frequency sound waves are classified as infrasonics, audible sound and ultrasonics.
- ▶▶ Based on physiological effect on the human ear, sound waves are classified as musical sound and noise.
- ▶▶ Musical sound produces pleasant effect on the human ear.
- ▶▶ Noise produces unpleasant effect on the ear.
- ▶▶ Pitch, quality and loudness are the characteristics of musical sound.
- ▶▶ Pitch is associated with frequency.
- ▶▶ Quality is associated with timbre.
- ▶▶ Loudness is associated with intensity.
- ▶▶ Frequency, timbre and intensity are physical quantities.
- ▶▶ Pitch, quality and loudness are physiological quantities.
- ▶▶ Intensity is defined as the sound energy flowing normally across unit area per second.
- ▶▶ The ratio intensity of a sound to standard intensity is called intensity level.
- ▶▶ The intensity level of a sound is measured in Bel or decibel.
- ▶▶ Weber-Fechner law relates the loudness and intensity of a sound.
- ▶▶ The loudness of a sound is proportional to the logarithm of its intensity is Weber-Fechner law
- ▶▶ The absorption coefficient of a material is defined as the ratio of sound energy absorbed by the material to the ratio of sound energy incident on the material.
- ▶▶ Different materials have different absorption coefficients.
- ▶▶ Absorption coefficient is measured by OWU or Sabine.
- ▶▶ Sabine's formula is used to calculate reverberation time.
- ▶▶ Reverberation time, loudness, echo, echelon effect, resonance, creep, focusing, interference and noise are the factors that affect the acoustics of buildings.

✓ SOLVED PROBLEMS

5.1 How does the intensity level of a sound wave change when its amplitude is increased by a factor of 3?

Solution   Given Data

$$\frac{I_2 = 9I_1}{\Delta L = ?}$$

Intensity level of sound is given by

$$L = 10\log\left(\frac{I}{I_0}\right) \text{ dB}$$

where $I_0 = 10^{-12}$ W/m^2 is the reference intensity.
Then, the initial intensity level is

$$L_1 = 10\log\left(\frac{I_1}{I_0}\right) \text{ dB} \tag{1}$$

Similarly, the final intensity level is

$$L_2 = 10\log\left(\frac{I_2}{I_0}\right) \text{ dB} \tag{2}$$

From Eqs (1) and (2), we get the change in the intensity level as

$$\Delta L = L_2 - L_1 = 10\log\left(\frac{I_2}{I_1}\right) \text{ dB} \tag{3}$$

Substituting the given data into Eq. (3), we get

$$\Delta L = 10 \log 9$$

i.e.,   **ΔL = 9.54 dB**

Thus, the intensity level of the sound increases by 9.54 dB.

5.2 The sound from a machine produces an intensity level of 100 decibels at a point a few metres away. Determine the intensity level at this point when two such machines are producing sound at the same distance away.

Solution   Given Data

$$\Delta L = 100 \text{ dB}$$

$$\frac{N = 2}{\Delta L' = ?}$$

Intensity level of sound is given by

$$\Delta L = 10 \log (I/I_0) \tag{1}$$

where I is the intensity of sound; $I_0 = 10^{-12} \text{ W/m}^2$ is the standard intensity.

In the case of one machine producing sound, we have

$$\Delta L = 100 \text{ dB}$$

i.e., $10 \log (I/I_0) = 100 \text{ dB}$ \hfill (2)

In the case of two machines producing sound, we have

$$\Delta L' = 10 \log (2I/I_0)$$

i.e., $\Delta L' = 10 \log 2 + 10 \log (I/I_0)$ \hfill (3)

From Eqs (2) and (3), we get

$$\Delta L' = 10 \log 2 + 100$$

i.e.,   **$\Delta L' = 103.01$ dB**

5.3   A source produces sound of frequency 500 Hz and of amplitude 5 mm. The velocity of sound in air is 340 m/s and the density or air is 1.29 kg/m^3. Determine the flow of sound energy across 1 m^2 per second.

Solution   Given Data

$$v = 500 \text{ Hz}$$
$$a' = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$
$$v = 340 \text{ m/sec}$$

$$\frac{\rho = 1.29 \text{ kg/m}^3}{I = ?}$$

Intensity of sound is given by

$$I = 2 (\pi v a')^2 v \rho \tag{1}$$

Substituting the given data into Eq. (1), we get

$$I = 2 \times (3.14 \times 500 \times 5 \times 10^{-3})^2 \times 340 \times 1.29$$

i.e., $\qquad$ **$I = 5.406 \times 10^4$ W/m^2**

5.4 A music hall of 7000 m^3 volume is expected to have a reverberation time of 1.4 seconds. Determine the total absorption to be provided in the music hall.

Solution Given Data

$$V = 7000 \text{ m}^3$$

$$\frac{T = 1.4 \text{ s}}{A = ?}$$

According to the Sabine's formula, we have

$$T = \frac{0.167V}{A}$$

Therefore, the total absorption in the hall is

$$A = \frac{0.167V}{T} \qquad (1)$$

Substituting the given data into Eq. (1), we get the total absorption to be provided in the hall as

i.e., $\qquad$ **$A = 835$ Sabine**

5.5 A room of 1000 m^3 volume has a wall area of 400 m^2, a floor area of 100 m^2 and a ceiling area of 100 m^2. The sound absorption coefficients of the wall, floor and ceiling are 0.02, 0.1 and 0.05 OWU, respectively. Determine the reverberation time in the room.

Solution Given Data

$$V = 1000 \text{ m}^3$$
$$S_1 = 400 \text{ m}^2$$
$$a_1 = 0.02 \text{ OWU}$$
$$S_2 = 100 \text{ m}^2$$
$$a_2 = 0.1 \text{ OWU}$$
$$S_3 = 100 \text{ m}^3$$

$$a_3 = 0.05 \, \text{OWU}$$
$$T = ?$$

According to Sabine's formula, we have

$$T = \frac{0.167V}{A} = \frac{0.167V}{\sum a_i S_i} \tag{1}$$

Substituting the given data into Eq. (1), we get the reverberation time as

$$T = \frac{0.167 \times 1000}{(0.02 \times 400) + (0.1 \times 100) + (0.05 \times 100)}$$

i.e.,   **T = 7.26 s**

5.6 If the intensity of a source of sound is increased to 20 times its value, by how many decibel does the intensity level increases?

(A.U, B.E/B.Tech, Jan. 2006)

Solution   Given Data

$$I_2 = 20 I_1$$
$$\Delta L = ?$$

The change in the intensity level of sound is

$$\Delta L = 10 \log\left(\frac{I_2}{I_1}\right) \text{ dB} \tag{1}$$

Substituting the given values for I_1 and I_2 into Eq.(1), we get

$$\Delta L = 10 \log\left(\frac{20 I_1}{I_1}\right) \text{ dB}$$

$$\Delta L = 10 \log (20) \text{ dB}$$

i.e.,   **ΔL = 13.01 dB**

That is, the intensity level of sound increases by 13.01 dB.

5.7 The average reverberation time of a hall is 1.5 seconds and the area of the inner surface is 3340 m². If the volume of the hall is 12000 m³, find the absorption coefficient.   *(A.U, B.E/B.Tech, Jan. 2005)*

Solution   Given Data

$$T = 1.5 \text{ s}$$

$$S = 3340 \text{ m}^2$$

$$\frac{V = 12000 \text{ m}^3}{a = ?}$$

The Sabine's formula for reverberation time is

$$T = \frac{0.167V}{aS}$$

i.e., $$a = \frac{0.167V}{ST} \qquad (1)$$

Substituting the given values for *V*, *S* and *T* into Eq. (1), we get

$$a = \frac{0.167 \times 12000}{3340 \times 1.5}$$

$$a = \textbf{0.4 OWU}$$

QUESTIONS AND ANSWERS

5.1   How are sound waves classified?

(A.U, B.E/B.Tech, May/June 2005)

Depending on the frequency, sound waves are classified as infrasonics, audible sound and ultrasonics. Moreover, based on the physiological effect on the human ear, sound waves are classified as musical sound and noise.

5.2   What is meant by musical sound?

Musical sound is a sound having the waveform of regular shape, amplitude and period. It produces a pleasing effect on the human ear.

5.3   What is meant by noise?

Noise is a sound having the waveform of irregular shape, amplitude and period. It produces an unpleasant effect on the human ear.

5.4   Define audible sound.

Sound wave of frequencies in the range of 20 Hz to 20 kHz is called audible sound.

5.5 What are the characteristics of musical sound?
Pitch, Quality and Loudness.

5.6 Define intensity of sound. *(A.U, B.E/B.Tech, Jan. 2006)*
Intensity of sound is defined as sound energy flowing normally across unit area per second. Its unit of measurement is W/m^2.

5.7 Define pitch of sound.
The characteristic property by virtue of which a sound appears to be grave or shrill is called the pitch. It is determined by the frequency.

5.8 Define quality of sound.
It is the property of sound which distinguishes two sounds of the same frequency and same loudness. It depends on the overtones produced by the sound.

5.9 Define loudness of sound.
The characteristic property due to which a sound appears to be feeble or loud is called the loudness. According to Weber–Fechner law, the loudness L of a sound is proportional to the logarithm of its intensity I. That is, $L = k \log I$ where k is the proportionality constant.

5.10 Mention the unit of measurement of loudness.
Bel or decibel is the unit of measurement of loudness.

5.11 Define intensity level of sound.
The ratio of the intensity of a sound to the standard intensity is called the intensity level or relative intensity of the sound.

5.12 State the Weber–Fechner law. *(A.U, B.E/B.Tech, Jan. 2005)*
According to the Weber–Fechner law, the loudness L of a sound is proportional to the logarithm of its intensity I. That is, $L = k \log I$ where k is the proportionality constant.

5.13 Define bel.
Bel is defined as the intensity level of a sound having the intensity ten times the standard intensity.

5.14 Define decibel.
It is defined as one tenth of the intensity level of a sound having the intensity ten times the standard intensity. It is given as
$$\Delta L = 10 \log(I/I_0) \, \text{dB}$$
where I is the intensity of sound and I_0 is the standard intensity.

5.15   Define phon.
The loudness level of a sound is said to be one phon, if the intensity level of a standard tone with equal loudness is 1 dB above the standard intensity.

5.16   Define sone.
Sone is defined as the loudness of 1000 Hz tone of 40 dB intensity level.

5.17   What is meant by acoustics of buildings?
Acoustics of buildings deals with the design and construction of acoustically good buildings.

5.18   How is sufficient loudness of sound achieved in a hall?
To ensure sufficient loudness in the hall, amplifying units and loudspeakers are used. False ceilings also enhance uniform loudness. Loudness can be increased by keeping plane polished boards above the speaker and by providing racked seating arrangement.

5.19   How is the absence of echoes ensured in a hall?
By covering the curved surfaces with sound absorbing materials.

5.20   What is meant by the echelon effect? Give the remedial measures for controlling it.
When regular structures such as stairs are in the hall, the listener receives regular and successive echoes produced by these structures. This is called the echelon effect. It results in poor clarity of sound.

Remedy
* By avoiding such structures
* By covering such structures with carpets

5.21   How can the focusing and interference of sound in a hall be avoided?
* By avoiding cylindrical and spherical structures
* By covering such structures with sound absorbing materials

5.22   What is meant by resonance in a hall? How can it be controlled?
Hollows and crevices select their natural frequencies from the sound produced in the hall and reinforce them. It leads to a jarring effect.

Remedy
- By covering cylindrical structures with sound absorbing materials
- By providing convex structures in the hall

5.23   What is creep in a hall? How can it be controlled?

It occurs due to the reflections of sound along a curved surface. Creep is produced when the sound source is located close to curved surfaces like domes.

5.24   What is air-borne noise in a hall? How can it be controlled?

The noise from outside reaches the hall through open windows, doors, and ventilators. This is called the air-borne noise.

Remedy
- By providing double doors and windows with sound insulating material between them
- By keeping the doors and windows closed. Then, the hall should be air-conditioned

5.25   What is structure-borne noise in a hall? How can it be controlled?

The sound conveyed through the building structure is called structure-borne noise.

Remedy
- By providing double walls with air in between them
- By breaking the continuity in the structure by sound insulating materials

5.26   What is inside noise in a hall? How can it be controlled?

The noise produced by the working machineries in the hall such as fan, air cooler, air conditioner, typewriter, etc., is called the inside noise

Remedy
- By covering the floor with carpets
- By covering the walls and ceiling with sound absorbing materials
- By placing the working machineries on absorbent pads

5.27   What is meant by reverberation?

When produced in a hall, a pulse of sound experiences multiple reflections by various objects present in the hall. As a result, the

listener receives a series of sound waves of decreasing intensities. This persistence of sound is called reverberation.

5.28   Define reverberation time.   *(A.U, B.E/B.Tech, May/June 2005)*
Reverberation time is defined as the time taken by the sound intensity to get reduced by a factor 10^6 from its maximum steady-state value.

5.29   Give Sabine's formula for reverberation time.
Sabine's formula gives the reverberation time of a hall as

$$T = \frac{0.167V}{A}$$

where V is the volume of the hall and A is the total absorption of sound in the hall.

5.30   Define absorption coefficient of a material.
(A.U, B.E/B.Tech, May/June 2005)
The absorption coefficient of a material is defined as the ratio of the sound energy absorbed by the material to the sound energy incident on the material.

$$a = \frac{I_a}{I_i}$$

where, I_a is the absorbed intensity of sound and I_i is the incident intensity of sound.

5.31   Define absorption coefficient of a material in terms of Open Window Units.
The absorption coefficient of a surface is defined as the reciprocal of its area which absorbs the same amount of sound energy as absorbed by the unit area of an open window.

EXERCISE PROBLEMS

5.1   The intensity levels due to two sound waves of the same frequency are 25 dB and 60 dB. Determine the ratio of their amplitudes.   *[$A_2/A_1 = 56.23$]*

5.2   Determine the change in the loudness when the sound experiences a 40% change in its intensity.   *[$\Delta L = 1.46dB$]*

5.3   The intensity of a sound is 7×10^{-10} W/m². Determine its intensity level, if the intensity of reference sound is 10^{-12} W/m².   *[$L = 2.845\ dB$]*

5.4 The intensity of sound in a busy street is 10×10^{-5} W/m² Calculate the intensity level of the sound. $[\Delta L = 80\ dB]$

5.5 A hall of volume 10000 m³ has a total absorption of 2200 Sabine. When filled with audience, the total absorption of the hall increases by another 2200 Sabine. Determine the change in the reverberation time. $[T_1 - T_2 = 0.380\ seconds]$

5.6 A hall of volume 8000 m³ is expected to have a reverberation time of 1.5 seconds. Determine the required total absorption in the hall. $[A = 891\ Sabine]$

6

ULTRASONICS

6.1   INTRODUCTION

Sound waves are mechanical disturbances in an elastic medium. On the basis of frequency, they are classified into audible, infrasonic and ultrasonic waves. The human ear can hear sound waves between 20 Hz and 20 kHz. This range is known as audible range. Sound waves which have frequencies less than the audible range are called infrasonic waves. Sound waves whose frequency lies above the audible frequency of 20 kHz are called ultrasonic waves. Normally they are called high frequency waves and are not audible to the human ear. As ultrasonic waves have very high frequencies, it follows that they also have very short wavelengths. As a result, ultrasonic waves can be focused in narrow, straight beams and also exhibit some unique phenomena in addition to the properties of the audible sound waves. The compressibility (k) and density of a material (ρ), combined with the laws of conservation of mass and momentum, directly imply the existence of acoustic waves. Ultrasound waves travel at a speed of sound c, given by $c = 1/\rho k$. The number of applications for ultrasound seems to be limited only by human imagination. There are literally dozens of ways that people have already found to make use of ultrasound. The first unsuccessful practical application in 1912 was the search for Titanic. World War II brought massive military research of SONAR. In 1940 they were used for nondestructive testing of materials. In 1942 they were used as a diagnostic tool for localising brain tumors. 2D gray scale images were obtained by using ultrasonic waves in 1950, and in 1965 for real-time imaging.

6.1.1   Characteristic Properties of Ultrasonics

- They have negligible diffraction due to their small wavelengths.
- They are highly energetic.

- Their speed of propagation depends on the frequency, i.e., speed increases with increase in frequency.
- They have high penetrating power.
- Intense ultrasonics produce cavitation (i.e., tiny bubbles) in liquids.
- A diffraction grating is formed when ultrasonics pass through a liquid.

6.1.2 Production of Ultrasonic Waves

Ultrasonic waves can be produced by various methods. These methods are shown in Table 6.1.

Table 6.1

Production Method	Principle Used	Maximum Available Frequency (kHz)
Galton's whistle	Principle of organ pipe	30
Magnetostrictive transducer	Magnetostriction	100
Piezoelectric transducer	Converse piezoelectric effect	500×10^3

6.1.3 Magnetostriction Effect

In 1847, Joule discovered this effect. In a ferromagnetic material like Ni, Co, Fe, etc., the magnetic moments are aligned in the same direction along the direction of the applied magnetic field. Under this condition, the material end close to the north pole of the magnet will be the south pole and vice versa. When an alternating magnetic field is applied, magnetic moments alternatively reorient along the field direction, which leads to the contraction and elongation of the material. This phenomenon is known as magnetostriction.
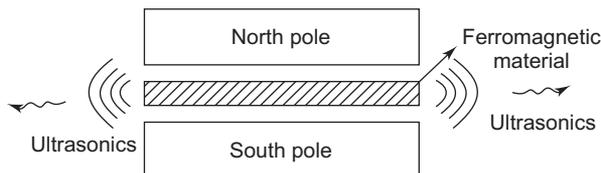
Fig. 6.1 Magnetostriction Effect

The change, i.e., either increase or decrease in the length of the rod depends upon the nature of the material and the strength of the applied magnetic field. The change in length is of the value of one part in a million.

6.1.4   Piezoelectric Effect

When a piezoelectric crystal is subjected to mechanical stress on the opposite faces, electric charges appear on the other pair of the opposite faces of the crystal. This effect is called the piezoelectric effect. The crystal which produce piezoelectric effect are called piezoelectric crystals. Quartz, rochelle salt and zinc blende are few examples of piezoelectric crystals.
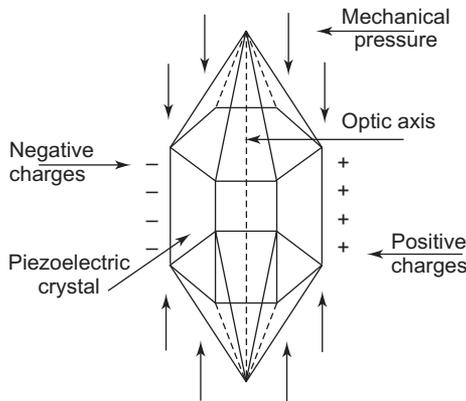
Fig. 6.2   Piezoelectric Effect

6.1.5   Inverse Piezoelectric Effect

The piezoelectric effect is reversible. When an electric field is applied on the opposite faces of a quartz crystal, alternative mechanical deformation, i.e., expansion or contraction is produced across the other pair of opposite faces of the crystal. This effect is known as inverse piezoelectric effect.
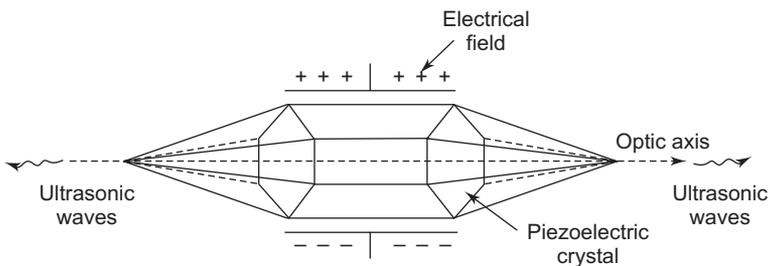
Fig. 6.3   Inverse Piezoelectric Effect

6.1.6   Piezoelectric Crystals

Quartz crystal has a hexagonal structure with a pyramid attached to each end. It has three axes, namely the optic axis (z axis) which joins the edges of the pyramid, the electrical axis (x axis) which joins the corners of the hexagon and the mechanical axis (y axis) which joins the centre or sides of the hexagon.
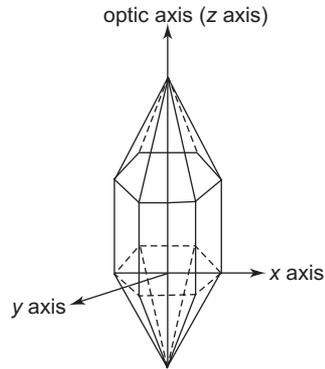
Fig. 6.4   Piezoelectric Crystals

6.1.7   X-Cut Crystal

When a crystal is cut perpendicular to the x axis, it is called an x-cut crystal. Generally they are used to produce longitudinal ultrasonic waves.
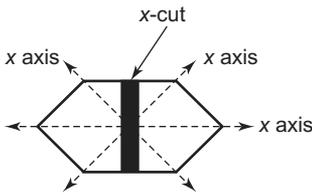
Fig. 6.5   X-Cut Crystal
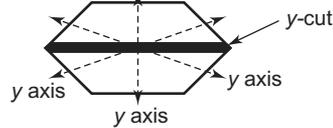
Fig. 6.6   Y-Cut Crystal

6.1.8   Y-Cut Crystal

When a crystal is cut perpendicular to the y axis, it is called a y-cut crystal. Generally they are used to produce transverse ultrasonic waves.

6.2   MAGNETOSTRICTIVE ULTRASONIC TRANSDUCER

This technique employs the phenomenon of magnetostriction exhibited by ferromagnetic materials.

6.2.1   Principle

When a ferromagnetic rod is placed in a strong magnetic field, its length changes. This phenomenon is called magnetostriction. It can be used to produce ultrasonics.

6.2.2   Construction and Working

A ferromagnetic rod *AB* is kept in a solenoid fed by a dc supply (Fig. 6.7). It leads to the permanent magnetisation of the ferromagnetic rod *AB*. Moreover, the rod material and its dimensions are chosen so that the natural frequency of the rod *AB* is in the desired ultrasonic range. Coils L_1 and L_2 wound up on the rod are connected to the base and the collector of a transistor *T*, respectively. A high-frequency current flows through the collector to have an alternating magnetic field produced in the coil L_2. It produces changes in the length of the rod. That is, the rod is alternatively compressed and stretched. Thus, the rod is set into vibrations. It causes a variation in the magnetic flux through the coil L_1. It results in an emf set up in the coil L_1. The induced emf acts on the base of the transistor and amplified current variations are produced in the coil L_2.
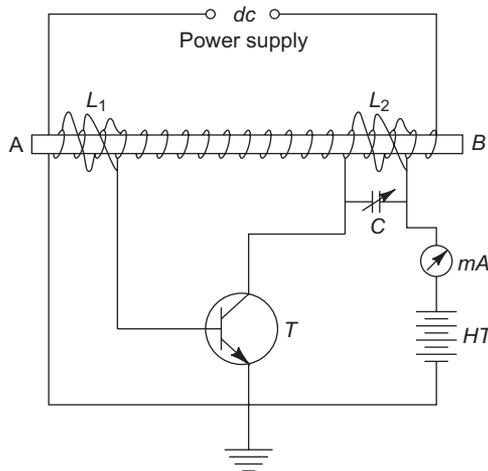
Fig. 6.7 Magnetostrictive Ultrasonic Transducer

By adjusting the variable capacitor *C*, the frequency of the alternating magnetic field (i.e., of the oscillatory circuit) is made equal to the natural frequency of the rod. That is,

$$\frac{1}{2\pi\sqrt{L_2 C}} = \frac{1}{2\ell}\sqrt{\frac{Y}{\rho}} \tag{6.1}$$

where ℓ is the length of the rod, Y is the Young's modulus of the rod material and ρ is the density of the rod material. Under this condition,

the rod executes resonant vibrations. It is marked by a peak value in the collector current shown by the milliammeter. Then, the surrounding air medium is set into vibrations resulting in the production of intense ultrasonic waves.

Merits
- Simple construction
- Low cost
- Large output at low ultrasonic frequencies without any risk to the oscillatory circuit

Demerits
- Ultrasonics of frequency above 100 kHz cannot be produced
- Frequency of ultrasonics depends greatly on temperature
- Ultrasonics of definite frequency cannot be produced since the breadth of the resonance curve is large

6.3   PIEZOELECTRIC ULTRASONIC TRANSDUCER

This technique employs the converse piezoelectric effect exhibited by piezoelectric materials.

6.3.1   Principle

When certain crystals called piezoelectric crystals, such as quartz, tourmaline and rochelle salt, are cut perpendicular to the optic axes and subjected to pressure along their mechanical axes, an electrical potential difference develops along their electrical axes. This phenomenon is known as the piezoelectric effect. In a piezoelectric crystal, mechanical, electrical and optic axes are mutually perpendicular to each other.

The converse piezoelectric effect is also true. That is, when a piezoelectric crystal is subjected to an electric field along its electrical axis, a mechanical pressure is produced along its mechanical axis. This phenomenon is called converse piezoelectric effect. It can be used for the production of ultrasound.

6.3.2   Construction

The schematic construction of a piezoelectric transducer is shown in Fig. 6.8. A piezoelectric crystal and its dimensions are chosen so that its natural frequency is in the desired ultrasonic range. This piezoelectric

crystal Q is placed between the two plates and it acts as a parallel-plate capacitor. This is coupled to an oscillatory circuit through the primary coil L_3 of a transformer. The coils L_2 and L_1 are secondary coils of the transformer. The collector coil L_2 is inductively coupled to the base coil L_1. The coil L_1, connected in parallel to the variable capacitor C, forms the tank circuit.
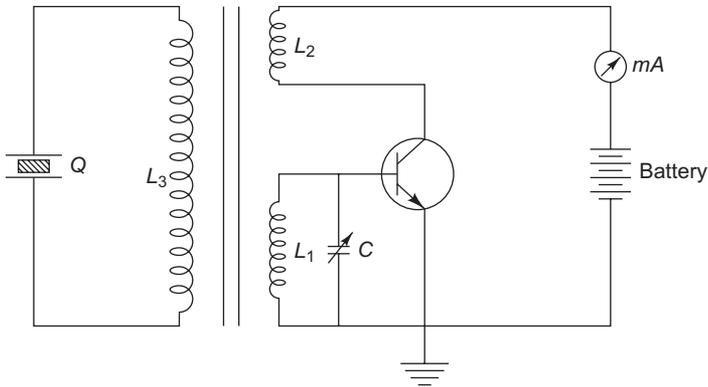
Fig. 6.8 Piezoelectric Ultrasonic Transducer

6.3.3 Working

When the battery is switched on, high-frequency oscillations are produced by the oscillator. Due to transformer action, an oscillating emf is induced in the coil L_3. Then, the piezoelectric crystal along its electrical axis is subjected to a high-frequency alternating voltage.

The capacitance of the variable condenser C is adjusted to make the frequency of the oscillatory circuit equal to the natural frequency of the piezoelectric crystal. That is,

$$\frac{1}{2\pi\sqrt{L_1 C}} = \frac{1}{2t}\sqrt{\frac{Y}{\rho}} \tag{6.2}$$

where t is the thickness of the crystal, Y is its Young's modulus and ρ is its density.

Under these conditions, the piezoelectric crystal is set into resonant vibrations. It is marked by the peak value in the current shown by the milliammeter. Then, the surrounding air medium is set into resonant vibrations to produce intense ultrasonic waves.

Merits
- Production of ultrasonics of frequencies up to 500 MHz
- Stable and definite frequency of the produced ultrasonics
- Output insensitive to temperature
- Possibility of using cheap synthetic piezoelectric crystals to produce ultrasonics
- Possibility of producing ultrasonics having a wide range of frequencies

Demerits
- High cost of piezoelectric crystals
- Complex procedure of cutting and shaping piezoelectric crystals

6.4   DETECTION OF ULTRASONICS

Ultrasonics can be detected by one of the following methods:
  (i)   Kundt's-tube method
 (ii)   Sensitive-flame method
(iii)   Piezoelectric method
(iv)   Thermal-detector method

6.4.1   Kundt's Tube Method

A Kundt's tube consists of a long glass tube fitted with a movable piston at one end, as shown in Fig. 6.9. The lycopodium powder is spread uniformly inside the bottom of the tube. Now, the open end of the Kundt's tube is kept in the path of the ultrasonic waves. Then, the lycopodium powder forms heaps at the nodes and is blown off at the antinodes. This confirms that the ultrasonic waves are present in the medium. If the lycopodium powder does not form heaps, then there are no ultrasonic waves in the medium. In this way, we can detect ultrasonics.
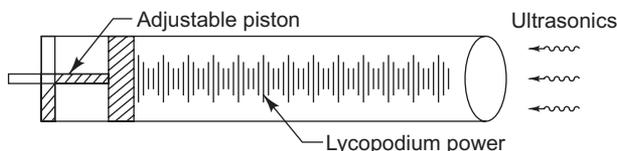
Fig. 6.9 Kundt's Tube Method

6.4.2 Sensitive-Flame Method

A sensitive steady flame moves in a medium where ultrasonic waves are present (Fig. 6.10). It flickers at the nodes and remains stationary at the antinodes. The flame alternatively flickers remains and steady in the medium. Hence, ultrasonic waves are detected.
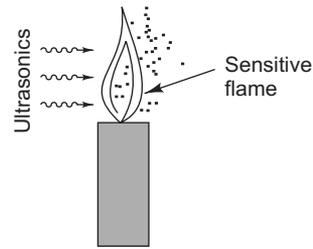
Fig. 6.10 Sensitive-flame Method

6.4.3 Piezoelectric Method

If one pair of opposite faces of a piezoelectric crystal (i.e., along the mechanical axis) is subjected to ultrasonics which are present in the medium, varying electric charges are developed in another pair of opposite faces (i.e., along the electrical axis) of the crystal. If the charges are amplified and detected by suitable means, ultrasonic waves can be detected in the medium.

6.4.4 Thermal-detector Method

A platinum wire is moved in a medium where ultrasonic waves are present. If ultrasonic waves are passing through the medium, the temperature of the medium changes due to their alternate compressions and rarefactions. Platinum wire has a high positive temperature coefficient of resistance. Hence, there is a change in resistance of the platinum wire at the nodes and it remains constant at the antinodes. The sensitive bridge arrangement can detect even a small change in resistance of the platinum wire; hence the ultrasonic waves are detected.

6.5 DETERMINATION OF VELOCITY OF ULTRASONICS—ACOUSTIC GRATING METHOD

6.5.1 Principle

If stationary wave pattern of ultrasonics (due to the superposition of direct and reflected ultrasonics) is produced in a transparent liquid, then the liquid density becomes greater at nodal planes and negligible at antinodal planes. Thus, the liquid acts at a diffraction grating called **acoustic grating**. The interplanar spacing d of the grating is equal

to the wavelength λ_u of the ultrasonics. When a collimated light beam normally falls on the acoustic grating, it is diffracted. Let λ_L be the wavelength of light, θ be the angle of diffraction and n be the corresponding order of diffraction. Then, we have

$$d \sin \theta = n\lambda_L$$

On the other hand, we have

$$v_u d = v_u$$

where v_u and v_u are frequency and velocity of the ultrasonics, respectively.

From the above two equations, we get

$$v_u = \frac{n\lambda_L v_u}{\sin \theta}$$

Using the above equation, the velocity of ultrasonics can be determined.

6.5.2 Experimental Arrangement

It consists of a monochromatic light source S and a glass vessel G with a quartz crystal Q at its bottom (Fig. 6.11). The quartz crystal is fed by an RF oscillator to produce ultrasonics. The superposition of the produced ultrasonics and the ultrasonics reflected from the walls of the vessel produces an acoustic grating.

1 - Collimator; 2 - Slit; 3 - Liquid; 4 - Focusing lens; 5 - Screen
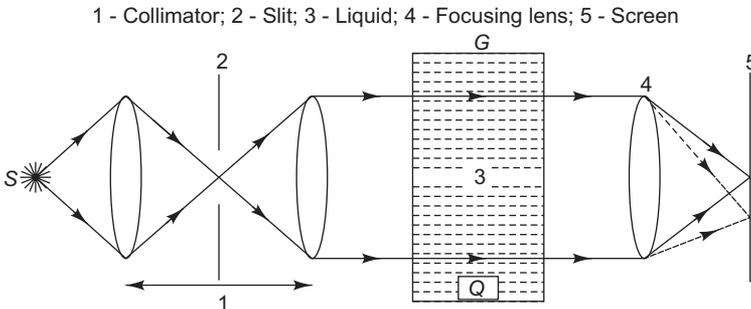
Fig. 6.11 Determination of Ultrasonic Velocity—Acoustic Grating

The monochromatic light from the source S is collimated. A parallel light beam leaving the collimator passes through the liquid filled in the

vessel. The light gets diffracted and a diffraction pattern is formed on the screen. The angle of diffraction θ for a particular diffraction order is determined. The frequency v_u of the ultrasonics is determined by a wavemeter. The wavelength of the monochromatic light is also determined. Thus, knowing the values of v_u, λ_L and θ, the velocity of ultrasonics is calculated by

$$v_u = \frac{n\lambda_L v_u}{\sin\theta}$$

6.6   ENGINEERING APPLICATIONS OF ULTRASONICS

6.6.1   Industrial Applications

Ultrasonic waves are used to detect surface, internal and external faults. In ultrasonic flaw detection, the frequency range commonly used is 1–15 megahertz only. The intensity of transmitted and reflected ultrasonic energy at an interface of two media is determined by the specific acoustic impedances of the media. The product of density of the medium and velocity of ultrasonic waves in that medium is defined as acoustic impedance.

6.6.2   Ultrasonic Welding (Cold Welding)

In electric or gas welding, the properties of some metals change on heating. Metals that cannot be welded by electric or gas welding can be done by ultrasonic welding at room temperature using ultrasonic waves. A hammer is made to vibrate ultrasonically. The tip of the hammer presses the two metal sheets very rapidly. The molecules of one metal diffuse into the molecules of the other. Hence the two sheets are welded without heating.
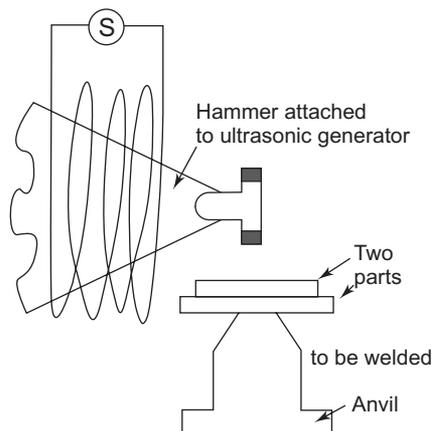
Fig. 6.12 Ultrasonic Welding

6.6.3 Ultrasonic Soldering

*S*oldering of aluminum foil condensers, aluminium wires and plates without using any fluxes can be performed by using ultrasonic soldering. It consists of an ultrasonic generator having a tip fixed at its end which can be heated by an electrical heating element. The tip of the soldering iron melts solder on the aluminium and the ultrasonic vibrator removes the aluminium oxide layer.
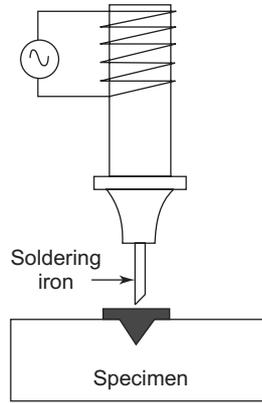
Fig. 6.13 Ultrasonic Soldering

6.6.4 Ultrasonic Drilling and Cutting

When ultrasonic waves pass through metals, the phenomenon of cavitation is produced. Due to cavitation, creation and collapse of air bubbles along the path of ultrasonic waves results in releasing a large amount of pressure and temperature used for cutting and drilling.

A suitable drilling tool bit is fixed at the end of an ultrasonic generator. Some abrasive sherry is made to flow between the bit and the plate in which the hole is to be made. The ultrasonic generator causes the tool bit to move up and down very quickly and the sherry particle below the bit just removes some material from the plate. This process continues and a hole is drilled in the plate. Through the same action takes place during ultrasonic cutting or machining. This technique, ultrasonics are used for making holes in very hard materials such as glass, diamonds, gems and ceramics.
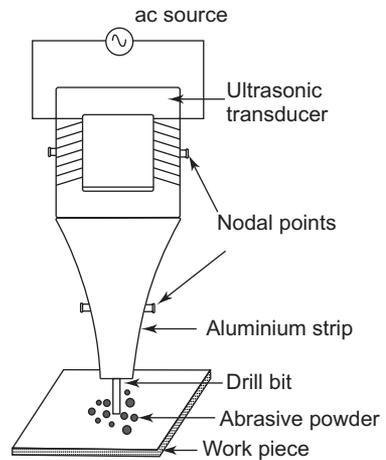
Fig. 6.14 Ultrasonic Drilling

6.6.5 Ultrasonic Cleaning

A cleaning tank is partially filled with a cleaning liquid, like a water-detergent solution, where the articles to be cleaned are placed.

Ultrasonic waves when passed through the bottom or side walls of that tank impart high acceleration within the cleaning liquid. As a result, the molecules of the cleaning liquid get enough kinetic energy and it strikes the dirt particles and separates them from the article. Ultrasonic cleaners are extensively employed for cleaning a wide range of articles

Fig. 6.15 Ultrasonic Cleaning

like parts of motors, airplanes, machines and electronic assemblies.
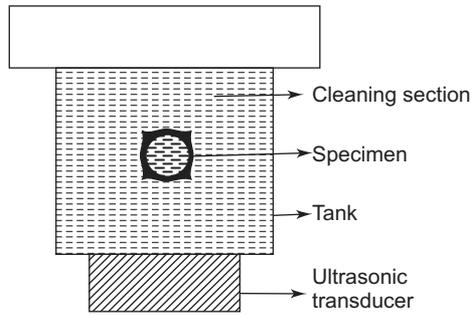
6.7 SONAR

SONAR is an acronym for Sound Navigation and Ranging. Due to their high frequencies, ultrasonics can travel as parallel beams over many kilometres. For depth ranging, the echo-sounding technique is adopted. A beam of ultrasonics is directed towards the bottom of the sea from where it is reflected back to a receiver (Fig. 6.16).

1 - Ultrasonic transducer/receiver; 2 - Sea level; 3 - Sea bed
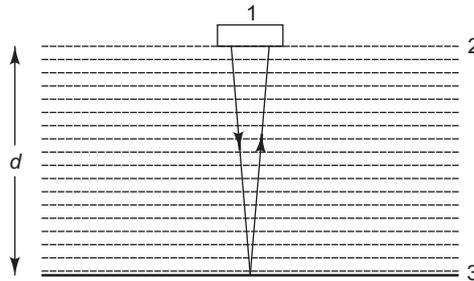
Fig. 6.16 SONAR

The time t taken by the ultrasonics for a to-and-fro journey is accurately recorded. If the ultrasonic velocity v in sea water is known, the sea depth d can be calculated as

$$d = \frac{vt}{2}$$

The SONAR technique can also be used to detect a submarine, iceberg or a school of fish in the sea. An industrial version of SONAR is used for nondestructive testing of various industrial components.

6.8 CAVITATION

Any liquid consists of nanobubbles of size 1–10 nm. When ultrasonic waves are passed through a liquid, alternate compression and rarefaction are created by the nodes and antinodes of the ultrasonic waves. At rarefaction, the pressure of the liquid suddenly drops and causes evaporation of the gas inside the bubble, which results in growth of the bubble. Thus the bubble is grown. Thereafter the liquid will have very high pressure and its temperature will increase enormously. When these bubbles collapse, the local pressure increases up to thousands of atmospheres and consequently the local temperature increases by about as much as 10,000°C, resulting in shock waves in the liquid. Hence, cavitation occurs within milliseconds.

Cavitation is the process of creation and collapse of bubbles, due to the sudden decrease in pressure created inside the bubble by the ultrasonic waves. Hence, cavitation can be artificially induced with the help of ultrasonics.

6.9 NONDESTRUCTIVE TESTING

Nondestructive testing is used to find out whether any flaws or defects exist in a specimen without causing any damage to it.

6.9.1 Pulse-Echo System through Transmission and Reflection Modes—Flaw Detector

Principle   Whenever there is a change in medium, the transmitted ultrasonic waves will be reflected. This principle is utilised in an ultrasonic flaw detector. Thus, from the intensity of the reflected echoes, the flaws are detected. In this method, the flaws are detected without causing any damage to the material and it is known as nondestructive testing method.

Description   It consists of a piezoelectric transducer coupled to the upper surface of the specimen to be tested for flaws without any air gap between the specimen and the transducer. a pulse generator is connected to the transducer (Transmitter) to produce ultrasonics and

the same is also connected to the CRO in order to record transmitted pulse. An amplitude is connected in between the transducer (receiver) and the CRO in order amplify the received signals. The timing section gives the time interval between the transmitted and received signals in the CRO.
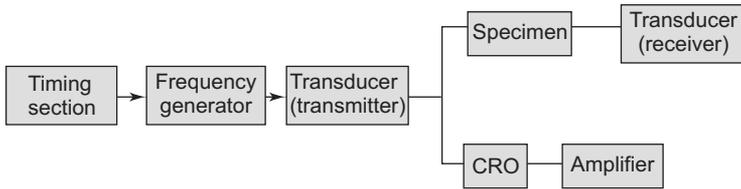
Fig. 6.17 Block Diagram of Ultrasonic Flaw Detector

Theory and Working The transmitting transducer produces pulses of ultrasonic waves and these are sent through the specimen to be tested. When the pulses reach the rear side of the specimen, they are reflected and picked up by the receiving transducer. At the location of the flaw, there is a change in the acoustic impedance and hence the pulses are partly reflected as weak echo pulses.

The transmitted pulse *A*, the pulse from flaw *B* and the pulse reflected by the rear side of the specimen *C* are displayed on the cathode ray oscilloscope screen. If the specimen contains no flaws, only two pulses *A* and *C* are displayed.
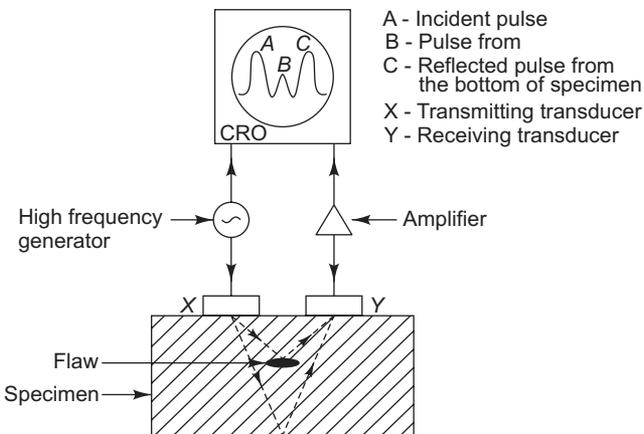
Fig. 6.18 Ultrasonic Flaw Detector

The location of a flaw inside the specimen can be calculated by the formula

$$d = vt/2$$

where, d is the distance of the flaw from the surface of the specimen, v is the velocity of the sound waves in the specimen and t is the time taken between two pulses A and B.

Advantages of Ultrasonic Testing
1. External and internal defects in metals can be located
2. Minute flaws can be detected
3. Location, nature and size of a flaw can be accurately determined
4. Simple and elegant mode of operation
5. Low cost and high speed inspection
6. Large-sized specimens can be inspected in a very short duration

Disadvantages of Ultrasonic Testing
1. No permanent record of the flaw is available
2. Only skilled and well-trained technicians can perform this testing
3. There should be good mechanical coupling between the piezoelectric crystal (called probe) and specimen to be tested

6.10   ULTRASONIC SCANNINGS

There are different ways of scanning the specimen by the ultrasonic transducer to get detailed information about the specimen. The three most common displays are
   (a) A-scan
   (b) B-scan
   (c) C-scan

6.10.1 A-scan

A-scan means amplitude-modulated time-base scan. In this scan, a single transducer is used to transmit and receive pulses from the specimen and is kept in a fixed position. The reflected echoes are displayed as

Voltage

Penetration depth
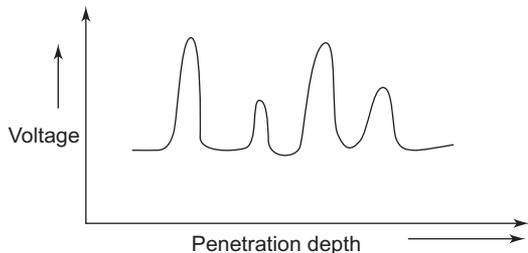
Fig. 6.19 A-scan

vertical spikes along the horizontal axis on the display screen. A one-dimensional image displaying the amplitude of the returning echo signals along the vertical axis and the time along the horizontal axis is A-scan display. The height of the vertical spikes corresponds to the strength of the echo from the specimen and the position of the vertical spike from the left to the right along the x axis corresponds to the depth of penetration.

6.10.2   B-scan

B-scan means brightness-modulated time-base scan. The reflected echoes are displayed as dots on the display screen. A two-dimensional cross section of the specimen is obtained whose dot's brightness and size varies with the strength and intensity of the reflected echo pulses,
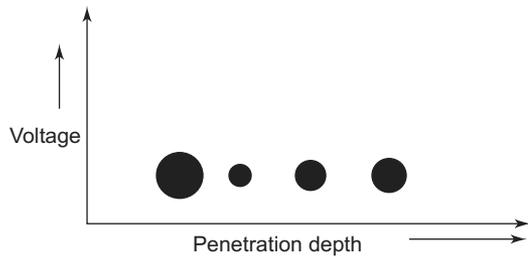
Fig. 6.20 B-scan

respectively. The distance between the two dots gives the penetration depth. The principle of B-scan is similar to that of the A-scan except with a difference that the transducer can be moved rather than keeping it in a fixed position as in A-scan. When the transducer is moved across the body, dots are created and are useful for further studies. B-scan provides the exact information about the internal structures of the specimen.

6.10.3   C-scan

C-scan means time motion scan or time position scan, also known as T-M scan. In C-scan, the transducer is held stationary and echoes appear as dots. Hence it combines the features of A-scan and B-scan. Here, the x axis shows the dots which indicate the position of the defect which depends on the depth of the reflection and the y axis shows the movement of the object. If the specimen moves during the scan, the pattern of movement can be recorded as the movement of the dots.
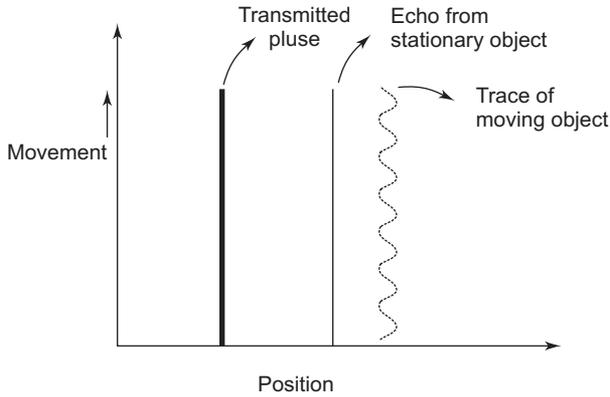
Fig. 6.21 C-scan

6.11 MEDICAL APPLICATIONS

6.11.1 Sonogram

A sonogram is a non-invasive technique of getting ultrasonic images of body parts. However, no harm is caused to our body during exposure to ultrasonic waves.
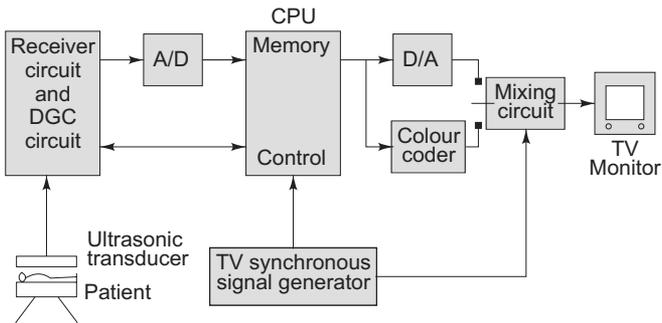
Fig. 6.22 Block Diagram of a Sonogram

Principle Acoustical impedance is the principle used in sonograms. During scanning, ultrasonic waves are transmitted from the piezoelectric transducer through the skin and then into the interior sections of the organs or tissues. When the ultrasonic waves strike an interface between two tissues of different acoustical impedance, reflections (echoes) are produced and are received by the transducer. The transducer converts these reflections into electrical signals.

These electrical signals are converted into optical signals which are displayed on the screen of the cathode ray oscilloscope. Thus one can get an ultra-sonogram.

It is different from X-ray radiography as it produces linear images rather than shadow images produced by the former. X-rays are harmful ionising radiations whereas ultrasonic is a non-invasive technique.

The ultrasonic waves are transmitted into the patient's body. The receiver circuit collects the echoes, i.e., reflected ultrasonic waves from the body. If the echoes are weak, they are compensated by the depth gain compensation circuit. They are stored in a computer by converting the obtained analog signals to digital form using an analog to digital convertor. The stored digital images are processed and colour coded, and sent to the digital to analog converter, which converts digital data to analog form. Mixing circuit combines the analog signals and TV synchronous scanning signals and feeds it to the TV monitor that displays the ultrasonic image of the internal parts of the body.

6.11.2　Other Medical Applications

Ultrasonic therapy has been used to treat diseases such as bursitis, abscesses, lumbago, etc. It is used in surgery to remove kidney stones and brain tumors without shedding blood. Ultrasonics are used in diagnosis for detecting tumors and for locating abnormal growths in our body. It is also used in dental cutting as it is almost painless for the patient. Echocardiography is also useful technique for diagnosis of heart diseases. Obstetric ultrasound is the use of ultrasound scans in pregnancy. Ultrasonics are useful in many more applications in the field of medicine.

SUMMARY

▶▶ Sound waves of frequencies between 20 Hz and 20 kHz are classified as audible sound wave range.

▶▶ Sound waves of frequencies less than 20 Hz are called infrasonics.

▶▶ Sound waves of frequencies greater than 20 kHz are called ultrasonics.

▶▶ The two commonly used methods for the generation of ultrasonic waves are magnetostriction oscillator and piezoelectric oscillator.

- ▶▶ In magnetostriction oscillator, we can generate ultrasonic waves of frequencies up to 100 kHz only.
- ▶▶ Crystals which produce piezoelectric effect are termed as piezoelectric crystals.
- ▶▶ A piezoelectric crystal can be classified as an x-cut crystal or *y*-cut crystal on the basis of its cut and shape.
- ▶▶ In piezoelectric oscillator method, the principle behind the production of ultrasonics is inverse piezoelectric effect.
- ▶▶ In piezoelectric oscillator, we can generate ultrasonic waves of frequencies up to 500 MHz.
- ▶▶ Kundt's-tube method, sensitive-flame method, piezoelectric method and thermal-detector method are some of the common methods used to detect ultrasonics.
- ▶▶ Cavitation is the process of creation and collapse of bubbles, due to the principle of negative local pressure created inside the bubble.
- ▶▶ When ultrasonic waves are passed through a liquid, the density of the liquid varies layer by layer due to the variation in pressure and hence the liquid acts at a diffraction grating, called as acoustical grating.
- ▶▶ Ultrasonic drilling, ultrasonic welding, ultrasonic soldering and ultrasonic cleaning are some important industrial applications of ultrasonics.
- ▶▶ Cavitation is the phenomenon used in ultrasonic drilling.
- ▶▶ Ultrasonic welding is also known as cold welding.
- ▶▶ Ultrasonic soldering is done without the use of any flux.
- ▶▶ SONAR is a device whose acronym is Sound Navigation and Ranging.
- ▶▶ SONAR operates on the principle of echo sounding and it is used to find the depth of sea.
- ▶▶ NDT is a method to test the specimen whether it contains any flaws, without destructing or damaging the specimen by passing ultrasonic waves through the specimen.
- ▶▶ A-scan, B-scan and C-scan are the three different types of scans performed by the ultrasonic transducer to get detailed information about specimen.
- ▶▶ Sonogram is an instrument used to monitor and visualise images of interior parts of the body using ultrasonic waves.

✓ SOLVED PROBLEMS

6.1 Ultrasonic pulse-echo technique is used to detect the defect in a steel beam of 0.5 m thickness. The arrival times for the echoes are 25 and 70 microseconds. Determine the location of the defect.

Solution Given Data

$$d = 0.5 \text{ m}$$
$$t_1 = 25 \ \mu s = 25 \times 10^{-6} \text{ s}$$
$$t_2 = 70 \ \mu s = 70 \times 10^{-6} \text{ s}$$
$$x = ?$$

Let the defect be located at a distance x from the top surface of the steel beam. Let v be the velocity of ultrasound in steel.

For the first echo reflected from the defect, we have

$$t_1 = \frac{2x}{v} \tag{1}$$

For the second echo reflected from the bottom surface of the steel beam, we have

$$t_2 = \frac{2d}{v} \tag{2}$$

From Eqs (1) and (2), we get

$$x = \frac{t_1 d}{t_2} \tag{3}$$

Substituting the given data into Eq. (3), we get

$$x = \frac{25 \times 10^{-6}}{70 \times 10^{-6}} \times 0.5$$

i.e., **x = 0.18 m**

Thus, the defect is located at a distance of 0.18 m below the top surface of the steel beam.

6.2 The oscillatory circuit in a piezolectric transducer consists of an inductance of 2 Henry. Determine the capacitance needed to generate ultrasonic waves of 10^5 Hz frequency.

Solution   Given Data

$L = 2$ Henry

$v = 10^5$ Hz

_____

$C = ?$

The output frequency of the oscillatory circuit is given by

$$v = \frac{1}{2\pi \sqrt{LC}}$$

i.e., $\quad C = \dfrac{1}{4\pi^2 v^2 L}$ $\qquad\qquad\qquad\qquad$ (1)

Substituting the given data into Eq. (1), we get the capacitance as

$$C = \frac{1}{4 \times (3.14)^2 \times 10^{10} \times 2}$$

$C = 0.013 \times 10^{-10}$ **Farad**

6.3 A piezoelectric crystal of 1.6 mm thickness produces ultrasonic waves. If the velocity of the longitudinal ultrasonic waves is 5760 m/s, determine the fundamental frequency of the crystal.

Solution   Given Data

$t = 1.6$ mm $= 1.6 \times 10^{-3}$ m

$v = 5760$ m/s

_____

$v = ?$

In the lowest mode of vibration of the piezoelectric crystal, we have

$$\lambda = 2t \qquad\qquad\qquad\qquad (1)$$

where λ is the wavelength of the ultrasonic wave. Therefore, the fundamental frequency of the crystal is

$$v = \frac{v}{\lambda} \qquad\qquad\qquad\qquad (2)$$

From Eqs (1) and (2), we get

$$v = \frac{\upsilon}{2t} \tag{3}$$

Substituting the given values for n and t into Eq. (3), we get the fundamental frequency of the crystal as

$$v = \frac{5760}{2 \times 1.6 \times 10^{-3}}$$

$$v = \textbf{1.8 MHz}$$

6.4 A quartz crystal in an ultrasonic interferometer produces stationary waves of 1.5 MHz frequency. If the distance between 6 consecutive nodes is 2.75 mm, find the velocity of the ultrasonic wave.

(A.U, B.E/B.Tech, Jan. 2005)

Solution Given Data

$v = 1.5 \text{ MHz} = 1.5 \times 10^6 \text{ Hz}$

$2.5\lambda = 1.75 \text{ mm} = 2.75 \times 10^{-3} \text{ m}$

$\lambda = 1.1 \times 10^{-3} \text{ m}$

_____

$\upsilon = ?$

For any wave, we have

$$\upsilon = v\lambda \tag{1}$$

Substituting the values of n and l for the given ultrasonic wave into Eq (1), we get its velocity as

$$\upsilon = 1.5 \times 10^6 \times 1.1 \times 10^{-3} \text{ m}$$

$$\upsilon = \textbf{1650 m/s}$$

6.5 An ultrasonic beam is used to determine the thickness of a steel plate. It is noticed that the difference between the two adjacent harmonic frequencies is 50 kHz. The velocity of sound in steel is 5000 m/s. Determine the thickness of the steel plate.

(A.U, B.E/B.Tech, Jan. 2006)

Solution Given Data

$\Delta v = 50 \text{ kHz} = 50 \times 10^3 \text{ Hz}$

$\upsilon = 5000 \text{ m/s}$

_____

$t = ?$

The difference in frequency between the two adjacent harmonics is

$$\Delta v = v_2 - v_1 = \frac{v}{2t}$$

i.e., $\quad t = \dfrac{v}{2\Delta v}$ (1)

Substituting the given values for v and Δv into Eq (1), we get

$$t = \frac{5000}{2 \times 50 \times 10^3}$$

$t = 0.05$ m

6.6 Calculate the frequency to which a piezoelectric oscillator circuit should be tuned so that a piezoelectric crystal of thickness 0.1 cm vibrates in its fundamental mode to generate ultrasonic waves. (Young's modulus and density of material of crystal are 80 Gpa and 2654 kgm^{-3}). *(A.U, B.E/B. Tech, Apr. 2003)*

Solution Given Data

$t = 0.1$ cm $= 0.1 \times 10^{-2}$

$E = 80$ Gpa $= 80 \times 10^9$ Pa

$\rho = 2654$ kg m^{-3}

$f = ?$

The frequency of vibration is given by

$$f = \frac{P}{2t}\sqrt{\frac{E}{\rho}}$$

For fundamental mode, $P = 1$

$\therefore \quad f = \dfrac{1}{2 \times 0.1 \times 10^{-2}}\sqrt{\dfrac{80 \times 10^9}{2654}}$

$f = 2.7451 \times 10^6$ **Hz**

6.7 A quartz crystal of thickness 1 mm is vibrating at resonance. Calculate the frequency of the second mode of vibration of the crystal. Given Young's modulus of crystal is 7.9×10^{10} N/m^2 and density of quartz is 2650 kg/m^3.

Solution   Given Data

$t = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

$E = 7.9 \times 10^{10} \text{ N/m}^2$

$\rho = 2650 \text{ kg/m}^3$

$f = ?$

The frequency of vibration is given by

$$f = \frac{P}{2t} \sqrt{\frac{E}{\rho}}$$

For second mode of vibration, $P = 2$

$$\therefore \qquad f = \frac{2}{2 \times 1 \times 10^{-3}} \sqrt{\frac{7.9 \times 10^{10}}{2650}}$$

$$f = 5.4599 \times 106 \text{ Hz}$$

6.8 An ultrasonic source of 0.07 MHz sends down a pulse towards the sea bed. Which returns after 0.7 s. The velocity of sound in sea water is 1700 m/s. Calculate the depth of the sea and the wavelength of the pulse.

Solution   Given Data

$v = 0.07 \text{ MHz} = 0.07 \times 10^6 \text{ Hz}$

$t = 0.7 \text{ s}$

$v = 1700 \text{ m/s}$

$d = ?$

$\lambda = ?$

Depth of the sea,

$$d = \frac{vt}{2} = \frac{(1700)(0.7)}{2} = 595 \text{ m}$$

Wavelength of the pulse,

$$\lambda = \frac{v}{v} = \frac{1700}{0.07 \times 10^6}$$

$$\lambda = 0.0243 \text{ m}$$

6.9 Calculate the frequency of the fundamental note and the first overtone emitted by a piezoelectric crystal using the following data. Vibrating thickness is 6 mm; Young's modulus is 8.8×10^{10} Nm^{-2}; and density of the crystal is 2650 kg m^{-3}.

Solution Given Data

$t = 6$ mm $= 6 \times 10^{-3}$ m

$E = 8.8 \times 10^{10}$ Nm^{-2}

$\rho = 2650$ kg m^{-3}

$f_1 = ?$

$f_2 = ?$

The frequency of vibration is given by

$$f = \frac{P}{2t} \sqrt{\frac{E}{\rho}}$$

where P = 1, 2, 3 etc., for the fundamental first overtone and second overtone, respectively.

The frequency of fundamental vibration is

$$f_1 = \frac{1}{2 \times 6 \times 10^{-3}} \sqrt{\frac{8.8 \times 10^{10}}{2650}}$$

$$\boldsymbol{f_1 = 4.8022 \times 10^5 \text{ Hz}}$$

The frequency of the first overtone is

$$f_2 = \frac{2}{2 \times 6 \times 10^{-3}} \sqrt{\frac{8.8 \times 10^{10}}{2650}}$$

$$\boldsymbol{f_2 = 9.6043 \times 10^5 \text{ Hz}}$$

6.10 Find the depth of a submerged submarine if an ultrasonic wave is received after 0.7 from the time of transmission. The velocity of ultrasonic waves is sea water is 1440 m/s.

Solution Given Data

$v = 1440$ m/s

$t = 0.7$ s

$d = ?$

The depth of a submerged submarine is

$$d = \frac{vt}{2}$$

$$= \frac{1440 \times 0.7}{2}$$

$$d = 504 \text{ m}$$

6.11 Calculate the fundamental frequency of a quartz crystal of thickness 1.5 mm, which is vibrating at resonance. The Young's modulus for quartz is 7.9×10^{10} Nm^{-2} and density is 2650 kg m^{-3}.

(A.U, B.E/ B. Tech, Jan. 2011)

Solution   Given Data

$$t = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$\rho = 2650 \text{ kg m}^{-3}$$

$$y = 7.9 \times 10^{10} \text{ Nm}^{-2}$$

$$v = ?$$

$$v = \frac{1}{2t} \sqrt{\frac{y}{\rho}}$$

$$= \frac{1}{2 \times 1.5 \times 10^{-3}} \sqrt{\frac{7.9 \times 10^{10}}{2650}}$$

$$v = 1.819 \text{ MHz}$$

6.12 Calculate the natural frequency of 40 mm length of a pure iron rod. Given the density of pure iron is 7.25×10^3 kg/m^3 and its Young's modulus is 115×10^9 N/m^2. *(A.U, B.E/B. Tech, Jan. 2011)*

Solution   Given Data

$$l = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\rho = 7.25 \times 10^3 \text{ kg/m}^3$$

$$y = 115 \times 10^9 \text{ N/m}^2$$

$$v = ?$$

$$v = \frac{1}{2l} \sqrt{\frac{y}{\rho}}$$

$$= \frac{1}{2 \times 40 \times 10^{-3}} \sqrt{\frac{115 \times 10^9}{7.25 \times 10^3}}$$

$$= \frac{10^9 \times 3.982}{80}$$

$$= 0.0497 \times 10^9$$

v = 49.7 MHz

6.13 An ultrasound pulse sent by a source in sea is reflected by a submerged target at a distance 597.50 m and reaches the source after 0.83 seconds. Find the velocity of sound in sea water.

(A.U, B.E/B. Tech, Jan. 2011)

Solution Given Data

$d = 597.50$

$t = 0.83$ s

$v = ?$

$d = \dfrac{vt}{2}$

$v = \dfrac{2d}{t}$

$V = \dfrac{2 \times 597.50}{0.83}$

$V = 1.439 \times 10^{-3}$ m/s

QUESTIONS AND ANSWERS

6.1 What are ultrasonics?

Sound waves of frequency exceeding 20 kHz are called ultrasonics.

6.2 Mention a few properties of ultrasonics.

- Negligible diffraction
- Highly energetic
- Speed of propagation depends on frequency
- High penetrating power
- Intense ultrasonics produce cavitation in liquids

- Formation of diffraction grating when ultrosonics pass through a liquid

6.3 Define magnetostriction effect.
When a ferromagnetic rod is placed in a strong magnetic field, its length changes. This phenomenon is called magnetostriction.

6.4 State the merits of a magnetostrictive ultrasonic transducer.
- Simple construction
- Low cost
- Large output at low ultrasonic frequencies without any risk to the oscillatory circuit

6.5 State the demerits of a magnetostrictive ultrasonic transducer.
- Ultrasonics of frequency above 100 kHz cannot be produced
- Frequency of ultrasonics depends greatly on temperature
- Ultrasonics of definite frequency cannot be produced

6.6 Define piezoelectric effect.
When piezoelectric crystals are subjected to pressure along their mechanical axes, electrical potential difference develops along their electrical axes. This phenomenon is known as the piezoelectric effect.

6.7 Define converse piezoelectric effect.
When a piezoelectric crystal is subjected to an electric field along its electrical axis, a mechanical pressure is produced along its mechanical axis. It is called converse piezoelectric effect.

6.8 State the merits of a piezoelectric ultrasonic transducer.
- Production of ultrasonics of frequencies up to 500 MHz
- Stable and definite frequency of the produced ultrasonics
- Output is insensitive to temperature
- Possibility of using cheap synthetic piezoelectric crystals to produce ultrasonics
- Possibility of producing ultrasonics having a wide range frequencies

6.9 State the demerits of a piezoelectric ultrasonic transducer.
(A.U, B.E/B.Tech, Jan. 2005)
- High cost of piezoelectric crystal
- Complex procedure of cutting and shaping the piezoelectric crystal

6.10 Write a note on SONAR. *(A.U, B.E/B.Tech, Jan. 2006)*
SONAR is an acronym for Sound Navigation and Ranging. A beam of ultrasonics is directed towards the bottom of the sea from where it is reflected back to a receiver. The time t taken by ultrasonics for its to-and-fro journey is accurately recorded. If the ultrasonic velocity v in sea water is known, the sea depth d can be calculated as $d = vt/2$.

6.11 State the principle involved in ultrasonic nondestructive testing.
Ultrasonic pulse-echo technique is used to detect cracks or cavities in metal castings. A strong pulse of ultrasonic waves is transmitted through the specimen to be tested. At the cracks, the ultrasonic pulse is reflected to produce an echo. The echo is detected by the CRO. The time for an ultrasound's to-and-fro journey is noted. Knowing the ultrasonic velocity in the specimen, the location of the flaw can be determined.

6.12 Mention a few uses of ultrasonic flaw nondestructive testing.
- Testing the turbine blades
- Concrete inspection, corrosion inspection and inspection of ships and supports under water
- Identification of cracks in a large concrete structure such as dam
- Testing the spares of missiles

6.13 Mention the methods of ultrasonic detection.
Kundt's-tube method; Sensitive-flame method; Piezoelectric method; Thermal-detector method

6.14 Write a note on the Kundt's-tube method of ultrasonic detection.
Kundt's tube consists of a long glass tube fitted with an adjustable piston at one end. Before keeping the Kundt's tube in the path of ultrasonic waves, lycopodium powder is smeared uniformly inside the tube. Now, the Kundt's tube is kept in the path of ultrasonic waves. Then, the lycopodium powder spreads and produces heaps at the nodes and is blown off at the antinodes. This confirms that the waves are ultrasonics. If the heaps are not formed, then there are no ultrasonic waves. In this way, we can detect ultrasonics.

6.15 Write a note on the sensitive-flame method of ultrasonic detection.

A sensitive flame is placed in the path of ultrasonics. Then it flickers at the nodes and remains stationary at the antinodes. Hence, ultrasonic waves are detected.

6.16 Write a note on the piezoelectric method of ultrasonic detection.

One pair of faces (perpendicular to the mechanical axis) of a piezoelectric crystal is subjected to ultrasonics. Then, on another pair of faces (perpendicular to the electrical axis), varying electric charges are developed. The charges are amplified and detected by suitable means.

6.17 Write a note on the thermal-detector method of ultrasonic detection.

A platinum wire is moved in a medium where ultrasonic waves are present. The temperature of the medium changes due to alternate compressions and rarefactions. There is a change in temperature at the nodes. At the antinodes, the temperature remains constant. Hence, the resistance of the platinum wire changes at the nodes and remains constant at the antinodes. The change in the resistance of the platinum wire is detected by a sensitive bridge arrangement.

6.18 Define cavitation.

Cavitation is the process of creation and collapse of bubbles, due to the negative local pressure created inside the bubble. Cavitation can be artificially induced with the help of ultrasonics.

6.19 Write a note on A-scan display mode of ultrasonic imaging.

Transmitted pulse triggers the sweep of an oscilloscope. This pulse and the returned echoes are displayed as vertical deflections of the trace. The sweep is calibrated in units of distance and it determines the distance to the echo-producing target. Thus, the A-scan method plots the amplitude of the echo against the distance to the echo-producing target. Usually, the transducer is kept stationary so that any movement of echoes along the trace will be the result of moving targets.

6.20 Write a note on B-scan display mode of ultrasonic imaging.

It uses the echo signal to brighten the oscilloscope beam. With this type of presentation, a linear sweep is applied to the x axis of the oscilloscope. The beam's brightness is adjusted so that it becomes

visible only when an echo is present. Thus, the position of the bright spot represents the range of the target.

6.21 Write a note on the M-scan display mode of ultrasonic imaging.

Transmitted pulse triggers the oscilloscope sweep. The received pulses are used to brighten the trace. The echoes appear as dots with brightness proportional to their intensity.

The transducer is held stationary so that the movement of the dots along the sweep represents the movement of the echo-producing targets. A moving photographic plate S is placed in front of the oscilloscope. When developed, the excursions of the bright spots, representing the target motion, are clearly seen as curved lines.

6.22 Define sonogram.

It is an instrument used to monitor and visualise the image of the interior parts of the body using high-frequency ultrasonic waves.

EXERCISE PROBLEMS

6.1 The shortest wavelength of ultrasonics emitted by a bat is 0.33 cm. What is the highest frequency of emitted ultrasound?
$$[v_{max} = 0.103 \text{ MHz}]$$

6.2 A piezoelectric X-cut crystal is 0.002 m thick. The velocity of ultrasound through the crystal is 5750 m/s. Calculate the fundamental frequency of the crystal. $\quad [v_1 = 1.44 \text{ MHz}]$

6.3 A quartz crystal is 0.05 m thick. The velocity of ultrasound through the quartz crystal is 5.5×10^3 m/s. Determine the frequency of the second mode of vibration of the crystal.
$$[v_2 = 0.11 \text{ MHz}]$$

6.4 A quartz crystal of 2-mm thickness is vibrating at resonance. Determine the frequency of the produced ultrasound, if quartz has the Young's modulus of 7.9×10^{10} N/m^2 and the density of 2.65×10^2 kg/m^3. $\quad [v = 1.365 \text{ MHz}]$

6.5 Ultrasonic pulse-echo technique is employed to study the depth of sea. The echo arrival time is 6 seconds. Determine the depth of sea if the velocity of ultrasound in sea water is 330 m/s. $\quad [d = 990 \text{ m}]$

6.6 Find the depth of a submerged submarine if ultrasonic waves are received after 0.1 from the time of transmission. The velocity of ultrasonic waves in sea water is 1440 m/s.

[d = 72 m]

6.7 An ultrasonic source of 0.05 μm sends down a pulse towards the sea bed, which returns after 0.5 s. The velocity of sound in sea water is 1700 m/s. Calculate the depth of the sea and the wavelength of the rise. *[d = 425 m, λ = 0.034 m]*

6.8 Calculate the frequency of the fundamental note and the first overtone emitted by a piezoelectric crystal using the following data. Vibrating thickness is 1 mm; Young's modules is 7.8×10^{10} N/m²; and density of the crystal is 2650 kg m^{-3}.

[f$_1$ = 27.12 × 10^5 Hz, f$_2$ = 54.25 × 10^5 Hz]

6.9 A quartz crystal of thickness 1 mm is vibrating at resonance. Calculate the frequency of the first mode of vibration of the crystal. Given Young's modulus of the crystal is 5.9×10^{10} N/m² and the density of quartz is 2650 kg/m³.

[f = 2.359 × 10^6 Hz]

6.10 Calculate the frequency to whch a piezoelectric oscillator circuit should be tuned. So that a piezoelectric crystal of thickness 0.5 cm vibrates in its fundamental mode to generate ultrasonic waves. (Young's modulus and density of material of crystal are 20 Gpa and 2654 kg/m³). *[f = 5.490 × 10^5 Hz]*

QUESTION BANK

PART-A

6.1 Sound waves having frequency greater than _____ are called ultrasonic waves.

6.2 What is the frequency range of ultrasonic waves?

6.3 Mention any one characteristic property of ultrasonic waves.

6.4 The speed of propagation of ultrasonic wave depends on the _____ .

6.5 Intense ultrasonics produce _____ in liquids.

6.6 What happens when ultrasonics pass through liquids?

6.7 _____ waves are high energetic sound waves.

6.8 Ultrasonic waves show negligible _____ due to their smaller wavelength.

6.9 The speed of ultrasonic waves _____ with increase in frequency.

6.10 Ultrasonic waves are having high _____ power.

6.11 Intense ultrasonics produce _____ in liquids.

6.12 A _____ pattern is formed when ultrasonics pass through a liquid.

6.13 Write any one method for the production of ultrasonic waves.

6.14 Which method is used to produce high frequency ultrasonic waves?

6.15 What is the basic principle behind the production of ultrasonics using a magnetostrictive ultrasonic transducer?

6.16 What is the basic principle behind the production of ultrasonics using a piezoelectric ultrasonic transducer?

6.17 State any one demerit of a magnetostrictive ultrasonic transducer.

6.18 State any one merit of a magnetostrictive ultrasonic transducer.

6.19 State any one demerit of a piezoelectric ultrasonic transducer.

6.20 State any one merit of a piezoelectric ultrasonic transducer.

6.21 What is meant by resonance?

6.22 Which type of material is used in a magnetostrictive ultrasonic transducer?

6.23 Which type of material is used in a piezoelectric ultrasonic transducer?

6.24 When pressure is applied to a quartz crystal, what type of energy is released?

6.25 Mention any one industrial application of ultrasonics.

6.26 Mention any one method of detection of ultrasonics.

6.27 What is the type of transducer used in ultrasonic imaging?

6.28 What type of ultrasonic technique is used in cardiology?

6.29 Write any one use of echo cardiography.

6.30 Write any one use of ultrasonic tomography.

6.31 Mention any one medical application of ultrasonics.

6.32 Write any one advantage of nondestructive testing.

6.33 Write the formula for frequency of an oscillating circuit.

6.34 Write the formula for natural frequency of vibration of a rod.

6.35 Write the formula for frequency of a vibrating crystal.

6.36 Which type of wire is used in a thermal detector?

6.37 Which type of property is varied in a platinum wire when strike by ultrasound waves?

6.38 Cavitation can be artificially induced with the help of _____ .

6.39 _____ can be artificially induced with the help of ultrasonics.

6.40 Mention any one application of the phenomenon of cavitation produced by ultrasonics.

6.41 State any one merit of ultrasonic welding.

6.42 Cold welding process is used in _____ .

6.43 Mention any one type of scanning technique used in the pulse-echo method.

6.44 What is the principle used in ultrasonic nondestructive testing?

6.45 Mention any one application of ultrasonic nondestructive testing.

6.46 State the phenomenon that is used in ultrasonic nondestructive testing.

6.47 What is the basic principle used in ultrasonic imaging devices?

6.48 Mention any one medical application of ultrasonic cardiology.

6.49 Piezoelectric effect is _____ .

6.50 What is the effect used in a piezoelectric ultrasonic detector?

6.51 What are ultrasonic waves?

6.52 Mention any four properties of ultrasonics.

6.53 Define magnetostriction effect.

6.54 State the merits of a magnetostrictive ultrasonic transducer.

6.55 State the demerits of a magnetostrictive ultrasonic transducer.

6.56 Define piezoelectric effect.

6.57 Define inverse piezoelectric effect.

6.58 State the merits of a piezoelectric ultrasonic transducer.

6.59 State the demerits of a piezoelectric ultrasonic transducer.

6.60 Mention the various methods of ultrasonic detection.

6.61 Write a note on the Kundt's-tube method of ultrasonic detection.

6.62 Write a note on the sensitive-flame method of ultrasonic detection.

6.63 Write a note on the piezoelectric method of ultrasonic detection.

6.64 Write a note on the thermal-detector method of ultrasonic detection.

6.65 Define cavitation.

6.66 Mention any four industrial applications of ultrasonics.

6.67 Write a note on ultrasonic drilling.

6.68 Write a note on ultrasonic welding.

6.69 Write a note on ultrasonic soldering.

6.70 Write a note on ultrasonic cleaning.

6.71 Define the phenomenon that is used in ultrasonic nondestructive testing.

6.72 Mention any four applications of ultrasonic flaw nondestructive testing.

6.73 Write a note on ultrasonic cardiology.

6.74 Write a note on A-scan display mode of ultrasonic imaging.

6.75 Write a note on B-scan display mode of ultrasonic imaging.

6.76 Write a note on M-scan display mode of ultrasonic imaging.

6.77 Write a note on ultrasonic nondestructive testing.

6.78 Mention three types of scan modes used in ultrasonic pulse-echo method.

6.79 Define any one principle that is used in the production of ultrasonic waves.

6.80 Mention any two medical applications of ultrasonics.

6.81 Ultrasonic pulse-echo technique is used to detect the defect in a steel beam of thickness 0.5 m. The arrival time for the echoes are 25 and 70 microseconds. Determine the location of the defect.

6.82 The oscillatory circuit in a piezoelectric ultrasonic transducer consists of an inductance of 2 Henry. Determine the capacitance needed to generate ultrasonic waves of frequency 10^5 Hz.

6.83 A piezoelectric crystal of thickness 1.6 mm produces ultrasonic waves. If the velocity of the longitudinal ultrasonic waves is 5760 m/s, determine the fundamental frequency of the crystal.

6.84 A quartz crystal in an ultrasonic interferometer produces stationary waves of frequency 1.5 MHz. If the distance between 6 consecutive nodes is 2.75 mm, find the velocity of the ultrasonic wave.

6.85 An ultrasonic beam is used to determine the thickness of a steel plate. It is noticed that the difference between two adjacent harmonic frequencies is 50 kHz. The velocity of sound in steel is 5000 m/s. Determine the thickness of the steel plate.

6.86 Calculate the natural frequency of a 40 mm long pure iron rod. Given the density of pure iron is 7.25×10^3 kg/m^3 and its Young's modulus is 115×10^9 N/m^2. Can you use it in a magnetostrictive oscillator to produce ultrasonic waves?

6.87 A quartz crystal of length 1 mm is vibrating at resonance. Calculate the fundamental frequency. (Assume Y for quartz = 7.9×10^{10} N/m^2 and ρ for quartz = 2650 kg m^3)

6.88 The shortest wavelength of ultrasonics emitted by a bat is 0.33 cm. What is the highest frequency of emitted ultrasound?

6.89 A piezoelectric x-cut crystal is 0.002 m thick. The velocity of ultrasound through the crystal is 5750 m/s. Calculate the fundamental frequency of the crystal.

6.90 A quartz crystal is 0.05 m thick. The velocity of ultrasound through the quartz crystal is 5.5×10^3 m/s. Determine the frequency of the second mode of vibration of the crystal.

6.91 Define the basic principle behind the production of ultrasonics using a magnetostrictive ultrasonic transducer.

6.92 Define the basic principle behind the production of ultrasonics using a piezoelectric ultrasonic transducer.

6.93 Mention the advantages of ultrasonic welding.

6.94 Define the phenomenon of ultrasonic cleaning.

6.95 Mention any four applications of ultrasonics.

6.96 Define the basic principle used in ultrasonic imaging devices.

6.97 Define the effect used in a piezoelectric ultrasonic detector.

PART-B

6.1 Explain the production of ultrasonics by the magnetostriction method.

6.2 Explain the production of ultrasonics by the piezoelectric method.

6.3 Explain in detail the four methods of detection of ultrasonic waves.

6.4 Discuss in detail the industrial applications of ultrasonics in drilling, welding, soldering and cleaning.

6.5 Explain with a neat sketch, ultrasonic nondestructive testing by the pulse-echo method.

6.6 Explain with a neat sketch, the applications of ultrasonics in cardiology.

6.7 Explain the three modes of display systems used in ultrasonic imaging.

6.8 (i) Discuss in detail the industrial applications of ultrasonics in drilling and cleaning.
   (ii) Explain in detail the Kundt's-tube method and sensitive-flame method of ultrasonic detection.

6.9 Discuss in detail ultrasound imaging of the human body.

6.10 (i) Discuss in detail the industrial applications of ultrasonics in drilling and welding.
   (ii) Explain in detail the Kundt's-tube method and thermal-detector method of ultrasonic detection.

6.11 (i) Discuss in detail the industrial applications of ultrasonics in soldering and cleaning.
   (ii) Explain in detail the piezoelectric method and thermal-detector method of ultrasonic detection.

6.12 Discuss in detail ultrasonic nondestructive testing by pulse-echo method.

6.13 (i) Explain in detail any two methods of detection of ultrasonic waves.
   (ii) Discuss in detail any two industrial applications of ultrasonics.

7

LASERS

7.1 INTRODUCTION

Laser is an outstanding achievement of science and technology in the twentieth century. The first lasers made in 1960 paved the way for the vehement development of laser technology. Today, lasers find wide use in communication systems, computers, navigation equipments, measuring equipments, complicated technological processes and medicine.

The term LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. As early as in 1917, Einstein predicted the existence of stimulated emission of radiation by an atom. The process of stimulated emission was first successfully used in the construction of a coherent optical source (MASER) by Basov and Prokhorov of Russia and Townes of USA in 1954 and finally in 1960, Theodore Maiman of USA demonstrated the first laser using a ruby crystal as the active material.

7.2 CHARACTERISTICS OF LASER

The wide use of lasers in various walks of life is due to some specific characteristic of lasers. The striking characteristics of lasers are
- High monochromacity
- High directionality
- High coherence
- High intensity

Monochromacity Laser radiation has a definite wavelength or frequency. Laser radiation is more monochromatic than any other conventional monochromatic source.

Directionality Laser radiation travels as a parallel beam over very long distances. For a typical laser, the beam divergence is less than

0.01 milliradian. That is, the laser beam diverges less than 0.01 mm for every metre of its travel.

Coherence All the constituent photons of laser radiation have the same energy, same direction of momentum and same direction of polarisation. That is, laser radiation is characterised by a high degree of 'ordering' of the light field than those of conventional sources. When a laser beam is of continuous wave output, it travels for an infinite length and carries the same wave amplitude. Such a laser is perfectly spatially coherent.
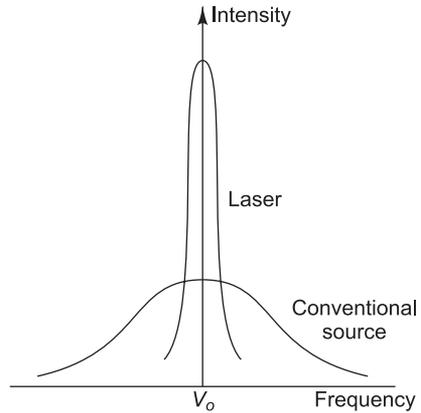
Fig. 7.1 Monochromacity of Laser

Intensity The coherent nature of laser radiation results in extremely high concentration of its energy in a small region. This concentration accounts for the great intensity of lasers.

7.3 PRINCIPLE OF LASER ACTION—EINSTEIN'S THEORY

Consider a matter irradiated by a radiation. When the radiation interacts with the matter, three different processes, namely, **absorption, spontaneous emission** and **stimulated emission of radiation** take place. Let hv be the energy of the incident radiation. Here, h is the Planck's constant and v is the frequency of the incident radiation. Let the matter consist of atoms having only two energy levels. Let the energy corresponding to the lower energy level and the higher energy level be E_1 and E_2, respectively.

7.3.1 Absorption of Radiation

Let a photon of energy $hv = E_2 - E_1$ be incident on the atom in the lower energy level. The atom absorbs the incident photon and gets excited to the energy level E_2 (Fig. 7.2). This process is called the **absorption of radiation**.
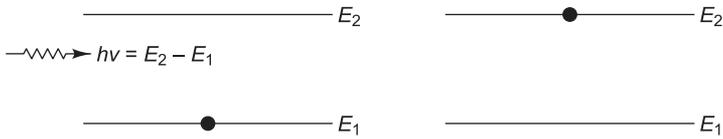
Fig. 7.2 Absorption of Radiation

The rate of absorption R_{12} is proportional to the population of the lower energy level N_1 and to the density of incident radiation ρ. Therefore, we have

$$R_{12} = B_{12}\,\rho\,N_1 \tag{7.1}$$

where B_{12} is the proportionality constant known as the **probability of absorption of radiation per unit time**.

7.3.2 Spontaneous Emission of Radiation

The higher energy level is an unstable one. Hence, the excited atom in the higher energy level E_2 spontaneously returns to the lower energy level E_1 (Fig. 7.3). It results in the emission of a photon of energy $hv = E_2 - E_1$. This process is called the **spontaneous emission of radiation**.

Fig. 7.3 Spontaneous Emission of Radiation

The rate of spontaneous emission of radiation $R_{21(sp)}$ is proportional to the population of the higher energy level N_2. Therefore, we have

$$R_{21(sp)} = A_{21}\,N_2 \tag{7.2}$$

where A_{21} is the proportionality constant known as the probability of spontaneous emission per unit time.

7.3.3 Stimulated Emission of Radiation

An incident photon of energy $hv = E_2 - E_1$ stimulates the excited atom in the higher energy level E_2 to return to the lower energy level E_1. It results in the emission of a photon of energy $hv = E_2 - E_1$ (Fig. 7.4). Both the inducing and induced photons have the same energy, same

direction of momentum and same direction of polarisation. This process is called the **stimulated emission of radiation**.
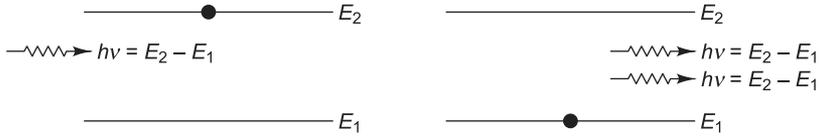
Fig. 7.4 Stimulated Emission of Radiation

The rate of stimulated emission of radiation $R_{21(sp)}$ is proportional to the population of the higher energy level N_2 and to the density of incident radiation ρ. Therefore, we have

$$R_{21(sp)} = B_{21} \, \rho \, N_2 \tag{7.3}$$

where B_{21} is the proportionality constant known as the probability of stimulated emission of radiation per unit time.

7.3.4 Einstein's A and B Coefficients–Derivation

Einstein was the first to calculate the probability of such transitions assuming the atomic system to be in equilibrium with electromagnetic radiation. Before Einstein found out the concept of stimulated emission, by considering absorption and spontaneous emission alone, he equated the two under the condition of thermal equilibrium as follows.

Under the condition of thermal equilibrium:

Rate of absorption = Rate of emission

i.e., $B_{12} \, \rho \, N_1 = A_{21} N_2$

$$\rho = \frac{A_{21}}{B_{12}(N_1/N_2)}$$

$$= \frac{A_{21}}{B_{12} \exp\left(\dfrac{hv}{kT}\right)} \qquad \left[\because \frac{N_1}{N_2} = \exp\left(\frac{hv}{kT}\right)\right]$$

The result obtained for ρ was compared with the value obtained for Planck's theory of black-body radiation. There was contradiction between the results. Then he concluded that one more emission is possible and discovered stimulated emission. In 1916, Albert Einstein proposed that there are essentially three processes occurring in the

formation of an atomic spectral line. The three processes are referred to as spontaneous emission, stimulated emission and absorption and with each is associated an Einstein coefficient and that is a measure of the probability of that particular process. The coefficients B_{12}, A_{21} and B_{21} in the expressions for the rates of absorption and emissions are called **Einstein's coefficients.** ρ is the density of the incident radiation. N_1 represents population of the lower energy level E_1 and N_2 represents population of the higher energy level E_2.

Under the condition of thermal equilibrium, we have

Rate of absorption = Rate of emission

i.e., $\qquad B_{12}\,\rho\,N_1 = A_{21}N_2 + B_{21}\,\rho\,N_2$

From the above equation, we get

$$\rho = \frac{A_{21}}{B_{12}(N_1/N_2) - B_{21}} \tag{7.4}$$

Moreover, under thermal equilibrium, the population of energy levels obeys the Boltzmann's distribution law. Hence, we have

$$\frac{N_1}{N_2} = \exp\left(\frac{E_2 - E_1}{kT}\right)$$

where k is the Boltzmann's constant and T is the absolute temperature.

In our case, $h\nu = E_2 - E_1$. Therefore, we get

$$\frac{N_1}{N_2} = \exp\left(\frac{h\nu}{kT}\right) \tag{7.5}$$

Substitution of Eq. (7.5) into Eq. (7.4) gives

$$\rho = \frac{A_{21}}{B_{12}\exp(h\nu/kT) - B_{21}} \tag{7.6}$$

On the other hand, Planck's theory of black-body radiation gives

$$\rho = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{\exp(h\nu/kT) - 1} \tag{7.7}$$

where c is the velocity of light in vacuum.

Comparing Eqs (7.6) and (7.7), we get

$$B_{12} = B_{21} \tag{7.8}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3} \qquad\qquad (7.9)$$

Equations (7.8) and (7.9) show that the Einstein's coefficients are interrelated. Moreover, Eq. (7.8) shows that the probability of absorption is equal to the probability of stimulated emission. That is, when a photon of energy $hv = E_2 - E_1$ is incident on an atom with two energy levels E_1 and E_2, the processes of absorption and stimulated emission are equally probable.

Equations (7.2), (7.3), (7.7) and (7.9) give

$$\frac{R_{21\,(sp)}}{R_{21\,(st)}} = \exp\left(\frac{hv}{kT}\right) - 1 \qquad\qquad (7.10)$$

Thus, when the energy of the incident photon is much greater than kT, the number of spontaneous emissions far exceeds the number of stimulated emissions. Hence, under this condition (i.e., $hv \gg kT$), laser action is not possible.

Conclusions

- **Stimulated emission concept was found out that lead to the concept of laser.**
- **Equations (7.8) and (7.9) show that the Einstein's coefficients are interrelated.**
- **Equation (7.8) shows that the probability of a absorption is equal to the probability of stimulated emission**. That is, when a photon of energy $hv = E_2 - E_1$ is incident on an atom with two energy levels E_1 and E_2, the process of absorption and stimulated emission are equally possible.
- When the energy of the incident photon is much greater than kT, the number of spontaneous emissions far exceed the number of stimulated emissions. **Hence under this condition i.e., ($hv \gg kT$), laser action is not possible.**

7.4   Basic Concepts in Laser Physics

7.4.1   Population Inversion

Population Inversion   Let us consider a two-level energy system in ground state and excited state. When the energy of a photon is equal

to the energy difference between the two energy levels incident on it, there are equal chances for stimulated emission and absorption to take place. When there are more number of atoms in the ground state than in the excited state, there is more chance for absorption to take place. When there are more number of atoms in the excited state than in the ground state, there is more chance for stimulated emission to takes place. Hence the number of atoms in the excited state must be more than the number of atoms in the ground state.

Population inversion is the condition in which the population of the higher energy level is greater than that of the lower energy level in the material.

Active Medium   It is the material with inverted population. It can be a solid, liquid, gas or plasma. Based on the state of active medium, the lasers are classified as solid-state lasers, liquid lasers, gaseous lasers and plasma lasers.

Active Centres   The atoms, ions or molecules of the active medium which emit the laser radiation are called the active centres.

Pumping   It is the mechanism of establishing population inversion in a material. The commonly used pumping mechanisms are optical pumping, electrical pumping, chemical pumping and heat pumping.

(i)   *Optical pumping*—Population inversion is achieved by means of light energy. An optical source like a xenon flash lamp is employed. This type of pumping is employed in ruby lasers.

(ii)   *Electrical pumping*—Population inversion is achieved by means of electrical energy. Here electrons are produced in the electrical discharge tube. These electrons are accelerated to high velocities by a strong electric field and they colloid with atoms or molecules. The energy is transferred from electrons to atoms or molecules resulting in population inversion. This type of pumping is employed in CO_2 lasers.

(iii)   *Chemical pumping*—Population inversion is achieved by means of exothermal chemical reactions. In this type of pumping, the atoms or molecules undergo chemical changes in which one of the product of the reaction is a molecule or an atom that is in the excited state under appropriate conditions. This type of pumping is employed in atomic iodine lasers.

(iv)   *Heat pumping*—Population inversion is achieved by means of quenching.

Optical Resonator It is a pair of reflecting surfaces with one being a perfect reflector and another being a partial reflector. It ensures an intense and coherent laser output.

Essential Components of a Laser Device

Laser has the three following basic components:
- Active medium
- Pumping source
- Optical resonator

The performance of these components determines the characteristics of the laser output.

7.5 TYPES OF LASER

7.5.1 Nd-YAG Laser

In an Nd-YAG laser, Yttrium Aluminium Garnet $(Y_3Al_5O_{12})$, commonly called YAG, doped with neodymium ions Nd^{3+} acts as the active medium. The neodymium ions Nd^{3+} are the active centres.

An Nd-YAG rod and a xenon flash lamp are placed inside an ellipsoidal reflector. The Nd-YAG rod is placed at one focal axis and the flash lamp at the other focal axis of the ellipsodial reflector (Fig. 7.5). This arrangement ensures that any light leaving the flash lamp will definitely be incident on the Nd-YAG rod.
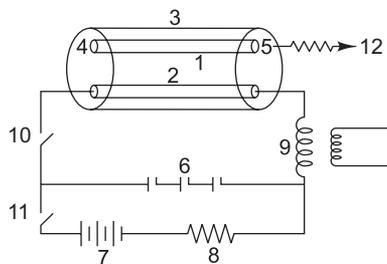
1.	Nd-YAG rod	2.	Xenon flash lamp
3.	Ellipsoidal reflector	4.	Perfect reflector
5.	Partial reflector	6.	Capacitor bank
7.	HT battery	8.	Ballast resistor
9.	Trigger pulse	10,11.	Key
12.	Laser output		

Fig. 7.5 Nd-YAG Laser Device

Working

The flash lamp is fired by discharging the capacitor bank. Only a part of the light energy emitted by the flash lamp is used to excite the Nd^{3+} ions, while the rest heats up the crystal. To prevent this, cooling is done by forcing air over the crystal. The end faces of the Nd-YAG rod are ground, polished and silvered to act as optical resonator mirrors. The energy-level diagram of the neodymium ions in the YAG crystal is shown in Fig. 7.6.
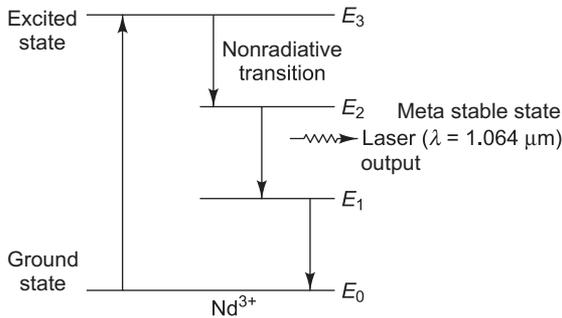
Fig. 7.6 Energy-Level Diagram of Nd-YAG Laser

The optical pumping excites the Nd^{3+} ions from the ground energy level E_0 to the higher energy level E_3. The excited Nd^{3+} ions, then, make a nonradiative transition to the lower metastable level E_2, which plays the role of the upper lasing level. Upon intense excitation, population inversion of Nd^{3+} ions is achieved at the metastable level E_2. By spontaneous emission of photons, the excited Nd^{3+} ions, then, make a transition $E_2 \rightarrow E_1$. The spontaneously emitted photons trigger the stimulated emission of photons. The photons travelling parallel to the resonator axis experience multiple reflections at the optical resonator mirrors. As a result, the transition $E_2 \rightarrow E_1$ yields an intense and coherent laser at a wavelength of 1.064 µm. The Nd^{3+} ions, then, make a transition $E_1 \rightarrow E_0$. In high power continuous operation, a krypton lamp is used as the pumping source.

Characteristics

- *Output wavelength* : 1.064 µm
- *Output power* : 10^4 watts (in pulsed operation)
- *Nature of output* : Pulsed or continuous

Applications

- *Material processing* : Drilling, spot welding and laser marking
- *Electronics* : Resistor trimming, circuit mask, memory repair and cutting out specialised circuits
- *Medicine* : Cataract surgery, gall-bladder surgery and cauterising gastrointestinal bleeding
- *Military* : Range finding and target designation
- Production of X-rays
- Study of inertial confinement fusion
- Scientific and general laboratory use

7.6   HELIUM–NEON LASER

This type of laser uses a mixture of helium and neon as the active medium. The partial pressure of helium is 1 mm of mercury, while that of neon is 0.1 mm of mercury. Neon atoms are the active centres and helium atoms play an important role in the excitation of neon atoms.

1. Active medium (He, Ne)
2. Electrodes
3. Brewster angle windows
4. Plano-concave perfect reflector
5. Plano-concave partial reflector
6. RF oscillator
7. Quartz discharge tube
8. Laser output
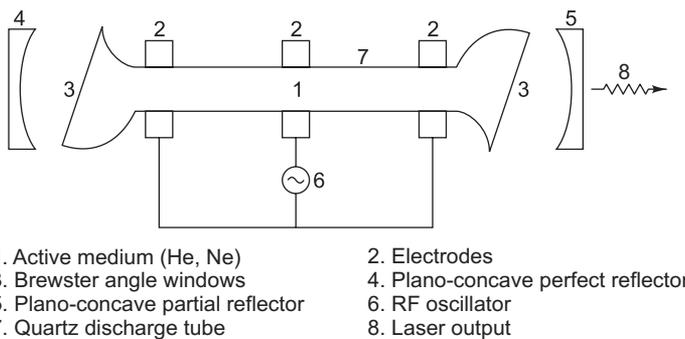
Fig. 7.7 Helium–Neon Laser Device

The helium–neon mixture is kept in a quartz discharge tube of 80 cm length and 1.5 cm diameter (Fig. 7.7). The discharge tube is aligned with the resonator axis. The windows of the discharge tube are placed at the Brewster angle to the resonator axis. It is the angle of incidence α of the light ray which satisfies the condition $\tan \alpha = n$,

where n is the refractive index of the quartz material. Brewster angle windows ensure plane polarised laser output and they also exclude the losses suffered by the laser beam in its reflections at the windows. A pair of plano-concave mirrors act as the optical resonator.

The helium–neon laser is excited by the discharge fired by a radio frequency current. Free electrons from the gas discharge inelastically collide with the helium atoms to excite them. That is,
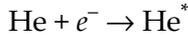
$$He + e^- \rightarrow He^*$$

The excited helium atoms, in turn, inelastically collide with the neon atoms to excite them. That is,

$$He^* + Ne \rightarrow Ne^*$$

The energy-level diagram illustrating the operation of a helium–neon laser is shown in Fig. 7.8.
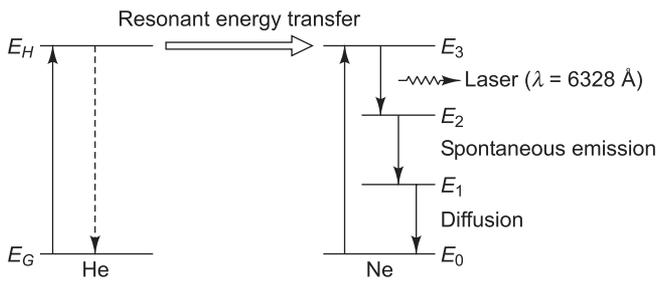
Fig. 7.8 Energy-Level Diagram of Helium–Neon Laser

As a result of resonant energy transfer by the excited He atoms, population inversion of Ne atoms is achieved at the higher energy level, E_3. By spontaneous emission of photons, the excited neon atoms, then, make a transition $E_3 \rightarrow E_2$. The spontaneously emitted photons trigger the stimulated emission of photons. The photons travelling parallel to the resonator axis experience multiple reflections at the optical resonator mirrors. Thus, the transition $E_3 \rightarrow E_2$ yields an intense and coherent laser output at a wavelength of 6328 Å. Then, the Ne atoms make the transition $E_2 \rightarrow E_1$ by spontaneous emission of radiation. The neon atoms, then, make a transition $E_2 \rightarrow E_1$ by inelastically colliding with free electrons and with the discharge tube walls.

Characteristics

- Output wavelength : 6328 Å
- Output power : 0.5 to 50 mW
- Nature of output : Continuous

Applications

- Interferometry
- Laser printing
- Barcode reading
- Pointing
- Fibre optic communication

7.7 CARBON DIOXIDE LASER

In a CO_2 laser, the transitions between various vibrational levels of CO_2 molecules lead to the production of a laser beam. Hence, the study of CO_2 laser requires the knowledge of various vibrational modes of CO_2 molecules.

7.7.1 Vibrational Modes of CO_2 Molecules

The CO_2 molecule, composed of two oxygen atoms and a carbon atom between them, executes three types of vibrational oscillations (Fig. 7.9).

In the symmetric stretching mode, the oxygen atoms oscillate along the molecular axis, by simultaneously departing and approaching the central carbon atom which is stationary. In the bending mode, the carbon and the oxygen atoms oscillate perpendicular to the molecular axis. In the bending mode, oxygen atoms move in one direction, while the carbon atom moves in the opposite direction. In the asymmetric stretching mode of vibration along the molecular axis, the oxygen atoms move in the same direction, while the carbon atom moves in the opposite direction. The frequencies of the symmetric stretching,

(i) Symmetric stretching   (ii) Bending   (iii) Asymmetric stretching
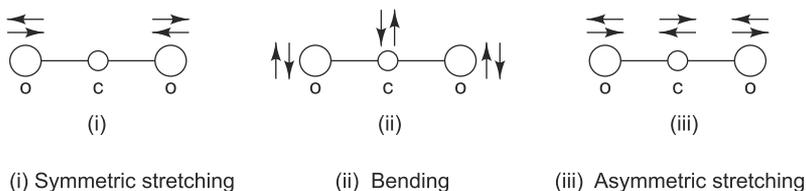
Fig. 7.9 Vibrational Modes of a CO_2 Molecule

bending and asymmetric stretching modes of vibration are denoted by v_1, v_2 and v_3, respectively. The energy contents of these vibrations are as follows:

$$hv_1 = 0.163 \text{ eV}; \quad hv_2 = 0.078 \text{ eV}; hv_3 = 0.276 \text{ eV}$$

At any instant, the CO_2 molecule can be vibrating in a linear combination of these three fundamental modes. The energy state of a CO_2 molecule is represented by a set of three integers ($i\ j\ k$). These numbers represent the amount of energy associated with each mode. For example, the set (0 2 0) means that the CO_2 molecule in this energy state is in a pure bending mode of vibrations with two units of energy.

7.7.2 Construction of CO_2 Laser

The active medium of the laser is a gaseous mixture of carbon dioxide, molecular nitrogen and helium. The active centres are the CO_2 molecules lasing on transitions between various vibrational levels of the electronic ground state.
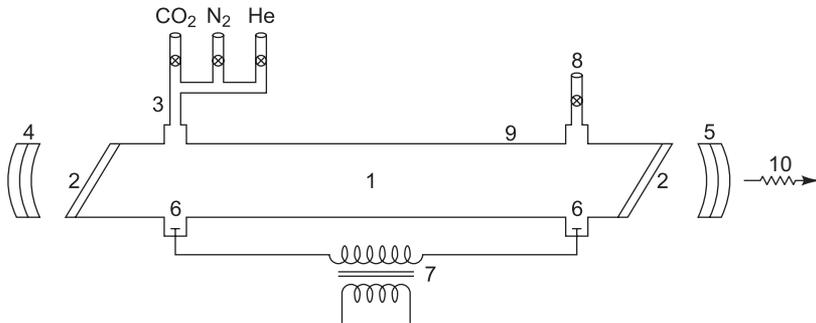
1. Active medium (CO_2, N_2, He) 2. NaCl Brewster angle windows
3. Flow system for active medium 4. Confocal perfect reflector
5. Confocal partial reflector 6. Electrodes
7. dc glow discharge 8. Vacuum pump
9. Discharge tube 9. Laser output

Fig. 7.10 Carbon Dioxide Laser Device

In this type of laser, the active medium is kept in a discharge tube of 5-m length and of 2.5-cm diameter (Fig. 7.10). The carbon dioxide, nitrogen and helium are under the partial pressures of 0.33 mm of mercury, 1.2 mm of mercury and 7 mm of mercury, respectively. After certain hours of operation, the CO_2 molecules of the active

medium dissociate into carbon monoxide. The dissociation products contaminating the active medium are periodically removed away by the vacuum pump. Then, the discharge tube is filled with a new portion of the active medium. The output windows of the discharge tube made of sodium chloride are placed at the Brewster angle to the resonator axis. These windows ensure the plane polarised laser output and the exclusion of reflection losses. A pair of confocal mirrors act as the optical resonator.

The CO_2 laser is excited by a dc glow discharge. The free electrons of the gas discharge inelastically collide with the nitrogen molecules to excite them. That is,

$$N_2 - e^- \rightarrow N_2^*$$
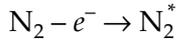
The excited nitrogen molecules, then, inelastically collide with the CO_2 molecules to excite them. That is,

$$N_2^* + CO_2 \rightarrow CO_2^*$$
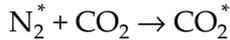
The energy-level diagram illustrating the operation of a CO_2 laser is shown in Fig. 7.11. In the energy-level diagram CO_2 portion of Fig. 7.11, the energy levels correspond to various vibrational modes. Due to resonant energy transfer by the excited N_2 molecules, population inversion of CO_2 molecules is achieved at the higher energy level (0 0 1). Then, by spontaneous emission of photons, CO_2 molecules make the transition (0 0 1 → 1 0 0). The spontaneously emitted photons trigger the stimulated emission of photons. The photons travelling along the resonator axis experience multiple reflections at the optical resonator mirrors. As a result, the transition (0 0 1 → 1 0 0) yields an intense and coherent laser output at a wavelength of 10.6 μm. Similarly, the transition (0 0 1) → (0 2 0) also yields an intense and coherent laser output at a wavelength of 9.6 μm.
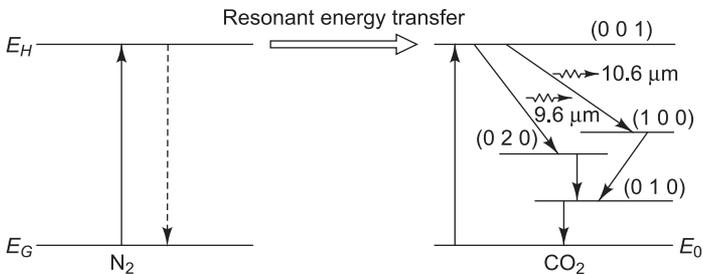
Fig. 7.11 Energy-Level Diagram of a Carbon Dioxide Laser

The CO_2 molecules, then, make the transition $(1\ 0\ 0) \rightarrow (0\ 1\ 0)$ and $(0\ 2\ 0) \rightarrow (0\ 1\ 0)$ through elastic collisions with the unexcited CO_2 molecules. Then, by elastic collisions with helium atoms, the CO_2 molecules make the transition $(0\ 1\ 0) \rightarrow E_0$. Due to their high thermal conductivity, the helium atoms conduct away the heat to keep the CO_2 molecules cold.

Characteristics

- *Output wavelength* : 9.6 µm and 10.6 µm
- *Output power* : 50 to 60 watts
- *Nature of output* : Continuous

Applications

- *Material processing* : Drilling, cutting, material removal, etching, melting, welding, alloying, submelting, annealing and hardening
- *Medicine* : Cutting and cauterising

7.8 SEMICONDUCTOR LASER

The semiconductor lasers lasing from their forward biased *p-n* junctions are also referred to as **injection lasers**. Electroluminescence is the basic principle behind the operation of a semiconductor laser. When a *p-n* junction is forward biased, the majority charge carriers are injected into the depletion region. These injected charge carriers recombine. As a result, photons are emitted. This is called **injection electroluminescence**. The semiconductor laser diodes of injection type are the cheapest and smallest lasers available. They are commercially significant since they can be mass produced and easily fabricated. They have a sufficiently high efficiency of 50 to 60%. These lasers have some disadvantages also. The spectral purity and monochromacity of the semiconductor laser output are poorer than that of the other types of lasers. Based on the number of semiconducting materials used in the fabrication of - junctions, the semiconductor lasers are classified as homojunction lasers and heterojunction lasers.

7.8.1 Homojunction Laser

In this type of laser, the *p-n* junction is fabricated in a single semiconductor. The typical homojunction laser diode built around a gallium arsenide (GaAs) chip is shown in Fig. 7.12. Its highly

degenerate *p* and *n* regions are achieved by doping the material with zinc and tellurium, respectively, in the concentrations of about 10^{19} atoms/cm^3.
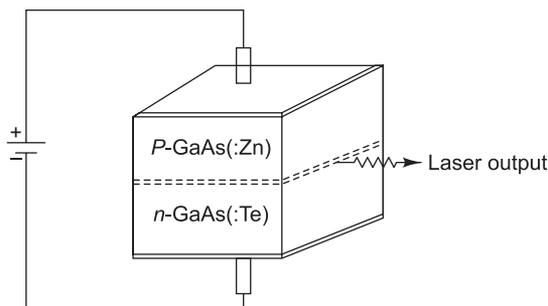
Fig. 7.12 Homojunction Laser Device

The *p-n* junction diode is in the form of a cube with each edge being 1 mm long. The *p-n* junction layer of 1-mm thickness lying in a horizontal plane passing through the centre of the diode acts as the active medium. The polished ends of the GaAs chip, cut at right angles to the *p-n* junction layer, take up the role of resonator mirrors. The other two faces are rough sawn. The diode is immersed in liquid nitrogen.
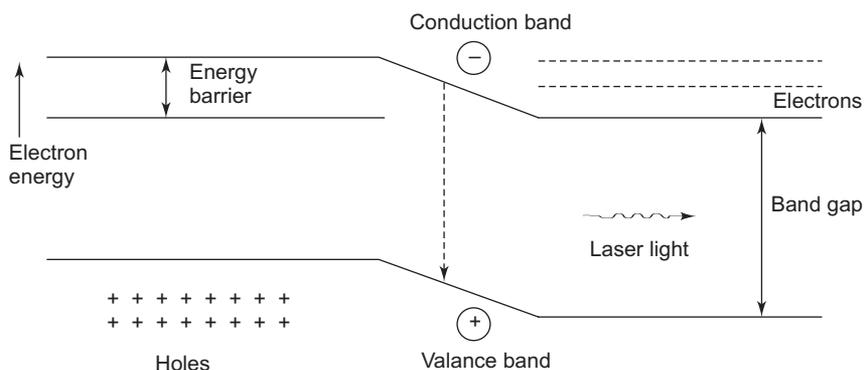
Fig. 7.13 Energy-Level Diagram of Homojunction Semiconductor Laser

As a result of doping, population inversion is achieved in the region of the *p-n* junction. The junction is forward biased. Then, electrons (from the region) and holes (from the *n* region) are injected into the junction region at a sufficiently high rate. The recombination of electrons and

holes in this region results in the emission of photons. These photons trigger the stimulated emission of photons. The photons travelling along the resonator axis experience multiple reflections at the optical resonator mirrors. It results in an intense and coherent laser output confined to a narrow range of wavelengths of 8380 to 8392 Å.

Characteristics

- *Output wavelength* : 8380 to 8392 Å
- *Output power* : 1 mW
- *Nature of output* : Continuous

Applications

- Long-distance communication
- Local Area Network (LAN)
- Compact Disc player
- High-speed printing
- Free space communication
- Pumping source for other solid-state lasers
- Laser pointers
- Medicine

Threshold current density (i.e., current density at which laser oscillation occurs) is sufficiently high for a homojunction laser. It leads to overheating of the semiconductor when operated continuously or at high peak power. Therefore, the homojunction lasers can be effectively operated only at very low temperatures. Moreover, in a homojunction laser, recombination and emission take place over an extensive region. This results in the poor confinement of emitted radiation and makes coupling to a small fibre inefficient. The first semiconductor lasers were operated with homojunction arrangement.

7.8.2   Heterojunction Laser

To have smaller threshold current density and improved confinement of emitted radiation, heterojunction lasers were developed. In heterojunction lasers, *p-n* junctions are formed by dissimilar semiconductors having different band-gap energies and different refractive indices. The changes in band-gap energies create a potential barrier for both holes and electrons. The free charge carriers can meet and recombine only in the narrow, well-defined active layer. As the active layer has a higher refractive index than the materials on either side, an optic waveguide is formed. The confined emission

improves the coupling efficiency, particularly for small fibres. The heterojunction lasers consist of several layers of various materials: semiconducting layers (both doped and undoped), insulating layers and metallic layers for current conduction. A single layer in the centre of these layers (called the active layer) is a direct band-gap material. It is an effective radiator. The adjacent layers (called the cladding layers) are indirect band-gap materials. Heterojunction lasers are fabricated from a range of lattice-matched semiconductor materials.

7.8.3 Simple Heterojunction Laser

Figure 7.14 shows a simple heterojunction laser configuration. It consists of an upper *p*-type layer of AlGaAs followed by a *p*-type layer of GaAs and substrate of *n*-type GaAs. The *p*-type layer of GaAs has an active region of only $0.1 - 0.2\,\mu\text{m}$ thickness. This is the only region where current can flow because AlGaAs serves as a barrier. The thin *p*-GaAs layer is also the only region where recombination radiation can occur. The energy-level diagram (Fig. 7.15) for this arrangement shows the additional 2-eV energy level provided by the *p*-type AlGaAs. This increased energy prevents the flow of current in the AlGaAs region since the energy level lies above the Fermi energy. Thus, the current is confined to the thin *p*-type GaAs region.
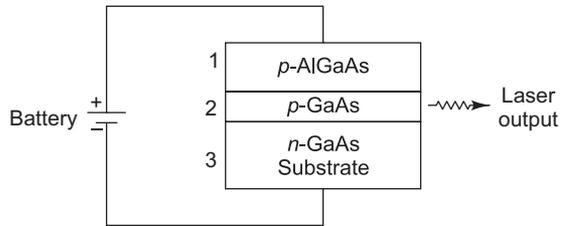
Fig. 7.14 Simple Heterojunction Laser Configuration
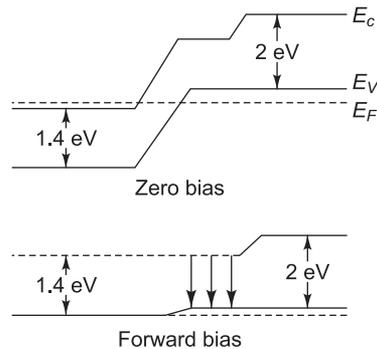
Fig. 7.15 Energy-Band Diagram of a Simple Heterojunction Laser

The *p-n* junction is forward biased. Then, holes (from *p*-AlGaAs, the confining layer) and electrons (from *n*-GaAs, the confining layer) are injected into the active layer *p*-GaAs. Then, holes and electrons recombine in the narrow, well-defined active layer *p*-GaAs.

The polished ends of the diode, cut at right angles to the active layer, take up the role of resonator mirrors. The electron–hole recombinations and multiple reflections of the emitted photons at the resonator mirrors lead to an intense and coherent laser output.

As the charge carriers in heterojunction lasers are confined to a much smaller region than in homojunction lasers, the heat deposition is much lower. Moreover, the change in refractive index at the interface between the *p*-type GaAs and the *p*-type AlGaAs provides a guiding effect for the laser beam.

7.8.4   Double Heterojunction Laser

Figure 7.16 shows a double heterojunction laser configuration composed of various doping combinations of GaAs and AlGaAs. It consists of two junctions J_1 and J_2 of dissimilar semiconductors. It offers more control over the size of the active region. It provides additional refractive index changes leading to an effective guiding effect for the laser beam. The *p-n* junction is forward biased. Then, holes (from *p*-AlGaAs, the confining layer) and electrons (from *n*-AlGaAs, the confining layer) are injected into the active layer *p*-GaAs. Then, holes and electrons recombine in the narrow, well-defined active layer *p*-GaAs. The polished ends of the diode, cut at right angles to the active layer, take up the role of resonator mirrors. The heterojunction J_2 confines within the active layer those minority carriers that are injected over the forward biased *p-n* junction.

1,2,4,5 – Confining layer   3 – Active layer

Fig. 7.16 Double Heterojunction Laser Configuration

Characteristics of Heterojunction Lasers
- High injection efficiency
- Confinement of minority carriers in a double heterostructure

Electrons

Band-gap E_{g_1} Recombination band-gap

Laser light E_{g_2}

Holes
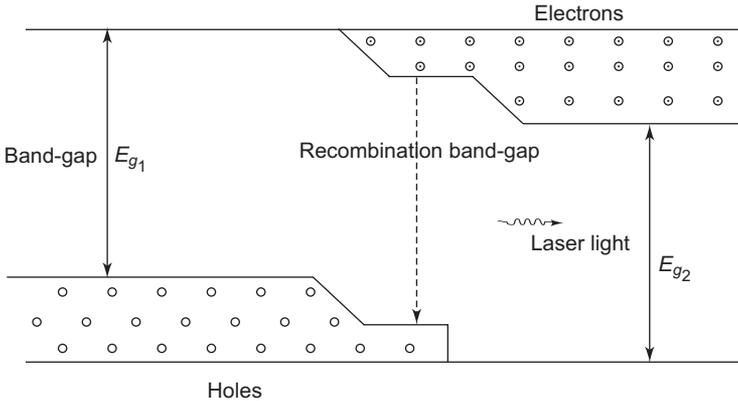
Fig. 7.17 Energy-Level Diagram of Heterojunction Semiconductor Laser

- Improvement of ohmic contacts
- Transparency of the wide band-gap material
- Optical guidance

7.9 INDUSTRIAL APPLICATIONS OF LASERS

The numerous fields of laser applications may be broadly categorised into two groups. One group involves applications where laser beams of high power are exploited to produce a targeted effect on the material. These applications include material processing (say, welding, heat treatment, cutting, hole burning, etc.), isotope separation, medical diagnosis, etc. The other group involves data transmission and processing, measurements and quality control.

Material Processing
Material processing is the science and technology by which a material is converted into a useful structure with properties that are optimised for the proposed application.

7.9.1 Laser Beam Machining
If refers to the machining process in which a laser beam is used to melt and vapourise materials. This process is also used for joining

(welding, crazing, soldering leads in microelectronic circuits and trimming). As shown in Fig. 7.18, the laser beam is focused down to 0.005 inches in diameter. High-energy solid-state lasers are used for machining.

The optical properties of the workpiece determine the wavelength of the laser to be used. Table 7.1 shows the commonly used lasers for material processing.
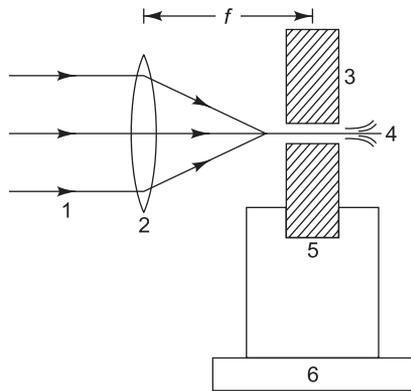
1. Laser beam   2. Lens   3. Workpiece
4. Vapourisation 5. Fixture   6. Table

Fig. 7.18 Laser Beam Machining

Table 7.1   Lasers for Material Processing

Laser Type	Applications
Ruby	Material removal, drilling diamonds, spot welding
Nd-YAG	Welding, resistor trimming, electronic circuit fabrication
CO_2	Cutting metals and organic materials, resistor trimming, welding, drilling
Nd:Glass	Material removal
Argon	Scribing thin films

7.9.2   Laser Beam Hardening

This technique is used to produce hardened surfaces on a variety of industrial components. An absorptive coating (such as zinc or manganese phosphate) is applied to the surface of the component to improve the efficiency of converting light energy into heat. The surface is then scanned with laser. The desired amount of heat input and depth of heating determine the beam size, beam intensity and scanning speed that should be used.

Merits
- High speed
- Little distortion
- Hardening the selected surface areas while leaving the remaining surfaces unaffected

7.9.3 Laser Welding

This type of welding offers a number of advantages which make it preferable. This welding is contactless. Therefore, there is no possibility for introducing deleterious impurities in the weldment. Unlike the electron-beam welding running in vacuum, laser welding is performed under atmospheric conditions. It offers a possibility to weld inaccessible regions too.

Laser welding is capable of fast and highly accurate local melting of a material. The heat-affected area is very small and hence laser welding is especially suitable for welding microelectronic components which are heat-sensitive.

Laser beam

Focusing lens

Work piece to be welded

Fig. 7.19 Laser Welding

High power CO_2 and Nd-YAG lasers are used for joining tips to blades of gas turbines, fastening and cutting steel edges to metal-cutting saws, connecting ciralloy tips to fuel elements of nuclear reactors, and so on.

Laser welding lends itself well to automatic processes in automotive production lines, for joining titanium and aluminium sheets in ship building, in trunk pipe lines, construction works, and so on. Welding nonmetallic materials is also an interesting application of lasers. Common glasses, quartz, etc., can also be laser-welded.

Laser welds can easily be performed between two dissimilar metals. Thus, a thermocouple may easily be welded to a substrate without much damage to the adjacent material.

7.9.4 Laser Heat Treatment

Treatment of the surface layer of an industrial component with a powerful laser beam considerably enhances the strength of the component. Laser heat treatment is advantageous as it provides selective heat treatment of the desirable areas, say, those subjected to intense wear.

In the automobile industry, laser heat treatment is used to strengthen cylinder blocks, valve guides, gears, etc. Laser heat treatment is
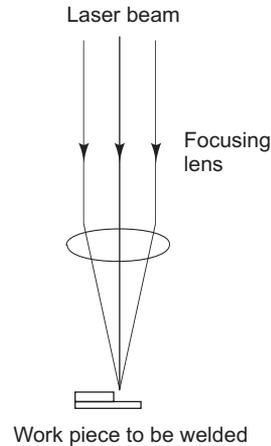
characterised by its considerable speed, high strengthening efficiency and least distortions of the components being processed. A laser widely used for heat treatment is the continuously operating CO_2 laser.

7.9.5 Laser Cutting

The most common laser that is used in the cutting process is the CO_2 laser due to its high output power. The efficiency of laser cutting can be increased by making use of a gas jet coaxial with the laser.

It offers the advantages such as
- Wide range of processed materials (paper, cloth, plywood, glass, asbestos products, ceramics, sheet metal, etc.)
- Possibility of fine and precise cuts
- Minimal amount of mechanical distortion and thermal damage introduced in the material being cut
- Chemical purity of the cutting process
- Possibility of cutting in two or three dimensions according to complicated profiles
- Easy automation of the process and high production rates

The continuously operating CO_2 laser is used for cloth-cutting in the garment industry. Laser cutting is a technology widely used in the aerospace industry, specifically in the fabrication of spacecrafts. A powerful CO_2 laser is used for cutting titanium, steel, and aluminium sheets.
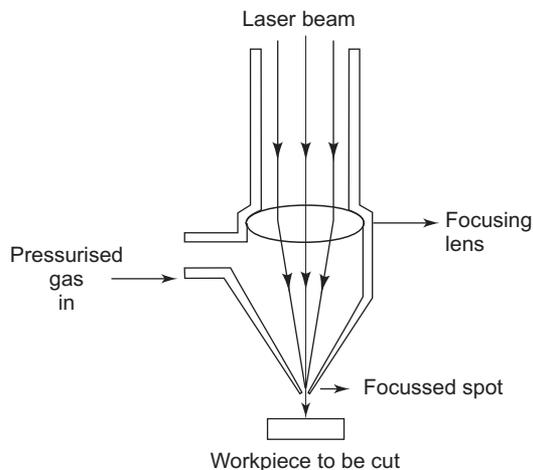
Fig. 7.20 Laser Cutting

7.9.6 Laser Drilling

Hole perforation by laser relies on the intense evaporation of material irradiated by a powerful laser beam. Laser perforation offers a number of advantages over traditional tools. It eliminates the problem of worn out and broken drills. Moreover, lasers perform drilling with extremely high precision. Lasers can perforate very small holes to extremely high depths. Drilling of hard materials is also possible. Another advantage is the possibility of drilling holes in close vicinity to each other and also near the edge of the component.

The CO_2 laser is suited for hole perforation in metallic and nonmetallic (say, plastics, ceramics, glass, etc.) materials. The Nd-YAG laser finds wide use in hole perforation in metals.

7.10 MEDICAL APPLICATIONS

Lasers are used for treating a wide spectrum of diseases involving both the anterior and posterior segments of the eye. Lasers are used in cancer diagnosis and therapy. Lasers are used in breaking kidney stones. Nd-YAG lasers are used for removal of the artery block. Laser scalpel is a scalpel for surgery, cutting or ablating living biological tissue by the energy of laser light. Lasers cause very precise tissue destruction of the lesion in focus and leaves the skin tissue in the immediate surrounding undamaged. Hence lasers have wide range of applications in the field of medicine.

7.11 HOLOGRAPHY

Conventional photography produces a two-dimensional image of a three-dimensional object. In this technique, only the intensity distribution of the light reflected from the object is recorded.

A qualitatively new method of recording optical images known as holography is available. It is a revolutionary technique which allows the three-dimensional (i.e., complete) image of an object to be recorded. The term **holography** is coined from the Greek words "*holos*", meaning whole and "*graphos*", meaning writing.

In the technique of holography, the light reflected from the object is the carrier of information, and both its intensity and phase distributions are recorded. The record is called a **hologram**. When observed through a microscope, a hologram seems to consist of dark

and bright regions only. It does not resemble the original object but it contains the complete record of the object.

7.11.1 Principle of Holography

The holographic method of optical imagery is a two-step method. In the first step, the hologram of the object is recorded. In the second step, the image is reconstructed from the hologram. Hologram recording is based on the interference of coherent light waves. The reconstruction relies on the diffraction of light waves. The light waves must be coherent to form a stable interference pattern.

7.11.2 Hologram Recording

The beam splitter S splits the laser beam into two components O and R (Fig. 7.21). One of them, i.e., beam O is directed towards the object, while the other beam R, to the photographic plate P. The wave illuminating the object is called the **object wave** or **signal wave**. It impinges on the photographic plate after being reflected by the object. The wave that proceeds directly to the photographic plate is called the **reference wave**. Since the object and reference waves originate from the same coherent source, a stable interference pattern is formed when

Fig. 7.21 Hologram Recording

they meet at the photographic plate. The detailed record of this interference pattern constitutes the hologram.

The developed hologram looks like an ordinary exposed negative. Even the sharpest eye cannot reveal anything resembling the object in this photographic plate. Nevertheless, the hologram contains a complete record of the original object. It is encoded in the fine and complex pattern of interference fringes produced by the object and reference waves. This pattern is very fine indeed, and the spacings between its interference fringes may be as small as 0.001 mm.

Conditions for Hologram Recording Holographic imagery requires certain conditions to be satisfied.

- The light should be highly coherent so that $c\tau \gg L$ where c is the velocity of light in vacuum, τ is the coherence time and L is the path difference between the object and reference waves.
- Optical components and the photographic plate must be absolutely still during exposure.
- The exposure time is about 5 seconds and it depends on the object's colour, brightness and the film used.
- The object is to be completely exposed to the laser radiation.
- The photographic plate should have high resolution, high sensitivity, wide spectral range, persistence of the hologram, easy handling and processing and high diffraction efficiency.

7.11.3 Holographic Image Reconstruction

The developed hologram is placed in the position it occupied while being exposed and the object is removed. The hologram is illuminated by a laser beam of the same wavefront and relative orientation as the reference beam making it. This laser beam is called the **readout wave** or **reconstruction wave**. It interacts with the interference pattern on the hologram and gets diffracted to produce two images of the original object (Fig. 7.22).

An observer facing the far side of the hologram will now see a three-dimensional image (called the **virtual image**) occupying the same spatial position as the original object. This virtual image is indistinguishable from the object and it appears in complete three-dimensional form. If one moves one's eye from the viewing position, the perspective of the image changes and it is possible to see the other sides of the object. This image manifests vivid realism.

Fig. 7.22 Holographic Image Reconstruction

The other image (called the **real image**) is formed between the observer and the hologram. The real image *B* appears inverted in depth, i.e., the foreground and background are reversed. Hence, it is also known as the **pseudoscopic image**.

The situation will reverse in another reconstruction arrangement with the illuminating wave travelling in a direction opposite to that of the reference wave. Then, the real image becomes virtual, while the virtual image becomes pseudoscopic.

Conditions for Image Reconstruction For reconstructing the image identical to the original object, certain conditions are to be satisfied.

- The relative distance between the source of illumination and the hologram is maintained to be the same, both during recording and reconstruction.
- The wavelengths of the object wave and of the reconstruction wave are maintained to be the same.
- The angle of diffraction is made equal to the angle of interference.

SUMMARY

- High directionality, high monochromaticity, high intense and high coherence are the characteristic properties of laser radiation.
- Einstien's coefficients *A* and *B* give the spontaneous emission probability per unit time and stimulated emission probability per unit time respectively.
- Under thermal equilibrium, laser action cannot be achieved.
- Population inversion is the condition in which the population of the higher energy level is greater than that of the lower energy level in a material.
- Laser action can be achieved by creating population inversion under nonthermal equilibrium.
- Pumping is the process of creation of population inversion.
- Optical pumping, heat pumping, electrical pumping and chemical pumping are some of the types of pumping process.
- Nd-YAG laser is a four-level solid-state laser.
- He-Ne and CO_2 lasers are gas lasers.
- In CO_2 laser, laser transitions take place between vibrational energy levels.
- Semiconductor lasers are made from direct band-gap semiconductors like GaAs and InP.
- Lasers are widely used in industries for welding, heat treatment, cutting and drilling.

▶ Lasers are also widely used in medicine for cancer diagnosis and therapy, removal of an artery block, cataract removal, etc.

▶ Holography is a three-dimensional lens-less photography, where information is recorded in the form of an interference pattern and retrieved by the principle of diffraction.

SOLVED PROBLEMS

7.1 Prove that no laser action is possible at a wavelength of 5893 Å when the active medium is at room temperature (i.e., 300 K).

Solution Given Data

$$\lambda = 5893 \text{ Å} = 5893 \times 10^{-10} \text{ m}$$

$$\underline{T = 300 \ K}$$

$$\frac{R_{21 \, (sp)}}{R_{21 \, (st)}} = ?$$

Einstein's theory of laser action gives

$$\frac{R_{21 \, (sp)}}{R_{21(st)}} = \exp\left(\frac{h\nu}{kT}\right) - 1$$

or

$$\frac{R_{21(sp)}}{R_{21(st)}} = \exp\left(\frac{hc}{\lambda kT}\right) - 1 \tag{1}$$

where c is the light velocity in vacuum, k is the Boltzmann constant and h is the Planck's constant.

Substituting the given data into Eq. (1), we get

$$\frac{R_{21(sp)}}{R_{21(st)}} = \exp\left(\frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5893 \times 10^{-10} \times 1.38 \times 10^{-23} \times 300}\right) - 1$$

i.e.,

$$\frac{R_{21 \, (sp)}}{R_{21 \, (st)}} = 2.25 \times 10^{35}$$

As the spontaneous emission predominates the stimulated emission, no laser action can take place under the given conditions.

7.2 A carbon dioxide laser emits light at a wavelength of 9.6 μm and has an output power of 55 W. How many photons are emitted in each minute?

Solution Given Data

$\lambda = 9.6\ \mu m = 9.6 \times 10^{-6}$ m

$P = 55$ W $= 55$ J/s

$N = ?$

Energy of a photon is $E \equiv h\nu = hc/\lambda$, where h is the Planck's constant and c is the velocity of light in vacuum. Then, we have

$$N \times E = P \times 60$$

i.e., $$\frac{Nhc}{\lambda} = P \times 60$$

or $$N = \frac{P \times 60 \times \lambda}{hc} \qquad (1)$$

Substituting the given data into Eq. (1), we get

$$N = \frac{55 \times 60 \times 9.6 \times 10^{-6}}{6.62 \times 10^{-34} \times 3 \times 10^{8}}$$

i.e., $N = 1595 \times 10^{20}$ **photons/minute**

7.3 A cadmium sulphide photodetector crystal is irradiated over a receiving area of 4×10^{-6} m^2 by Light of wavelength $0.4 \times 0\ 10^{-6}$m and intensity 200 W/m^2. Assuming that each quantum generates an electron hole pair, calculate the number of pairs generated per second.

(A.U, B.E/B.Tech, Dec. 2001, Apr. 2002)

Solution Given Data

$a = 4 \times 10^{-6}$ m^2

$\lambda = 0.4 \times 10^{-6}$ m

$I = 200$ W/m^2

$n = ?$

Intensity per unit are = Number of photons $\times$ Energy of a photon

$\therefore$ Number of photons $= \dfrac{\text{Intensity} \times \text{Area}}{\text{Energy of photon}}$

$$n' = \frac{Ia}{h\nu}$$

$$n' = \frac{Ia\,\lambda}{hc}$$

$$n' = \frac{200 \times 4 \times 10^{-6} \times 0.4 \times 10^{-6}}{6.625 \times 10^{-34} \times 3 \times 10^{8}}$$

$$n' = 1.6100 \times 10^{15}$$

Since each photon generates an electron-hole pair, the number of photons is equal to the number of electron-hole pairs.

$$n = n'$$

∴ $n = 1.6100 \times 10^{15}$

7.4 The first line of the principal series of sodium is the *D*-line at 590 nm. This corresponds to transition from the first excited state (3P) to the ground state (3S). What is the energy volt of the first excited state?

Solution Given Data

$$\lambda = 590 \text{ nm} = 5900 \times 10^{-10} \text{ m}$$

$$E = ?$$

The energy emitted is given by

$$E = h\nu = \frac{hc}{\lambda}$$

$$E = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{5.9 \times 10^{-7}}$$

$$E = 3.3686 \times 10^{-19} \text{ J}$$

∴ the energy of the first excited state

$$E = \frac{3.3686 \times 10^{-19}}{1.602 \times 10^{-19}} \text{ eV}$$

$$E = 2.10277 \text{ eV}$$

7.5 Calculate the number of photons from green light of mercury ($\lambda = 496/\text{Å}$) required to do one joule of work.

(A.U, B.E/B.Tech, May 2003)

Solution Given Data

$\lambda = 4961 \text{ Å} = 4961 \times 10^{-10}$ m

$N = ?$

Energy $= E = h\nu$

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{4961 \times 10^{-10}} \text{ J}$$

$$E = 4.006 \times 10^{-19} \text{ J}$$

Number of photons required to do one joule of work is given by

$$N = \frac{1 \text{ J}}{4.006 \times 10^{-19} \text{ J}}$$

$$N = 2.4961 \times 10^{18}/\text{m}^3$$

7.6 Calculate of wavelength of radiation emitted by an LED made up of a semiconducting material with band-gap energy 2.8 eV.

(A.U, B.E./B.Tech, May 2003)

Solution Given Data

$E_g = 2.8 \text{ eV} = 2.8 \times 1.602 \times 10^{-19} \text{ J}$

$\qquad = 4.4856 \times 10^{-19}\text{J}$

$\lambda = ?$

$$E_g = h\nu = \frac{hc}{\lambda}$$

$$E_g = \lambda = \frac{hc}{Eg}$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{4.4856 \times 10^{-19}}$$

$$\lambda = 4430.8 \text{ Å}$$

7.7 What is the ratio of stimulated emission to spontaneous emission at a temperature of 280 K for sodium D-line?

Solution Given Data

$T = 280$ K

$\lambda = 590$ nm $= 5.9 \times 10^{-7}$ m

$$\dfrac{R_{21\,(st)}}{R_{21(sp)}} = ?$$

$$\dfrac{R_{21(st)}}{R_{21(sp)}} = \dfrac{1}{\exp\left(\dfrac{h\nu}{KT}\right) - 1}$$

$$= \dfrac{1}{\exp\left(\dfrac{hc}{\lambda\, KT}\right) - 1}$$

$$= \dfrac{1}{\exp\left(\dfrac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.38 \times 10^{-23} \times 280 \times 5.890 \times 10^{-7}}\right) - 1}$$

$$\dfrac{R_{21(st)}}{R_{21(sp)}} = 1.14 \times 10^{-38}$$

7.8 For an InP laser diode, the wavelength of light emission is 1.55 μm. What is its band-gap in eV?   (*A.U, B.E./B. Tech, May 2003*)

Solution Given Data

$\lambda = 1.55$ μm $= 1.55 \times 10^{-6}$ m

$E_g = ?$

$$E_g = h\nu$$

$$E_g = \dfrac{hc}{\lambda}$$

$$E_g = \dfrac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.55 \times 10^{-6}}$$

$$E_g = 1.2855 \times 10^{-19} \text{ J}$$

$$E_g = \frac{1.2855 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$E_g = \textbf{0.8014 eV}$$

7.9 Calculate the long wavelength limit of an extrinsic semiconductor if the ionisation energy is 0.02 eV. *(A.U, B.E./B. Tech, Dec. 2003)*

Solution Given Data

$$E = 0.02 \text{ eV} = 0.02 \times 1.6 \times 10^{-19} \text{ J}$$

$$\lambda = ?$$

$$E - h\nu$$

$$E = \frac{hc}{\lambda}$$

$$\therefore \qquad \lambda = \frac{hc}{E}$$

$$\lambda = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{0.02 \times 1.6 \times 10^{-19}}$$

$$\lambda = \textbf{6.210} \times \textbf{10}^{-5} \textbf{ m}$$

7.10 Calculate the wavelength of emission from a GaAs semiconductor laser whose band-gap energy is 1.44 eV.

Solution Given Data

$$E_g = 1.44 \text{ eV} = 1.44 \times 1.6 \times 10^{-19} \text{ J}$$
$$\lambda = ?$$

$$E - h\nu$$

$$E = \frac{hc}{\lambda}$$

$$\therefore \qquad \lambda = \frac{hc}{E_g}$$

$$\lambda = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{1.44 \times 1.6 \times 10^{-19}}$$

$$\lambda = 8.626.3 \times 10^{-10} \text{ m}$$

$$\lambda = \mathbf{8626.3 \text{ Å}}$$

7.11 Calculate the energy in eV of a photon of wavelength 1.2 Å [Plank's content = 6.62×10^{-34} J/s, Speed of light = 3×10^{9} m/s)

(A.U, B.E./B. Tech, Jan. 2011)

Solution   Given Data

$$\lambda = 1.2 \text{ Å} = 1.2 \times 10^{-10} \text{ m}$$

$$E = ?$$

$$E = h\nu$$

$$E = \frac{hc}{\lambda}$$

$$E = 6.62 \times 10^{-34} \times \frac{3 \times 10^{8} \text{ m/s}}{1.2 \times 10^{-10} \text{ m}}$$

$$E = 1.65 \times 10^{-35} \text{ J/s}$$

$$E = \frac{1.65 \times 10^{-35}}{1.6 \times 10^{-19}}$$

$$E = \mathbf{1.03 \times 10^{-16} \text{ eV}}$$

QUESTIONS AND ANSWERS

7.1   Mention the characteristic properties of laser.

(A.U, B.Sc (C.T & I.T), Jan. 2005), (A.U, B.Sc (C.T & I.T), Jan. 2006)

Monochromacity; Directionality; Coherence; Intensity.

7.2   Write a note on monochromacity of laser.

Laser radiation has a definite wavelength or frequency.

7.3   Write a note on directionality of laser.

Laser radiation travels as a parallel beam over very long distances.

7.4 Write a note on coherent nature of laser.

All the constituent photons of laser radiation have the same energy, same direction of momentum and same direction of polarisation.

7.5 Write a note on high intensity of laser.

The coherent nature of laser radiation results in extremely high concentration of its energy in a small region.

7.6 What is meant by absorption of radiation?

Let a photon of energy $h\nu = E_2 - E_1$ be incident on an atom in the lower energy level E_1. The atom absorbs the incident photon and gets excited to the energy level E_2. This process is called the absorption of radiation.

7.7 What is meant by spontaneous emission of radiation?

The higher energy level is an unstable one. Hence, the excited atom in the higher energy level E_2 spontaneously returns to the lower energy level E_1. It results in the emission of a photon of energy $h\nu = E_2 - E_1$. This process is called spontaneous emission of radiation.

7.8 What is meant by stimulated emission of radiation?

(A.U, B.E / B.Tech, May/June 2005)

An incident photon of energy $h\nu = E_2 - E_1$ stimulates the excited atom in the higher energy level E_2 to return to the lower energy level E_1. It results in the emission of a photon of energy $h\nu = E_2 - E_1$. The inducing and induced photons are identical. This process is called the stimulated emission of radiation.

7.9 What are Einstein's coefficients and how they are interrelated?

The coefficients B_{12}, A_{21} and B_{21} in the expressions for the rates of absorption and emission are called the Einstein's coefficients. Moreover,

$$B_{12} = B_{21}; \quad \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

where B_{12} and B_{21} are the probabilities of absorption and stimulated emission, respectively; A_{21} is the probability of spontaneous emission; h is the Planck's constant; ν is the frequency of radiation; c is the light velocity in vacuum.

7.10   Define population inversion in a material.

It is the condition in which the population of the higher energy level is greater than that of the lower energy level in a material.

7.11   Define active medium.

It is the material with inverted population.

7.12   Define active centres.

The atoms, ions or molecules of the active medium which emit laser radiation are called the active centres.

7.13   What is meant by pumping? Mention the various types of pumping.   *(A.U, B.Sc (C.T & I.T), Jan. 2006)*

It is the mechanism of establishing population inversion in a material. The popular pumping mechanisms are optical pumping, electrical pumping, chemical pumping and heat pumping.

7.14   Define optical pumping.

It is the process of establishing population inversion by light energy.

7.15   Define electrical pumping.

It is the process of establishing population inversion by electrical energy.

7.16   Define chemical pumping.

It is the process of establishing population inversion by exothermal chemical reactions.

7.17   Define heat pumping.

It is the process of establishing population inversion by quenching.

7.18   What is meant by optical resonator? Give its role in a laser device.

It is a pair of reflecting surfaces with one being a perfect reflector and another being a partial reflector. It ensures an intense and coherent laser output.

7.19   Mention the essential components of a laser device.

Active medium; Pumping source; Optical resonator.

7.20   What are the active medium and the active centres in a Nd-YAG laser?

Yttrium Aluminium Garnet ($Y_3 Al_5 O_{12}$), i.e., YAG, doped with neodymium ions Nd^{3+} acts as the active medium. The neodymium ions Nd^{3+} are the active centres.

7.21 Draw the energy-level diagram showing the lasing transition in an Nd-YAG laser.
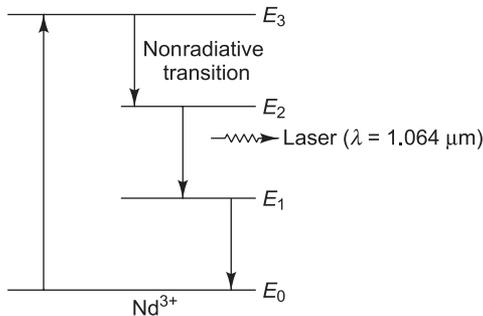
Fig. 7.23

7.22 Mention the characteristics of radiation emitted by an Nd-YAG laser.

Output wavelength : 1.064 μm
Output power : 10^4 W (in pulsed operation)
Nature of output : Pulsed or continuous

7.23 What are the active medium and the active centres in a ruby laser?

Aluminium oxide (Al_2O_3) doped with chromium ions acts as the active medium. The chromium ions Cr^{3+} are the active centres.

**7.24 Draw the energy-level diagram showing the lasing transition in a ruby laser.**
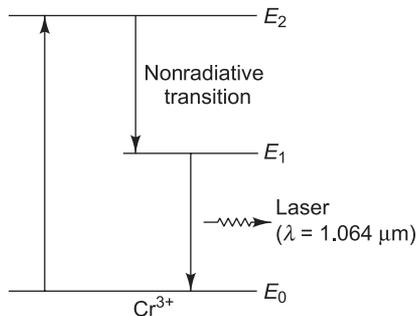
Fig. 7.24

7.25   Mention the characteristics of radiation emitted by a ruby laser.

Output wavelength	:	6943 nm
Output power	:	10^4 to 10^5 W
Nature of output	:	Pulsed

7.26   What is meant by Brewster angle?

It is the angle of incidence of a light ray which satisfies the condition $\tan \alpha = n$, where n is the refractive index of the material.

7.27   State the role of Brewster angle windows in a gaseous laser device.

Brewster angle windows ensure a plane polarised laser output. They also exclude the losses suffered by the laser beam in its reflections at the windows.

7.28   What are the various vibrational modes of a carbon dioxide molecule?

Symmetric stretching, bending and asymmetric stretching.

7.29   What is meant by symmetric stretching of a carbon dioxide molecule?

In the symmetric stretching mode, the oxygen atoms oscillate along the molecular axis, by simultaneously departing and approaching the central carbon atom which is stationary.

7.30   What is meant by bending of a carbon dioxide molecule?

In the bending mode, oxygen atoms move in one direction, while the carbon atom moves in the opposite direction. The carbon and oxygen atoms vibrate perpendicular to the molecular axis.

7.31   What is meant by asymmetric stretching of a CO_2 molecule?

In the asymmetric stretching mode of vibration along the molecular axis, the oxygen atoms move in the same direction, while the carbon atom moves in the opposite direction.

7.32   What are the active medium and the active centres in a CO_2 laser?

The active medium of the laser is a gaseous mixture of carbon dioxide, molecular nitrogen and helium. The active centres are the CO_2 molecules.

7.33 State the principle behind the emission of radiation by a CO_2 laser.

In a CO_2 laser, the transitions between various vibrational levels of CO_2 molecules lead to the production of laser beam.

7.34 What is the role of helium in the working of a carbon dioxide laser?

The helium atoms of high thermal conductivity conduct away the heat to maintain the active medium at a constant temperature.

7.35 Write a note on the two-step process of exciting the active centres in a CO_2 laser.

Free electrons of the gas discharge inelastically collide with the nitrogen molecules to excite them. That is,

$$N_2 + e^- \rightarrow N_2^*$$
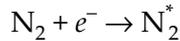
The excited nitrogen molecules, in turn, inelastically collide with the CO_2 molecules to excite them. That is,

$$N_2 + CO_2 \rightarrow CO_2^*$$
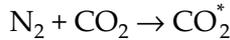
7.36 Draw the energy-level diagram showing the lasing transitions in a CO_2 laser.
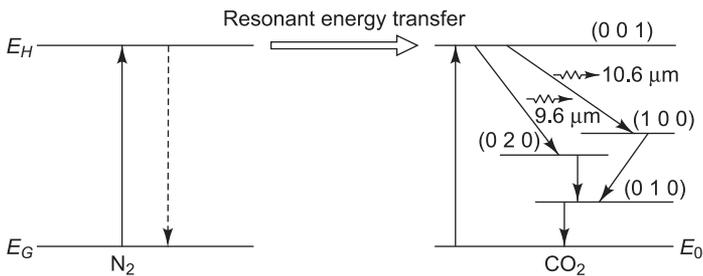
Fig. 7.25

7.37 Mention the characteristics of radiation emitted by a carbon dioxide laser.

Output wavelength : 9.6 μm and 10.6 μm
Output power   : 50 to 60 W
Nature of output : Continuous

7.38 What are the active medium and the active centres in He–Ne laser?

A mixture of helium and neon is the active medium. Neon atoms are the active centres.

7.39 Write a note on the two-step process of exciting the active centres in a He–Ne laser.
Free electrons of the gas discharge inelastically collide with the helium atoms to excite them. That is, $He + e^- \rightarrow He^*$ The excited helium atoms, in turn, inelastically collide with the neon atoms to excite them. That is, $He^* + Ne \rightarrow Ne^*$

7.40 Draw the energy-level diagram showing the lasing transition in an He–Ne laser.
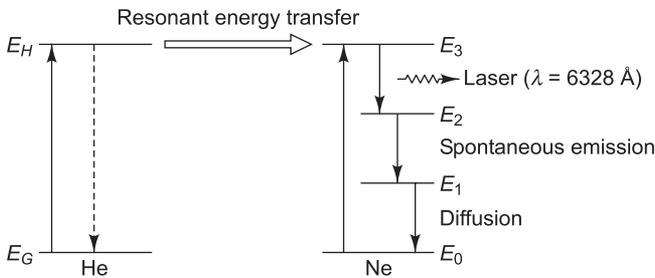
Fig. 7.26

7.41 Mention the characteristics of radiation emitted by He Ne– laser.
Output wavelength : 6328 Å
Output power   : 0.5 to 50 mW
Nature of output : Continuous

7.42 What is the active medium in a semiconductor laser?
The *p-n* junction is the active medium in a semiconductor laser.

7.43 How can population inversion be achieved in a semiconductor laser?
Population inversion can be achieved in a semiconductor laser by doping, irradiation by light or by an electron beam.

7.44 Mention the merits and demerits of a semiconductor laser.
The semiconductor lasers are the cheapest and smallest lasers available. They can be mass produced and easily fabricated. They have a sufficiently high efficiency of 50 to 60%. But, the spectral purity

and monochromacity of the semiconductor laser output are poorer than that of the other types of lasers.

7.45 Mention the characteristics of radiation emitted by a GaAs laser.

Output wavelength : 8380 to 8392 Å
Output power : 1 mW
Nature of output : Continuous

7.46 Define homojunction laser.

In this type of laser, the *p-n* junction is fabricated in a single semiconductor.

7.47 Write a note on laser beam machining.

It refers to the machining process in which a laser beam is used to melt and vapourise materials. This process is also used for joining (welding, crazing, soldering leads in microelectronic circuits and trimming).

7.48 Write a note on laser beam hardening.

This technique is used to produce hardened surfaces on a variety of industrial components. An absorptive coating is applied to the surface of the component to improve the efficiency of converting light energy into heat. The surface is then scanned with a laser. The desired heat input and depth of heating determine the beam size, its intensity and scanning speed used.

7.49 Write a note on laser welding.

This welding is contactless. Therefore, there is no possibility for introducing impurities in the weldment. Laser welding is performed under atmospheric conditions. It offers a possibility to weld inaccessible regions.

7.50 Write a note on the advantages of laser cutting.

Wide range of processed materials, possibility of fine and precise cuts, minimal mechanical distortion and thermal damage to the material, and possibility of cutting in two or three dimensions according to complicated profiles.

7.51 Write a note on laser heat treatment.

Treatment of surface layer of an industrial component with a powerful laser beam considerably enhances the strength of the component. Laser heat treatment is advantageous as it provides selective heat treatment of the desirable areas, say, those subjected to intense wear.

7.52   Write a note on laser drilling.

Hole perforation by the laser relies on the intense evaporation of material irradiated by a powerful laser beam. It eliminates the problem of wear and broken drills. Moreover, lasers perform drilling with extremely high precision. Lasers can perforate very small holes to extremely high depths. Drilling of hard materials is also possible. Another advantage is the possibility of drilling holes in close vicinity with each other and also near the edge of the component.

7.53   What is holography?   *(A.U, B.E / B.Tech, May/June 2005)*

It is a technique which allows the three-dimensional image of an object to be recorded. In this technique, both intensity and phase distributions are recorded.

7.54   What is a hologram?

In holography, the light reflected from the object is the carrier of information and both its intensity and phase distributions are recorded. The record is called the hologram.

7.55   State the principle behind hologram recording.

Interference is the principle behind hologram recording.

7.56   State the principle behind holographic image reconstruction.

Diffraction is the principle behind holographic image reconstruction.

7.57   State the conditions for hologram recording.

The light should be highly coherent. Optical components and the photographic plate must be absolutely still. The exposure time is about 5 seconds and it depends on the object's colour, brightness and the film used. The object is to be completely exposed to the laser radiation. The photographic plate should have high resolution, high sensitivity, wide spectral range, persistence of the hologram, easy handling and processing and high diffraction efficiency.

7.58   State the conditions for holographic image reconstruction.

The wavefronts of the object wave and of the reconstruction wave are to be the same. The relative distance between the source of illumination and the hologram is maintained to be the same both during recording and reconstruction. The wavelengths of the object wave and of the reconstruction wave are maintained to be the same. The angle of diffraction is made equal to the angle of interference.

7.59   How can image magnification be achieved in holography?

If a hologram is made using a laser of wavelength λ but is reconstructed with a laser of wavelength λ_R, the reconstructed image is magnified by a factor λ_R/λ with respect to the object. The image magnification may also be achieved by making the relative distance between the source of illumination and the hologram different compared to the case of hologram recording.

7.60   State a few applications of holography.

Data storing system, associative information search, information coding, pattern recognition, holographic display and holographic scanner.

EXERCISE PROBLEMS

7.1   A helium–neon laser emits light at a wavelength of 6328 Å and it has an output power of 3 mW. Determine the number of photons emitted in one minute.   *[5.74 × 10^{17} photons/minute]*

7.2   Find the intensity of a laser beam of 10 mW power and of a diameter of 1.3 mm. Assume the intensity to be uniform across the laser beam.   *[I = 7538 W/m^2]*

7.3   A laser beam can be focused on an area equal to the square of its wavelength. A helium–neon laser emits power of 1 mW at a wavelength of 6328 Å. Find the intensity of the focused beam.   *[I = 24.97 × 10^8 W/m^2]*

7.4   The first line of the principal series of sodium is the D-line at 5893 Å. This corresponds to transition from the first excited state to the ground state. What is the energy volt of the first excited state.   *[E = 2.1052 eV]*

7.5   Calculate the wavelength of radiation emitted by an LED made up of semi conducting material with band-gap energy 3.7 eV.   *[λ = 3353 Å]*

7.6   A cadmium sulphide photo detector crystal is irradiated over a receiving are of 2×10^{-4} m^2 by light of wavelength 6000 Å and intensity 150 watt/m^2. Assuming that each quantum generates an electron hole pair calculate the number of pairs generated per second.   *[n = 9.056 × 10^{16}]*

7.7 What is the ratio of the stimulated emission to spontaneous emission at a temperature of 230°C for sodium D-line?

$$\left[\frac{R_{21}(\text{st})}{R_{21}(\text{sp})} = 7.73 \times 10^{-22}\right]$$

7.8 Calculate the number of photons from green light of mercury ($\lambda = 2463$ Å) requires to do one joule of work.

$$[N = 1.239 \times 10^{18} \ m^{-3}]$$

7.9 Calculate the wavelength of emission from GaAs semiconductor laser whose band-gap energy is 3.9 eV. $\quad$ [$\lambda = 3185$ Å]

7.10 Calculate the long wavelength limit of an extrinsic semiconductor if the ionisation energy is 1.7 eV. $\quad$ [$\lambda = 7306$ Å]

✓ QUESTION BANK

PART-A

7.1 Which one of the following is an injection laser?
(a) He–Ne laser $\quad\quad$ (b) Nd-YAG laser
(c) GaAs laser $\quad\quad$ (d) CO_2 laser

7.2 The hologram is produced due to _____ of object beam and reference beam.

7.3 What is the active centre in a carbon dioxide laser?

7.4 A carbon dioxide laser can be used to cut certain materials because _____ .

7.5 GaAs doped with tellurium acts as a _____ semiconductor.

7.6 What is the unit of energy density of radiation?

7.7 The host medium in an Nd-YAG laser is_____ .

7.8 List the characteristics of laser.

7.9 What is the principle used in the reconstruction of hologram?

7.10 What is the role of N_2 in the CO_2 laser?

7.11 Holography means _____ .

7.12 Give an example of direct band-gap semiconductor material.

7.13 Give one example of indirect band-gap semiconductor material.

7.14 Why is the threshold current density for a homojunction laser is large?

7.15 What are the two types of coherence?

7.16 Give the need for coherent light in hologram recording.

7.17 Give the lasing transitions in a CO_2 laser.

7.18 Give the output power of a CO_2 laser.

7.19 Write the wavelength of an Nd-YAG laser.

7.20 What is the wavelength of a CO_2 laser?

7.21 What is the active medium of a CO_2 laser?

7.22 Name the type of pumping used in an Nd-YAG laser.

7.23 What is the wavelength of a semiconductor laser on GaAs?

7.24 Give the active medium of a semiconductor laser?

7.25 Is the laser an oscillator or amplifier?

7.26 Write the Brewster's law.

7.27 The word 'LIDAR' is an acronym for _____ .

7.28 What is the material used in Brewster angle windows in a CO_2 laser?

7.29 In which mode of vibration, two oxygen atoms of s CO_2 molecule stretch equally on both sides?

7.30 The Nd-YAG laser is a
(a) Solid laser   (b) Liquid laser
(c) Gaseous laser   (d) Glass laser

7.31 Name the vibrational modes of a CO_2 molecule.

7.32 What is the pumping technique used in a semiconductor laser?

7.33 Give one of the methods used for pumping.

7.34 What is the role of helium in the working of a carbon dioxide laser?

7.35 Name the two different types of LEDs.

7.36 In a laser, positive feedback may be obtained by _____ .

7.37 Which parameter of the used semiconductor determines the wavelength of the LED radiation?

7.38 Which of the following statement is correct?
(a) Semiconductor diode laser is a three-level laser.
(b) Nd-YAG is a four-level laser.
(c) Carbon dioxide laser is a three-level laser.
(d) Carbon dioxide laser is a two-level laser.

7.39 Name the laser which can be used in open air communication.

7.40 In which process upward transition of atom occurs on the energy scale?

7.41 _____ arrangement is made to get polarised laser beam in a gaseous laser system.

7.42 Which gas is used for laser cutting along with the CO_2 laser?

7.43 Expand the acronym 'LASER'.

7.44 Name the laser diode in which we have p and n layers made of different materials.

7.45 The three-dimensional image on hologram is in the form of _____ pattern.

7.46 What is the role of vacuum pump in CO_2 laser?

7.47 What is the active centre of an Nd-YAG laser?

7.48 In fiber optic communication, which type of laser is preferable?

7.49 What is the other name for optical resonator?

2 50 Write a note on coherent nature of laser.

7.51 Give an account on high intensity of laser.

7.52 What is meant by absorption of radiation?

7.53 Define spontaneous emission of radiation.

7.54 What is meant by stimulated emission of radiation?

7.55 Define population inversion in a material.

7.56 What are the different methods of achieving population inversion?

7.57 Define active medium.

7.58 Define active centres.

7.59 What is meant by pumping?

7.60 Define optical pumping.

7.61 Define chemical pumping.

7.62 Write a note on heat pumping.

7.63 What is meant by optical resonator? Give its role in a laser device.

7.64 Mention the essential components of a laser device.

7.65 What are the active medium and the active centres in an Nd-YAG laser?

7.66 Draw the energy level diagram showing the lasing transition in an Nd-YAG laser.

7.67 Mention the characteristics of radiation emitted by an Nd-YAG laser.

7.68 What is meant by Brewster angle?

7.69 State the role of Brewster angle windows in a gaseous laser device.

7.70 What are the various vibrational modes of a carbon dioxide molecule?

7.71 Write a note on symmetric stretching of a carbon dioxide molecule?

7.72 What is meant by bending of a carbon dioxide molecule?

7.73 Give an account of asymmetric stretching of a carbon dioxide molecule?

7.74 What are the active medium and the active centres of a carbon dioxide laser?

7.75 Write a note on the two-step process of exciting the active centres in a carbon dioxide laser.

7.76 Draw the energy-level diagram showing the lasing transitions in a carbon dioxide laser.

7.77 Mention the characteristics of radiation emitted by a carbon dioxide laser.

7.78 How can population inversion be achieved in a semiconductor laser?

7.79 Mention the merits and demerits of a semiconductor laser.

7.80 Mention the characteristics of radiation emitted by a GaAs laser.

7.81 Define homojunction laser.

7.82 Define heterojunction laser.

7.83 State the advantages of a heterojunction laser over a homojunction laser.

7.84 What is Light Emitting Diode (LED)?

7.85 Mention a few examples of LED materials.

7.86 What is surface-emitting LED?

7.87 What is edge-emitting LED?

7.88 In what way does a semiconductor laser differ from a light emitting diode?

7.89 Write a note on laser beam hardening.

7.90 Give an account on laser welding.

7.91 Write a note on laser cutting.

7.92 Write a note on laser drilling.

7.93 What is holography?

7.94 What is hologram?

7.95 Define the principle behind hologram recording.

7.96 State the conditions for hologram recording.

7.97 State the conditions for holographic image reconstruction.

7.98 How can image magnification be achieved in holography?

7.99 State a few applications of holography.

PART-B

7.1 Explain with neat sketches: stimulated absorption, spontaneous emission and stimulated emission with energy-level diagram.

7.2 Discuss the construction and working of a heterojunction semiconductor laser.

7.3 With a neat diagram, describe the construction and working of a CO_2 laser.

7.4 Describe the principle, construction and working of an Nd-YAG laser.

7.5 Explain the modes of vibrations of a CO_2 molecule. Describe the working of a CO_2 laser with necessary diagram.

7.6 Explain the construction and working of a semiconductor laser with a diagram.

7.7 Discuss the construction and working of a homojunction semiconductor laser.

7.8 Describe the construction and reconstruction of a hologram.

7.9 Discuss the applications of laser in industries.

7.10 Discuss the construction and working of a He–Ne laser.

7.11 Define the following terms:
(a) Population inversion   (b) Pumping
(c) Active medium   (d) Active centres

7.12 Compare the characteristics of laser with ordinary source of light. What are the conditions necessary for laser action?

8 FIBRE OPTICS AND APPLICATIONS

8.1 INTRODUCTION

Fibre optics is the science and technology of information transfer by means of light guided through dielectric waveguides. Communication may be defined as the transfer of information from one point to another. In a communication system, the information is usually superimposed onto an electromagnetic wave which acts as a carrier. At the receiver end, it is demodulated and the original information is obtained. Since the information carrying capacity depends on the frequency or bandwidth of the carrier, electromagnetic waves are more suitable for communication than microwaves or radiowaves. Great interest in communication was created in 1960 with the advent of the laser which made available a coherent optical source. Unguided optic communication systems were developed shortly after the discovery of the laser. The light signal, when transmitted through the atmosphere, is subjected to attenuation and distortion even when weather conditions are fair. Therefore, a better light wave communication system needs a light guide to preserve the signal and increase the reliability and distance of transmission.

As early as 1854, John Tyndall, a British physicist, demonstrated that light can be guided along a curved stream of water. However, it was not until 1910 that anything like a practical waveguide was envisaged. Experiments on glass fibres packed into bundles were carried out in the 1930s to make them work as light guides. The major breakthrough in the development of optical fibre came with its use in telecommunication in the mid sixties. In 1970, Karpon, Keck and Maurer of the Corning Glass Works fabricated a silica fibre having 20 dB/km attenuation. In the next two decades, researchers worked intensively to reduce the attenuation to 0.16 dB/km at a 1550 nm wavelength.

The development and application of optical fibre systems grew from the combination of semiconductor technology, which provided the necessary light sources, and photodetectors and optical waveguide technology.

8.2 LIGHT PROPAGATION THROUGH OPTICAL FIBRES

The process of propagation of light in an optical fibre is simple, because once the light enters the fibre, it does not encounter any new surface, but repeatedly hits the same surface. Total internal reflection is the principle of confining the light beam inside the fibre.

When a light ray travelling from a denser medium to a rare medium is incident on the boundary at an angle greater than the critical angle, the light ray is reflected back into the originating medium. When the light ray is incident on the boundary at an angle lesser than the critical angle, the light ray is refracted into the rarer medium. When the light ray is incident on the boundary at an angle equal to the critical angle, the light ray traverses along the interface of the denser and rarer medium. All the above processes are schematically shown in Fig. 8.1.
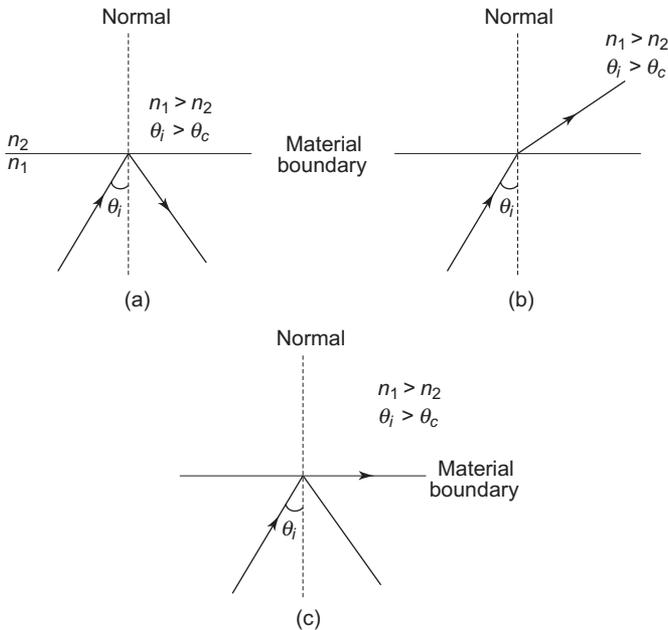

Fig. 8.1 Total Internal Reflection

8.2.1 Basic Optical Laws and Definitions

We shall now review some of the basic optical laws and definitions relevant to optical fibres. A fundamental optical parameter of a material is the refractive index defined as

$$n = c/v \qquad\qquad (8.1)$$

where c and v are the speeds of light in vacuum and in the given material, respectively.

Snell's Law When a light ray is incident on the boundary between two media, a part of the ray is reflected back into the first medium and a part is refracted into the second medium (Fig. 8.2). The transmission of light through a boundary is governed by Snell's law.
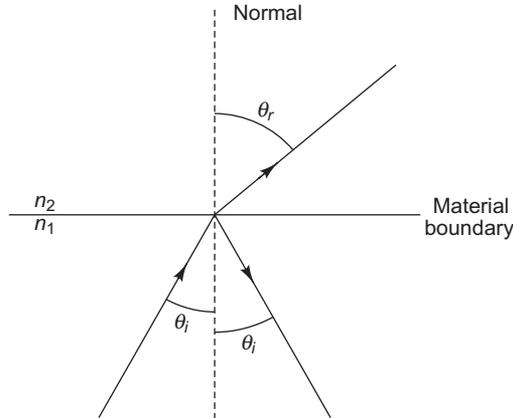
Fig. 8.2 Reflection and Transmission of Light

According to Snell's law, we have

$$n_1 \sin \theta_i = n_2 \sin \theta_r \qquad\qquad (8.2)$$

where n_1 and n_2 are the refractive indices of the first and the second medium, respectively; θ_i is the angle of incidence; θ_r is the angle of refraction.

Consider a light ray travelling from a denser medium (of refractive index n_1) to a rarer medium (of refractive index n_2). Then, according to Snell's law given by Eq. (8.2), the angle of refraction is greater than the angle of incidence. Hence, an increase in the angle of incidence results in an increase in the angle of refraction. At a particular angle of incidence, called the critical angle, the angle of refraction will be equal to 90°. Then, light travels along the material boundary.

Critical Angle It is the angle of incidence of a light ray travelling from a denser medium to a rarer medium, for which, the angle of refraction is 90°. Application of Snell's law to the light ray shown in Fig. 8.3 gives

$$n_1 \sin \theta_c = n_2 \sin 90°$$

i.e.,
$$n_1 \sin \theta_c = n_2$$

or
$$\sin \theta_c = \frac{n_2}{n_1} \tag{8.3}$$

Thus, the critical angle is given as

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \tag{8.4}$$
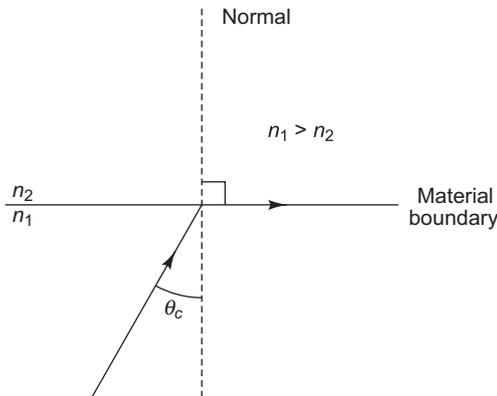
Fig. 8.3 Critical Angle of Incidence

Based on these studies, the following conditions for total internal reflection are established:

1. Light should travel from a denser medium to a rarer medium.
2. The angle of incidence at the material boundary should be greater than the critical angle.

8.2.2 Configuration of an Optical Fibre

Typically, an optical fibre consists of a central solid cylinder (called the **core**) made of a high-refractive-index material. In fact, the core is the actual fibre and it has the remarkable property of conducting a light beam. The core is surrounded by a cylindrical tube (called **cladding**) made of a low-refractive-index material (Fig. 8.4). The cladding ensures

- The condition for total internal reflection at the core-cladding boundary
- The reduction in scattering losses due to discontinuities at the core surface
- The addition of mechanical strength to the fibre
- The protection of core from surface contaminants

Moreover, an outer jacket of plastic or polymer protects the fibre from moisture, abrasion, crushing and other environmental changes.
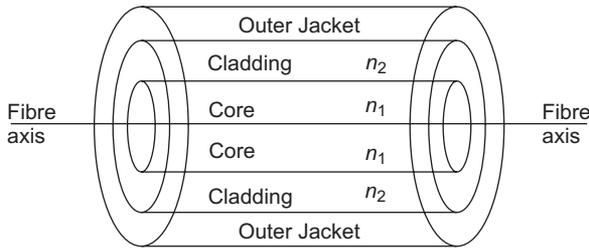
Fig. 8.4 Typical Configuration of an Optical Fibre

Conventionally, the size of an optical fibre is denoted by writing its core diameter and then its cladding diameter (both in micrometres) with a slash between them.

8.2.3   Basic Parameters of an Optical Fibre

An optical fibre is characterised by parameters such as the critical angle, acceptance angle, acceptance cone and numerical aperture.

Acceptance Angle   Consider an optical fibre (Fig. 8.5) of core refractive index n_1 and of cladding refractive index n_2. Let n_1 be greater than n_2. Let n_0 be the refractive index of the surrounding medium. Let θ_a be the angle of launching the light ray at the surrounding medium-core boundary, for which, the angle of incidence at the core-cladding boundary is equal to the critical angle θ_c.
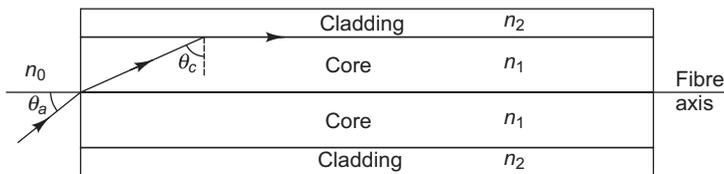
Fig. 8.5 Acceptance Angle of a Fibre System

Applying Snell's law to the given light ray, we get

$$n_0 \sin \theta_a = n_1 \sin (90° - \theta_c)$$

i.e.,
$$n_0 \sin \theta_a = n_1 \cos \theta_c$$

or
$$n_0 \sin \theta_a = n_1 \sqrt{1 - \sin^2 \theta_c}$$

Substituting Eq. (8.3) into the above equation, we get

$$n_0 \sin \theta_a = \sqrt{n_1^2 - n_2^2} \tag{8.5}$$

$$\theta_a = \sin^{-1} \left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0} \right) \tag{8.6}$$

An analysis of launching light into the optical fibre reveals three different cases as follows:

Case 1   When a light ray is launched into the fibre at an angle equal to the acceptance angle, it will travel along the core–cladding boundary and it will be delivered at the output end of the fibre.

Case 2   If a light ray is launched into the fibre at an angle less than the acceptance angle, it will experience total internal reflection at the core–cladding boundary and it will be delivered at the output end of the fibre.

Case 3   When a light ray is launched into the fibre at an angle greater than the acceptance angle, it is refracted into the cladding and is eventually lost by radiation.

Thus, the **acceptance angle of an optical fibre system is defined as the angle of launching of a light ray into the fibre core so that the light ray is transmitted along the core-cladding boundary and gets delivered at the output end of the fibre**. The acceptance angle is also called t**he maximum acceptance angle**.

Acceptance Cone   Obviously, for light rays to be transmitted by total internal reflection within the fibre core, they must be launched into the fibre core within the acceptance cone defined by the half conical angle θ_a (Fig. 8.6).

Numerical Aperture   The numerical aperture (NA) of an optical fibre is a measure of its light collecting ability. It is defined as

$$NA = n_0 \sin \theta_a \tag{8.7}$$
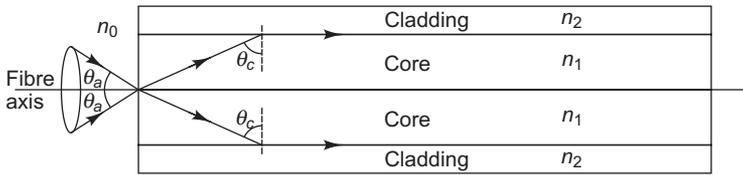
Fig. 8.6 Acceptance Cone of a Fibre System

Then, Eqs (8.5) and (8.7) give

$$NA = \sqrt{n_1^2 - n_2^2} \qquad (8.8)$$

In terms of the relative refractive index difference $\Delta \equiv (n_1^2 - n_2^2)/2n_1^2$, an alternative expression for the numerical aperture is given as

$$NA = n_1\sqrt{2\Delta} \qquad (8.9)$$

Equation (8.7) shows the advantage of a fibre having a larger numerical aperture. It implies a larger acceptance angle and thus improved light collection.

8.3   TYPES OF OPTICAL FIBRES

The refractive index profile, modes of propagation and the fibre materials can serve as the criteria for classifying fibres into various types.
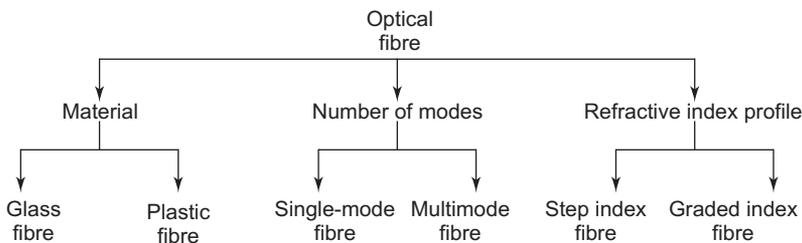
8.3.1   Classification I

Based on the refractive index profile of the core, optical fibres are classified as
   • Step index fibres
   • Graded index fibres

Step Index Fibre (SI Fibre)   In step index fibre, the variation of refractive indices of air, cladding and care varies step by step, and hence it is

called step index fibre. It consists of a core of constant refractive index surrounded by a cladding whose refractive index is less than that of the core [Fig. 8.7(a)].

i.e., $n(r) = \begin{cases} n_1, & \text{for } r < a \\ n_2, & \text{for } r \geq a \end{cases}$ (8.10)

where a is the core radius and r is the radial distance.

Fig. 8.7(a) Refractive Index Profile of an SI Fibre

As seen, there is an abrupt change between the refractive indices of the core and the cladding. This abrupt change gives this fibre the name step index. The values of core refractive index n_1 and cladding refractive index n_2 are chosen such that the relative refractive index difference is about 0.01. Since the core refractive index is larger than the cladding refractive index, the light propagates along the fibre by total internal reflection at the core-cladding boundary. The light ray travels through the SI fibre in straight lines [Fig. 8.7(b)]. Typical dimensions of SI fibres are 50/125, 100/140 and 200/230.

Fig. 8.7(b) Ray Path in a Step Index Fibre

Due to the constant value of core refractive index of an SI fibre, the simultaneously launched signals will not emerge out simultaneously. That is, the SI Fibre exhibits dispersion. It results in the broadening of pulses transmitted through SI Fibres.

Common Forms of SI Fibres In general, various materials may be used for fabricating the core and the cladding of a step index fibre. It results in the availability of step index fibres in a number of forms.

There are
- A glass core cladded with another glass
- A glass core cladded with a plastic
- A plastic core cladded with another plastic

Performance Characteristics of All-Glass Fibres
- Lowest losses
- Smallest intermodal pulse spreading
- Useful at moderately high information rates
- Useful at fairly long range
- Low numerical aperture

Performance Characteristics of Plastic Clad Fibres
- Higher losses
- Larger pulse spreading
- Suitable for shorter links (few hundred metres of path length)
- Improved source coupling due to large core diameter

Performance Characteristics of All-Plastic Fibres
- High propagation losses
- Limited to very short paths (less than a few tens of metres)
- Large numerical aperture
- Higher coupling efficiency

Graded Index Fibre (GRIN Fibre)   The refractive index of the core varies radially from the axis of the fibre. The refractive index of the core has the maximum value at the core axis and it gradually decrease radially towards the outer edge of the core [Fig. 8.8(a)]. That is,

Fig. 8.8(a) Refractive Index Profile of a Graded Index Fibre

$$n(r) = \begin{cases} n_1\sqrt{1-2\Delta(r/a)^\alpha}, & \text{for} \quad r < a \\ n_2 & , \quad \text{for} \quad r \geq a \end{cases}$$

(8.11)

where α is a constant defining the shape of the refractive index profile of the core. The commonly used GRIN fibres have a parabolic refractive index profile with $\alpha = 2$.

Light ray travels through a GRIN fibre in an oscillatory fashion [Fig. 8.8(b)]. The changing refractive index of the core makes the light

rays to be continuously redirected toward the fibre axis. In this manner, the fibre traps the light ray, causing it to oscillate back and forth as it propagates down. The refractive index variations in a typical fibre given by Eq. (8.11) periodically refocuses the light rays.
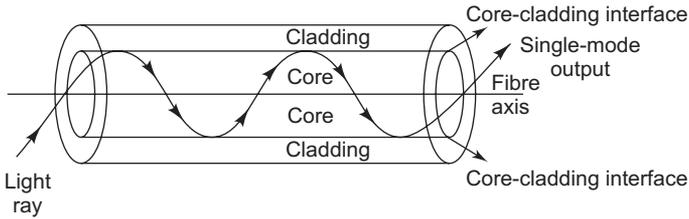
Fig. 8.8(b) Ray Path of a Graded Index Fibre

The numerical aperture of a graded index fibre is not constant across the core. The local numerical aperture is given as

$$NA(r) = \begin{cases} \sqrt{n^2(r) - n_2^2} \cong NA(0)\sqrt{1-(r/a)^\alpha} & \text{, for } r < a \\ 0 & \text{, for } r \geq a \end{cases} \quad (8.12)$$

where $NA(0)$ is the axial numerical aperture given by

$$NA\,(0) = \sqrt{n_1^2 - n_2^2} \cong n_1\sqrt{2\Delta} \quad (8.13)$$

Thus, the numerical aperture of a GRIN fibre has its maximum value at the fibre axis. And it drops to zero at the edge of the core. In case of a parabolic refractive index profile, the numerical aperture of a GRIN fibre is given as

$$NA = n_1(2\Delta)^{1/2}\sqrt{1-(r/a)^2} \quad (8.14)$$

Table 8.1 Performance Characteristics of SI and GRIN Fibres

Fibre	Merits	Demerits
SI	Low cost	Small bandwidth
	Simple fabrication	Large intermodal dispersion
	Constant numerical aperture	
	Higher coupling efficiency	
	Small intermodal dispersion	High cost
GRIN	Large bandwidth	Variable numerical aperture
		Smaller coupling efficiency

8.3.2 Classification II

When light rays are transmitted through optical fibres, the conditions for total internal reflection and constructive interference of light rays are to be satisfied. This can be achieved only when light rays are launched into the fibres at certain angles. That is, light rays can take only certain paths across the fibre core. In this respect, optical fibres are said to support only certain modes of propagation.

Modes of propagation refer to the various paths that light can take in the fibre core. Based on the mode of propagation of light ray, optical fibres are classified as

- Single-mode fibres
- Multimode fibres

A **single-mode fibre** allows only one mode of propagation of light through it. On the other hand, a **multimode fibre** allows more than one mode of propagation of light through it. The performance characteristics of single-mode and multimode propagation are compared in Table 8.2.

Table 8.2   Characteristics of Single-mode and Multimode Propagation

Single-mode propagation	*Multimode propagation*
Larger information capacity	Smaller information capacity
Absence of modal pulse spreading	Modal pulse spreading
Long-haul applications	Short-haul applications
Larger information-capacity applications	Smaller information-capacity applications

Single-mode step index, multimode step index and multimode graded index fibres are commercially available.

Single-Mode Step Index Fibre   It is a step index fibre and has a very small core diameter so that it can allow only one mode of propagation and hence it is called single-mode fibre. In this case, the optical loss is very much reduced.
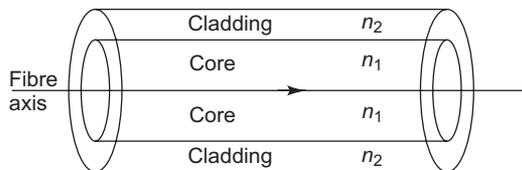
Fig. 8.9   Ray Path in a Single-Mode Step Index Fibre

Typically, the core diameter of a single-mode step index fibre is about 2 to 10 micrometer. And the cladding diameter is about 125 micrometer.

Multimode Step Index Fibre It is a step index fibre which has very large core diameter compared to single-mode fibres, and it can allow many modes of propagation of light rays through the fibre core.
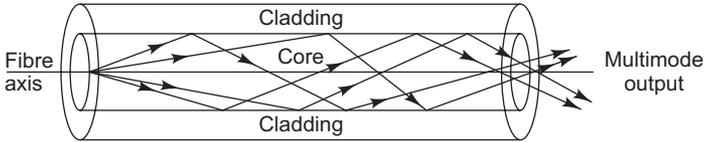
Fig. 8.10 Ray Paths in a Multimode Step Index Fibre

A typical multimode step index fibre has a core diameter of about 50 μm and a cladding diameter of about 125 to 400 μm.

By appropriately choosing the wavelength of the transmitted light, the given fibre can be operated as a single-mode one or a multimode one. An important parameter related to the modes supported by an optical fibre is the **normalised frequency** (also called the *V* number or the *V* parameter). It is given as

$$V = \frac{2\pi a \sqrt{n_1^2 - n_2^2}}{\lambda} \qquad (8.15)$$

where a is the core radius; n_1 and n_2 are the refractive indices of the core and cladding, respectively; and λ is the wavelength of transmitted light.

The *V* parameter determines how many modes a fibre can support. The number of modes supported by a multimode step index fibre with a large value of *V* parameter (i.e., $V > 10$) is given by

$$N = \frac{V^2}{2} \qquad (8.16)$$

A multimode fibre can also be operated as a single-mode fibre by appropriately choosing the wavelength of light transmitted through the fibre.

Single-mode propagation of light through an SI fibre is assured if

$$v = \frac{2\pi a \sqrt{n_1^2 - n_2^2}}{\lambda} \leq 2.405 \qquad (8.17)$$

$$\lambda = hc/E_g$$

Equation (8.17) gives the **single-mode condition** (also called the **cut-off condition**) for a step index fibre.

Multimode Graded Index Fibre   It is a graded index fibre and has large core diameter and it can allow many modes of propagation of light rays through the fibre core.
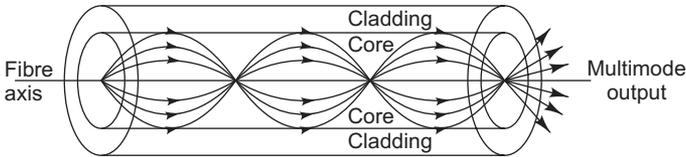
Fig. 8.11   Ray Paths in a Multimode Graded Index Fibre

A typical multimode graded index fibre has a core diameter of 50 to 100 μm and a cladding diameter of 125 to 140 μm.

The number of modes supported by a multimode graded index fibre with a large value of V parameter (i.e., $V > 10$) is given by

$$N = \frac{V^2}{4} \tag{8.18}$$

Single-mode propagation of light through a GRIN fibre is assured if

$$\frac{2\pi a \sqrt{n_1(n_1 - n_2)}}{\lambda} \leq 2.405 \tag{8.19}$$

Equation (8.19) gives the **single-mode condition** (also called the **cut-off condition**) for a graded index fibre.

The performance characteristics of multimode SI and multimode GRIN fibres are presented in Table 8.3.

Table 8.3   Characteristics of Multimode SI and GRIN Fibres

Mutimode SI fibre	*Multimode GRIN fibre*
Low information rate	High information rate
High source-coupling efficiency	Low source-coupling efficiency
------------------ Equal fibre losses-------------------	
High pulse distortion	Low pulse distortion
Short-range applications	Long-range applications

As seen from Eqs (8.16) and (8.18), it follows that a step index fibre can support two times the number of modes that are supported by a comparable graded index fibre.

8.3.3   Classification III

Materials used to produce the optical fibres must satisfy the following requirements. The fibre optic materials should have slightly different refractive indices for the core and the cladding, should be transparent at a particular optical wavelength and should have the malleability so that long, thin, flexible fibres can be drawn. The materials satisfying these requirements are glasses and plastics. Based on the materials used in their fabrication, optical fibres are classified as

- Glass fibres
- Plastic fibres

Glass Fibre   It is made of glass consisting of silica (SiO_2) or silicate. To produce materials that have slightly different refractive indices, dopants such as B_2O_3, GeO_2 or P_2O_5 are added to the silica.

Typical Glass-Fibre Composition
- $GeO_2 - SiO_2$ core; SiO_2 cladding
- $P_2O_5 - SiO_2$ core; SiO_2 cladding
- SiO_2 core; $B_2O_3 - SiO_2$ cladding
- $GeO_2 - B_2O_3 - SiO_2$ core; $B_2O_3 - SiO_2$ cladding

The variety of available glass fibres ranges from high-loss glass fibres used for short-haul applications to low-loss glass fibres used for long-haul applications.

Plastic Fibre   Plastic fibres are typically made of plastic due to their flexibility and law cost. The core of this fibre is made of polymethyl methacryolate or perfluorinated polymer. It has greater mechanical strength compared to a glass fibre. But, the plastic fibre has higher attenuation compared to a glass fibre. Hence, plastic fibres are used only in short-haul applications. The plastic fibres are handled without special case due to their toughness and durability.

Typical Plastic-Fibre Composition
- Polymethylmethacrylate core; co-polymer cladding
- Polystyrene core; methylmetha crylate cladding

8.4   FIBRE OPTIC COMMUNICATION LINK

Optical fibres act as waveguides for transmitting light signals from one place to another. A fibre optic communication link is schematically shown in Fig. 8.12. The main components of a fibre optic communication link are

1. Optical source
2. Means of modulating the optical output from the source with the signal to be transmitted
3. The transmission medium (optical fibre)
4. Couplers to couple the optical fibre to the source and to the detector
5. The photodetector which converts the received optical power back into an electrical wave form
6. Electronic amplification and signal processing required to recover the signal and present it in a form suitable for use
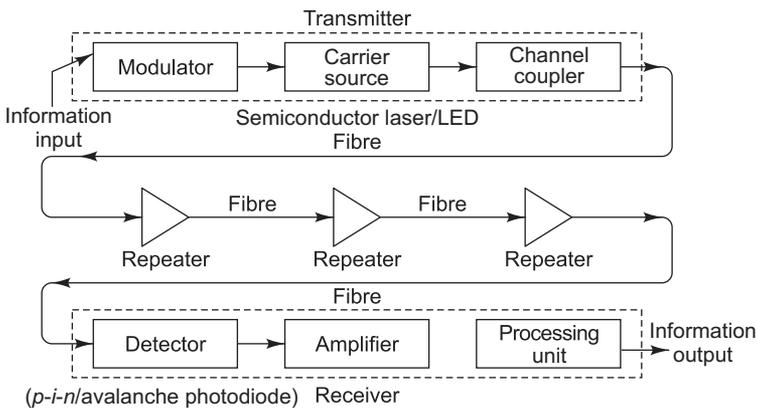
Fig. 8.12 Fibre Optic Communication Link

At the transmitter end, the input information is modulated and fed to the light source. Semiconductor lasers or light emitting diodes are used as the light source because of their compatibility with optical fibres. The output from the light source is coupled to the optical fibres by a connector. An optical fibre is used as the transmission medium. The repeaters, placed at regular intervals, boost the light transmitted through the fibre.

At the receiver end, the optical fibre is coupled to a photodetecor which converts the optical signal into an electrical signal.

Typically, a *p-i-n* or avalanche photodiode is used as a detector. The electrical signal is amplified and is fed to a processing unit which decodes the received information to give the output information.

The number of messages that can be simultaneously transmitted through the system is determined by

- The bandwidth requirement of the message
- The bandwidth of the carrier

In an optical communication link, light is the carrier of information. As light has a bandwidth of about 350×10^6 MHz, it can accommodate about 30×10^6 TV programmes at a time. Similarly, light can carry about 40×10^{10} telephone calls simultaneously. Thus, the use of light waves tremendously expands our communication capacities. Fibre optic communication links are widely used in telecommunication, data links, local area network, analog links for video and telemetry, broadband networks, etc.

8.5   ADVANTAGES OF FIBRE OPTIC COMMUNICATION

Large Information-Carrying Capacity   The optical fibre communication system has larger information-carrying capacity in comparison with other conventional systems.

Small Size and Weight   Optical fibres have very small diameters and are made of low-density materials such as glass and plastic. Hence, the optical fibres are far smaller and much lighter in weight than the copper-wire system.

Electrical Isolation   Optical fibres are fabricated from dielectric materials. Therefore, optical fibre communication is ideally suited for electrically hazardous environment as they do not create any short circuit.

Immunity to Interference and Cross Talk   Optical fibres are dielectric waveguides. Hence, they are free from electromagnetic interference. The absence of electromagnetic interference makes cross talk negligible, even when the fibres are cabled together.

Signal Security   There is no significant radiation of light from optical fibres. Therefore, they provide a high degree of signal security.

Low Transmission Loss   Optical fibres exhibit very low transmission loss (of about 0.2 dB km^{-1}). Hence, optical fibre communication is highly suitable for long-haul applications.

Flexibility and Ruggedness Optical fibres are flexible and extremely rugged.

System Reliability Optical fibre systems are highly reliable as only a few intermediate repeaters are employed to boost the signal strength.

Easy Maintenance Optical fibre systems are easy to maintain as their components have longer lifetime (of about 20 to 30 years).

Low Cost Typically, sand is the raw material used for fabricating optical fibres. As sand costs less, the optical fibres cost less.

8.6 FIBRE OPTIC SENSORS

Optical fibres can be used to acquire information about an environmental parameter of interest such as pressure, temperature, voltage, current, etc., and to transport the information to a convenient location for processing. Intrinsic or Active sensors and Extrinsic or Passive sensors are the two types of sensors.

Active Sensors In this type of sensors, the physical parameter to be sensed acts directly on the fibre itself to produce changes in the transmission characteristics.

Passive Sensors In this type of sensors, separate sensing will be utilized and the fibre will act as a guiding media to the sensors.

8.6.1 Fibre Optic Displacement Sensor

It is schematically shown in Fig. 8.13. It consists of two bundles of fibres. One bundle of fibres transmits light to the reflecting target whose location is to be determined.

1. Light source   2. Light detector   3. Transmitting fibre
4. Receiving fibre   5. Lens   6. Reflecting target
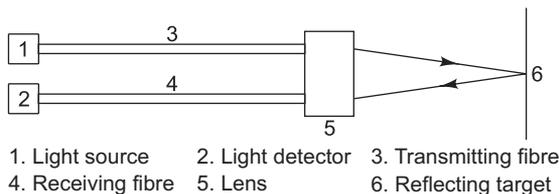
Fig. 8.13 Fibre Optic Displacement Sensor

The other bundle traps the reflected light and transmits it to the detector. The intensity of the detected light depends on the relative distance between the fibre optic probe and the target. The lens system expands the range of the sensor.

8.6.2 Fibre Optic Temperature Sensor

It is schematically shown in Fig. 8.14. The transmitting fibre carries the light to the reflecting mirror. In its path, the light is polarised and it passes through a birefringent crystal. In a birefringent crystal, the refractive index is different for the orthogonally polarised light waves. The light reflected from the mirror passes through the birefringent crystal and the polariser to enter the receiving fibre. The receiving fibre, then, carries the light to the detector.

| 1. Light source | 2. Light detector | 3. Transmitting fibre | 7. Mirror |
| 4. Receiving fibre | 5. Polariser | 6. Birefringent crystal | |
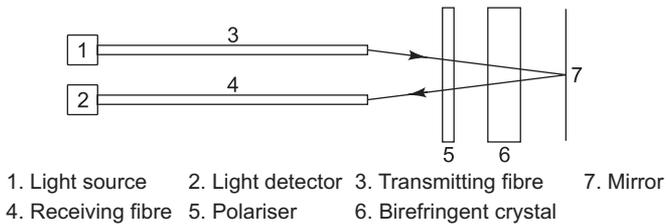
Fig. 8.14 Fibre Optic Temperature Sensor

The intensity of the received light is proportional to temperature. This is due to the fact that the refractive index and hence the birefringence is a strong function of temperature.

8.6.3 Fibre Optic Voltage Sensor

It is schematically shown in Fig. 8.15. The transmitting fibre carries the light to the reflecting mirror. An electro-optic material is placed in the light path. Electro-optic materials exhibit the Pockels effect or the Kerr effect.

1. Light source	2. Light detector
3. Transmitting fibre	4. Receiving fibre
5. Electro-optic material	6. Mirror
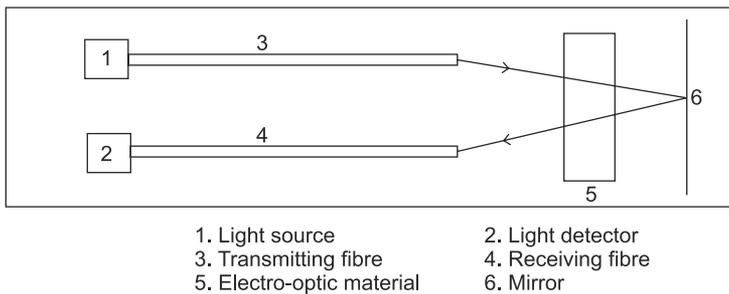
Fig. 8.15 Fibre Optic Voltage Sensor

In the Pockels effect, the refractive index change of an electro-optic material is proportional to the electric field and hence to voltage. In the

Kerr effect, the refractive index change of an electro-optic material is proportional to the square of the electric field and hence to the square of the voltage. The light reflected from the mirror passes through the electro-optic material to enter the receiving fibre. The receiving fibre, then, carries the light to the detector. The intensity of the received light is proportional to the refractive index of the electro-optic material and hence to the voltage.

8.6.4 Fibre Optic Magnetic Field Sensor

The fibre optic magnetic field sensor (Fig. 8.16) operates on Faraday rotation effect. A beam of polarised light propagates through the optical fibre in a direction parallel to that of the magnetic field to be determined.

1. Light source   2. Light detector
1. Light source   2. Polariser
3. Fibre   4. Magnetic field
5. Faraday transducer   6. Analyser
7. Light detector
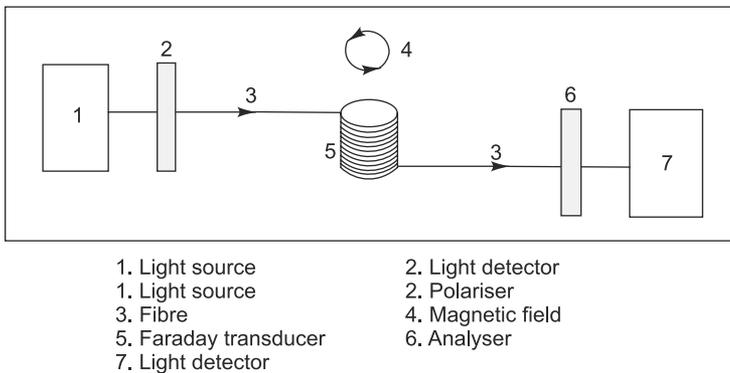
Fig. 8.16 Fibre Optic Magnetic Field Sensor

It causes the polarisation of the light to rotate in proportion to the strength of the magnetic field. Thus, by detecting the polarisation rotation by means of polarisation-sensitive optical receiver, the magnetic field strength can be determined.

8.6.5 Fibre Fabrication

At present, there are two main techniques of manufacturing low-loss optical fibres, namely, **double crucible method** or **crucible–crucible method** and **vapour deposition method**.

Crucible Method A crucible is a refractory container that can withstand extremely high temperatures to melt or otherwise alter its contents. It is used for metal, glass, and pigment production as also for a number

of modern laboratory processes including fibre fabrication. Crucibles are made up of high temperature-resistant materials. Platinum was used to make crucibles in the early state. Recently metals like nickel and zirconium are used. A crucible with a bottom perforated with small holes is designed for fibre fabrication. The raw material for manufacturing of fibre is kept in a crucible in the powder form and it is heated to a very high temperature using a furnace. An inert gas atmosphere is to be maintained inside the furnace. When the material inside the crucible is heated to a very high temperature, it goes to the molted state and it starts squeezing out through the small holes perforated at the bottom of the crucible. The diameter of the hole determines the thickness of the optical fibre.

Crucible–Crucible Method The apparatus for this technique is illustrated in Fig. 8.17. Pure glass is fed into two platinum crucibles. The bottom of each crucible has a circular nozzle. The nozzles of the crucibles are concentric. The nozzle of the inner crucible is slightly above that of the outer crucible. The inner crucible contains the core material and the outer crucible contains the cladding material. The temperature of the apparatus is sufficiently raised by using a furnace. Then, the core material flows through the inner nozzle into the centre of the flow stream from the outer crucible. As a result, fibre filament comes out of the outer crucible. It then passes through the plastic coating bath containing molten plastic. Thus, the fibre acquires a protective plastic coating. Now, after passing through the curing oven, the fibre is wound onto a rotating drum placed below the crucibles.
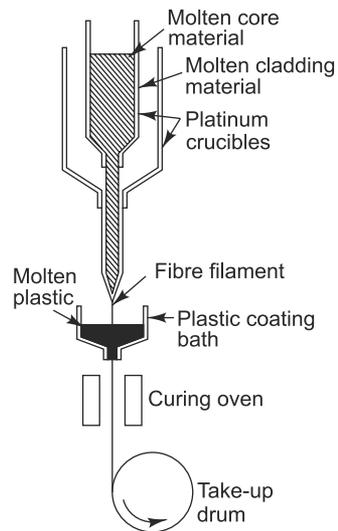

Fig. 8.17 Crucible-Crucible Technique

Merit Both step index and graded index fibres can be fabricated.

Demerit Fabricated graded index fibres certainly have intermodal dispersion.

8.6.6 Fibre Losses

Fibre loss is a fundamental limiting factor as it reduces the power reaching the receiver. The transmission distance is limited by fibre

losses. Fibre loss depends on the wavelength of transmitted light. The loss is considerably higher for shorter wavelengths and exceeds 5 dB/km in the visible region of the optical spectrum. Several factors contribute to the losses. The important factors are material absorption, scattering and waveguide imperfections.

Attenuation Coefficient The measure of fibre losses, the attenuation coefficient, is given as

$$\alpha \text{ (dB/km)} = -\frac{10}{L} \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

where P_{in} is the power launched at the input of a fibre of length L and P_{out} is the output power.

Losses due to Absorption Optical fibres are generally made of fused silica. Material absorption are classified into two categories. **Intrinsic material absorption** is due to the loss caused by pure silica. The **extrinsic absorption** corresponds to the loss caused by impurities. In the ultraviolet region and in the infrared region, the intrinsic absorption has peak loss. The vibrations of chemical bonds such as the silicon oxygen bond causes infrared loss.

The presence of impurities results in extrinsic material absorption. Transition a metal impurities (Fe, Cu, Co, Ni, Mn and Cr) absorb strongly in the wavelength range 0.6-1.6 μm. By using modern techniques, high purity silica can be produced with the impurity concentration reduced to below 1 part per billion to obtain a loss level below 1 dB/km.

The dopants such as GeO_2, P_2O_5 and B_2O_3 used during fibre fabrication to produce the required index step also lead to additional losses.

Rayleigh Scattering Scattering losses in glass arise from microscopic fluctuations in density, from compositional fluctuations and from structural inhomogeneities or defects occurring during fibre manufacturing. During fibre fabrication, silica molecules move randomly in the molten state and then freeze. Density fluctuations lead to fluctuations of the refractive index on a scale smaller than the optical wavelength 'λ'. Light scattering in such a medium is called **Rayleigh scattering**. In this case, the absorption coefficient varies as λ^{-4}. Rayleigh scattering is dominant at short wavelengths. At a wavelength near 3 micrometres, the Rayleigh scattering can be reduced to below 0.01 dB/km.

Losses due to Waveguide Imperfections **Mie scattering** refers to imperfections at the core-cladding interface that lead to losses due to index inhomogeneities. To reduce this, it has to be noted that the core radius of a fibre does not vary significantly along the fibre length.

Macrobending Losses When an optical fibre undergoes a bend of finite radius of curvature, radiative loss occurs. Normally, a guided ray hits the core-cladding interface at an angle greater than the critical angle to experience total internal reflection. In the bending region, the angle of ray incidence decreases and may become smaller than the critical angle for light bends. The ray escapes out of the fibre. The macrobending losses are negligible in practice.
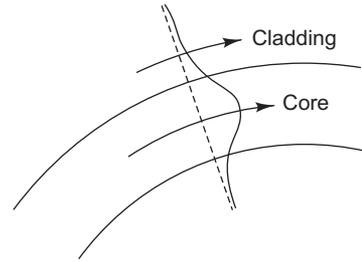
Fig. 8.18 Radiation at a Macrobend

Microbending Losses When the fibre is pressed against a surface that is not perfectly smooth, microbending loss occurs. Such losses are called microbending losses. One method of minimising microbending losses is by adding a compressible jacket over the fibre.
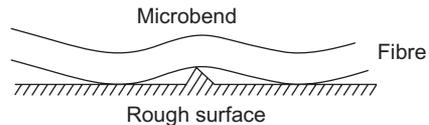
Fig. 8.19 Microbend in a Fibre

8.6.7 Dispersion

An optical signal becomes increasingly distorted as it travels along a fibre. This distortion is a consequence of intramodal dispersion and intermodal dispersion.

Intramodal Dispersion It depends on the wavelength, its effect on signal distortion by causing the pulse spreading that occurs within a single mode known as intramodal dispersion. The intramodal dispersion increases with the spectral width of the optical source. The two main causes of intramodal dispersion are material dispersion and waveguide dispersion.

Material Dispersion It results from the variation of the refractive index of the core material as a function of wavelength as certain light sources like LEDs emit a band of wavelengths.

Different wavelengths have different velocities in a material. As the result, all portions of the light pulse, which is a combination of wavelengths, will not arrive simultaneously at the output. The output is a distortion of the optical signal. If the wavelength range of 1200–600 nm is used, material dispersion is much reduced. By replacing an LED with an injection laser, dispersion can be reduced by a factor of 20. It is also known as **chromatic dispersion**.

Waveguide Dispersion   In a single-mode fibre, about 80 per cent of the optical power propagates along the core and the remaining 20 per cent of the light propagating in the cladding travels faster than the light confined to the core, waveguide dispersion occurs. The magnitude of this dispersion depends on the fibre design.

Intermodal Dispersion   It can be reduced by using a multimode graded index optical fibre and can be eliminated by using a single-mode step index fibre. In a step index fibre, light rays that travel parallel to the axis (fundamental mode) will have a shorter path length than rays that zigzag down the fibre (the highest-order mode). The highest-order mode will take longer time to reach the output and the fundamental mode will take a minimum time. The pulse broadening arising from intermodal dispersion is the difference between the highest-order mode and the fundamental mode of the light rays.

This time delay is a measure of the pulse dispersion. If this delay is comparable to the interval between pulses, the output pulses will overlap into adjacent time slots.

8.7   OPTICAL SOURCES

Optical sources convert an electrical input signal into the corresponding optical signal. Laser diodes and light-emitting diodes are the most commonly used sources. The advantages are compact size, high efficiency, reliability, right wavelength range, small emissive area compatible with fibre-core dimensions and possibility of direct modulation at relatively high frequencies.

8.7.1   Light Emitting Diode (LED)

Injection electroluminescence is the principle used here. LED is the semiconductor *p-n* junction that under proper forward-biased condition can emit electromagnetic radiation in the UV, visible and infrared regions. The direct band-gap semiconductors Si and Ge

give up the energy in the form of heat. But, in the direct band-gap semiconductors GaAs and GaAsP, a greater percentage of energy is given out in the form of light. The n type and p type are made by doping with impurities. When p and n type semiconductors are brought together, an energy barrier is produced. A forward voltage decreases the barrier, and free electrons and free holes now have sufficient energy to move into the junction. When a free electron meets a free hole, the electron can fall into the valence band and recombine with the hole. Then, light is emitted. The wavelength of the emitted light is $\lambda = hc/E_g$ where E_g is the forbidden energy gap of the semiconductor. The construction of the typical LED is shown in Fig. 8.20.

If the same material is used on both sides of the junction, it is called a **homojunction**. A homojunction LED does not confine its radiation very well because the electron–hole recombination occurs over a wide region (~1 – 10 μm) and after the photons are created, they diverge over unrestricted paths. The carrier confinement problem can be solved by sandwiching a thin layer of semiconductor material with comparatively small band-gap between the p-type and n-type layers. Hence, the junction formed by dissimilar semiconductors is known as a **heterojunction**. The changes in band-gap energies between the two semiconductors confine electrons and holes to the middle layer called the **active layer**. The active layer also has a slightly larger refractive index than the surrounding p-type and n-type layers. As a result, the active layer acts as a dielectric waveguide.
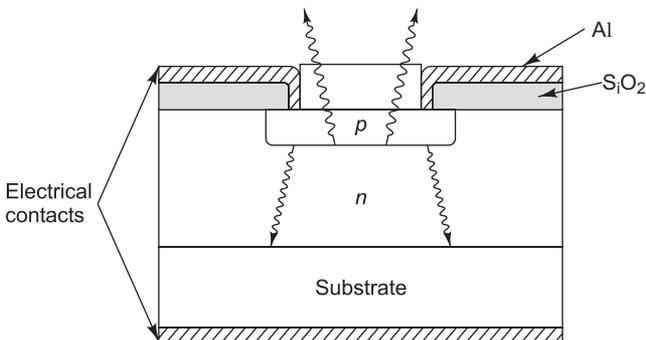

Fig. 8.20 Light Emitting Diode

LED Structures There are two arrangements of an LED: **surface emitting** and **edge emitting**.

Surface Emitting LED In this structure, the light produced by the electron-hole recombinations in the depletion region radiates in all directions. However, only a transparent window of the upper electrode allows light to escape from the LED structure. All other possible directions are blocked from light by the LED's packages. The other name for surface emitting diode is Burrus or etched-well LED.

Edge Emitting LED In this structure, the light produced by the electron-hole recombinations in the depletion region radiates in all directions. However, only an open edge allows light to escape from the LED structure and the opposite edge is blocked from light by the LED's packages.
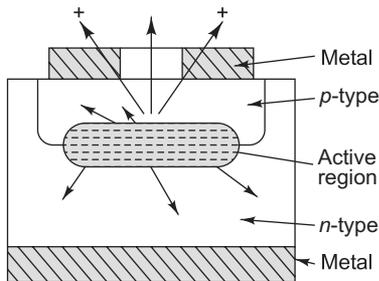
Fig. 8.21 Surface Emitting LED   **Fig. 8.22** Edge Emitting LED
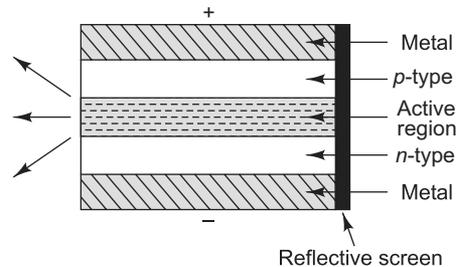
This device radiates over a smaller cone than does the Burrus diode. It has a divergence of about 30° in the direction perpendicular to the junction plane. The emitting area is rectangular rather than circular. Considerable light can be coupled into a fibre of even low numerical aperture (< 0.3) because of reduced divergence and high radiance at the emitting face.

8.7.2 Homojunction Laser Diode

In this type of laser, the *p-n* junction is fabricated in a single semiconductor. The typical homojunction laser diode built around a gallium arsenide (GaAs) chip is shown in Fig. 8.23. Its highly degenerate *p* and *n* regions are achieved by doping the material with zinc and tellurium, respectively, in the concentrations of about 10^{19} atoms/cm^3.

The *p-n* junction diode is in the form of a cube with each edge being 1 mm long. The *p-n* junction layer of 1 mm thickness lying in a horizontal plane passing through the centre of the diode acts as the

active medium. The polished ends of the GaAs chip, cut at right angles to the *p-n* junction layer, take up the role of resonator mirrors. The other two faces are rough sawn. The diode is immersed in liquid nitrogen.
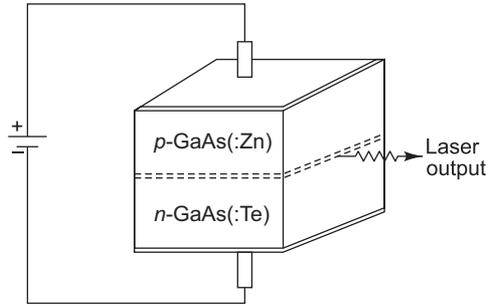
Fig. 8.23   Homojunction Laser Diode

As a result of doping, population inversion is achieved in the region of the *p-n* junction. The junction is forward biased. Then, electrons (from the *n* region) and holes (from the *p* region) are injected into the junction region at a sufficiently high rate. The recombination of electrons and holes in this region results in the emission of photons. These photons trigger the stimulated emission of photons. The photons travelling along the resonator axis experience multiple reflections at the optical resonator mirrors. It results in an intense and coherent laser output confined to a narrow range of wavelengths of 8380 to 8392 Å.

Characteristics

- Output wavelength   :   8380 to 8392 Å
- Output power   :   1 mW
- Nature of output   :   Continuous

Threshold current density (i.e., current density at which laser oscillation occurs) is sufficiently high for a homojunction laser. It leads to overheating of the semiconductor when operated continuously or at high peak power. Therefore, homojunction lasers can be effectively operated only at very low temperatures. Moreover, in a homojunction laser, recombination and emission take place over an extensive region. It results in the poor confinement of emitted radiation. This makes coupling to a small fibre inefficient. The first semiconductor lasers were operated with homojunction arrangement.

8.7.3   Heterojunction Laser Diode

To have smaller threshold current density and improved confinement of emitted radiation, heterojunction lasers were developed. In heterojunction lasers, *p-n* junctions are formed by dissimilar semiconductors having different band-gap energies and different

refractive indices. The changes in band-gap energies create a potential barrier for both holes and electrons. The free charge carriers can meet and recombine only in the narrow, well-defined active layer. As the active layer has a higher refractive index than the materials on either side, an optic waveguide is formed. The confined emission improves the coupling efficiency, particularly for small fibres. Heterojunction lasers consist of several layers of various materials: semiconducting layers (both doped and undoped), insulating layers and metallic layers for current conduction. A single layer in the centre of these layers (called the active layer) is a direct band-gap material. It is an effective radiator. The adjacent layers (called the cladding layers) are indirect band-gap materials. Heterojunction lasers are fabricated from a range of lattice-matched semiconductor materials.

8.7.4   Simple Heterojunction Laser Diode

Figure 8.24 shows a simple heterojunction laser diode configuration. It consists of an upper *p*-type layer of AlGaAs followed by a *p*-type layer of GaAs and substrate of *n*-type GaAs. The *p*-type layer of GaAs has an active region of only $0.1 - 0.2\,\mu m$ thickness. This is the only region where current can flow because AlGaAs serves as a barrier. The thin *p*-GaAs layer is also the only region where recombination radiation can occur. The energy-level diagram for this arrangement shows the additional 2 eV energy level provided by the *p*-type AlGaAs. This increased energy prevents the current flow in the AlGaAs region since the energy level lies above the Fermi energy. Thus, the current is confined to the thin *p*-type GaAs region.

1,3 → Confining layer   2 → Active layer
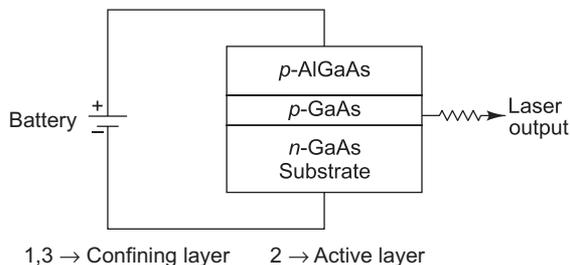
Fig. 8.24 Homojunction Laser Diode

The *p-n* junction is forward biased. Then, holes (from *p*-AlGaAs, the confining layer) and electrons (from *n*-GaAs, the confining layer) are injected into the active layer *p*-GaAs. Then, holes and electrons

recombine in the narrow, well-defined active layer *p*-GaAs. The polished ends of the diode, cut at right angles to the active layer, take up the role of resonator mirrors. The electron-hole recombinations and multiple reflections of the emitted photons at the resonator mirrors lead to an intense and coherent laser output.

As the charge carriers in heterojunction lasers are confined to a much smaller region than in homojunction lasers, the heat deposition is much lower. Moreover, the change in refractive index at the interface between the *p*-type GaAs and the *p*-type AlGaAs provides a guiding effect for the laser beam.

8.7.5 Double Heterojunction Laser Diode

Figure 8.25 shows a double hetero-junction laser diode configuration composed of various doping com-binations of GaAs and AlGaAs. It consists of two junctions J_1 and J_2 of dissimilar semi-conductors. It offers more control over the size of the active region. It provides addi-tional refractive index changes lead-ing to an effective guiding effect for the laser beam. The *p-n* junction is forward biased. Then, holes (from *p*-AlGaAs , the confining layer) and electrons (from *n*-AlGaAs, the con-fining layer) are injected into the active layer *p*-GaAs. Then, holes and electrons recombine in the narrow, well-defined active layer *p*-GaAs. The polished ends of the diode, cut at right angles to the active layer, take up the role of resonator mirrors. The heterojunction J_2 confines within the active layer those minority carriers that are injected over the forward biased *p-n* junction.


1,2,4,5 → Confining layer   3 → Active layer

Fig. 8.25 Double Heterojunction Laser Diode

Characteristics of Heterojunction Laser Diodes

- High injection efficiency
- Confinement of minority carriers in a double heterostructure
- Improvement of ohmic contacts
- Transparency of the wide band-gap material
- Optical guidance

8.8　PHOTODETECTORS

The role of an optical receiver is to convert optical signal into electrical form and recover the data transmitted. Its main component is a photodetector that converts light into electricity through the photoelectric effect. A good photodetector should have high sensitivity at the operating wavelengths, fast response, low noise, low cost, high reliability and stability of performance characteristics.

8.8.1　*p-n* Photodiode

The fundamental mechanism behind the photodetection process is optical absorption. A reverse biased *p-n* semiconductor junction consists of a region known as the **depletion region**. Having no free charges, the resistance of the depletion region is high.

When a *p-n* junction is illuminated (Fig. 8.26) with light on one side, electron-hole pairs are created through absorption. Because of the large built-in electric field, electrons and holes generated inside the depletion region accelerate in opposite directions and drift to *n* and *p* sides, respectively.

Fig. 8.26 *p-n* Photodiode

The resulting flow of current is proportional to the incident optical power. The quantum efficiency of the photodiode is defined as

$$\eta = \frac{\text{Electon-hole generation rate}}{\text{Photon-incident rate}}$$

When a photon is absorbed in the *p* or *n* regions, an electron-hole is created but these free charges will not move quickly because of the weak electrical forces outside the junction. Most of the free charges will diffuse slowly through the diode and recombine before reaching the junction. Thus, the detector's responsivity decreases. To increase its responsivity, the width of the depletion layer is increased by introducing an intrinsic layer between the *p* and *n* regions. This improved photodetector is known as *p-i-n* photodiode.

For wavelength ranges of 1–1.6 µm, the materials used for optical detection purposes are InGaAs and InGaAsP.
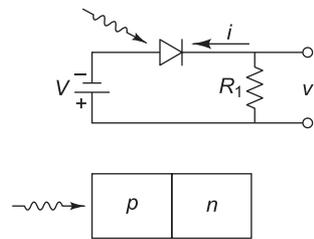
8.8.2   *p-i-n* Photodiode

The schematic of a *p-i-n* photo-diode is shown in Fig. 8.27. An intrinsic layer is sandwiched between the *p* and *n* layers. The intrinsic layer has no free charges, so its resistance is high. Most of the voltage appears across it and the large electric field exists in the middle region. Because the width of the depletion region extends throughout the *i* region, most of the incident power is absorbed inside the *i* region rather than in the *p* or *n* regions. To create an electron-hole pair, an incoming photon must have enough energy to raise an electron across the band-gap. The high electric field causes the carriers to separate and be collected across the junction. This causes a current flow in an external circuit. The quantum efficiency is not a constant at all wavelengths of the incident light, since it varies according to the photon energy. The quantum efficiency can be made 100% by using an InGaAs layer several micrometers thick operating near 1.3 and 1.55 μm wavelengths.
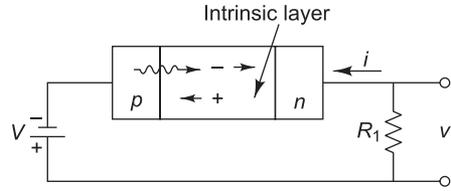
Fig. 8.27   *p-i-n* Photodiode

8.8.3   Avalanche Photodiode

To overcome the limitations in receiver performance caused by pre-amplifier noise, an Avalanche Photodiode (APD) is used. Detectors with large responsivity are preferred, since they require less optical power. APDs can have much larger values of responsivity.

An additional layer is added to the *p-i-n* photodiode. Figure 8.28 shows the APD structure. Under reverse bias, a high electric field exists in the *p*-type layer sandwiched between *i*-type and *n*-type layers. This region is known as the **gain region** or the **multiplication region** since secondary electron-hole pairs are generated here through impact ionisation. The *i* layer acts as the depletion region in which most of the incident photons are absorbed and primary electron-hole pairs are generated. The generated electron
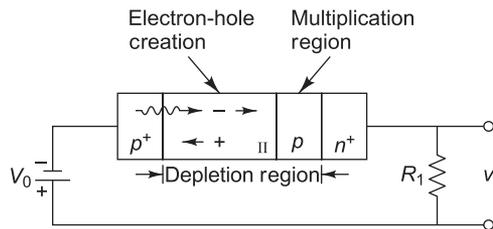
Fig. 8.28   Avalanche Photodiode

cross the gain region and generate secondary electron-hole pairs responsible for the current gain.
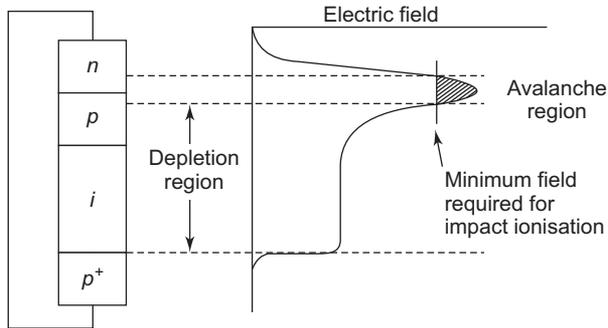
Fig. 8.29 Working of Avalanche Photo Diode (APD)

As the reverse bias is increased, the high field region continues to deplete and the fields within that region begin to be sufficient to cause multiplication. If the reverse bias voltage is sufficiently high, one obtains a condition of infinite multiplication corresponding to sustained avalanche breakdown. A commonly used material for APD is InGaAs with a cut-off wavelength of about 1.65 μm.

8.9 SPLICES

The permanent or semipermanent joint between two fibres are called **splices**. There are two main methods of splicing fibres together:
1. Fusion splicing
2. Mechanical splicing

The reasons for splicing fibres are to connect preterminated tails to the ends of fibres, to join cables, to join two fibres with a low loss joint and to repair damaged cables

To ensure a good, low loss connection, the following are the steps involved in the procedure of splicing.

Fibre Preparation   By using tools like Miller Stripper, the fibres are to be stripped down to the cladding at the ends. Now, the fibre ends are to be thoroughly cleaned. This is best done by a degreasing solvent such as isopropyl alcohol.

Fibre Clearing   The fibre ends are cut squarely, known as fibre clearing, and it is an essential factor. It is done with a good-quality clearer.

The clearing process produces fibre ends that are clean and flat with no rough edges.

Mounting the Fibres   Fibres are clamped to place in the splicer.

Alignment of Fibres   The fibres are brought close together and aligned. The older splicers required manual alignment of fibres, but now, this is automatically done by the splicer with an image of the fibre ends displayed on a screen.

8.9.1   Fusion Splices

The schematic of a fusion splice is shown in Fig. 8.30. The aligning of fibres is done mechanically and by using an electric arc produced by the electrodes heats the fibres evenly. As a result, the fibres are fused, i.e., welded together.

Fig. 8.30 Fusion Splice

By using materials such as epoxy and heat-shrinkable tubing, the splice area is protected by covering. Fusion splices are suitable for all-glass fibres, both single mode and multimode.

The fusion splices are characterised by losses up to several hundredth of a dB and large misalignment when fibres of different radii are spliced.
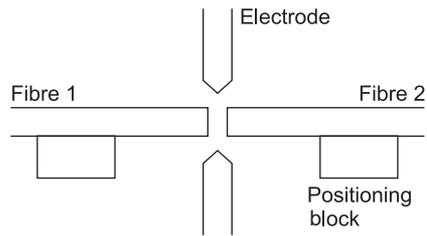
8.9.2   Adhesive Splices

A lot of fibre alignment structures are available. These structures align the fibres and provide strength to the joint. The fibres are held in place by epoxy. After the epoxy is cured, these splices can be used immediately. The commonly used adhesive splices are V-blocks, precision sleeves, loose tubes, three-rod splices, four-rod splices and rotary mechanical splices.

V-block   The fibres are placed on the groove and the two fibres slide in the groove until they touch. Then, they are permanently epoxied. To protect the splice, a cover plate may be placed over the V-block.
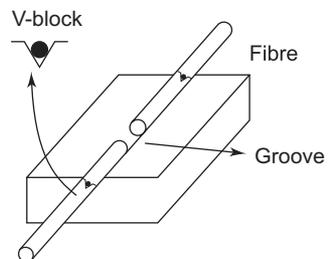
Fig. 8.31 V-Block Splice

Precision Sleeve   The fibre is inserted into the precision sleeve made of metal or plastic. An index-matching epoxy is applied to the fibre ends before insertion into the sleeve.

Fig. 8.32   Precision-Sleeve Splice

Loose Tube   Two fibres are inserted into the freely suspended tube. Bending the fibres causes the tube to rotate. The fibres are aligned and then epoxied.
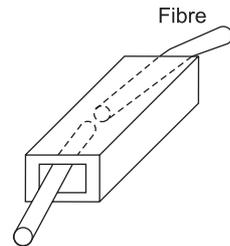
Fig. 8.33   Loose-Tube Splice

Three-Rod Splices   Three glass or metal rods are positioned as shown in Fig. 8.34. A hole is formed at the junction. The fibres are applied index-matching epoxy and they are inserted into the hole until they touch. A heat-shrinkable sleeve is placed over the assembly. The rods squeeze against the fibres when they are heated.
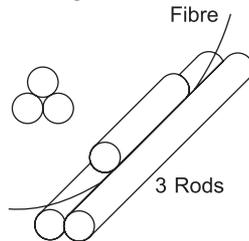
Fig. 8.34   Three-Rod Splice

Four-Rod Splices   Four glass rods are positioned (Fig. 8.35) to form four *V*- grooves. The ends of the bundle are bent so that the entering fibre is forced into one of the grooves. The fibres are pushed until they touch. Then, the fibres are epoxied.

Fig. 8.35   Four-Rod Splice

Elastomer Splice   The two fibres of unequal diameters can be joined by the elastomer splice joints. A hole is cut into it so as to insert the fibres. The elastomer is covered by a glass sleeve with ends in such a way that it aligns the fibres into the elastometric splice. The aligned fibres can be connected well into the splice with an adhesive that may serve as an index-matching gel. Thus, the fibres are connected.

Fig. 8.36   Construction of Four-Red Splice

Rotary Mechanical Splices   This splice directly aligns the fibres. Three rods in the alignment clip secure the ferrules. The fibres are aligned by rotating the ferrules. After alignment, the ferunles are fixed with an UV-curable epoxy.
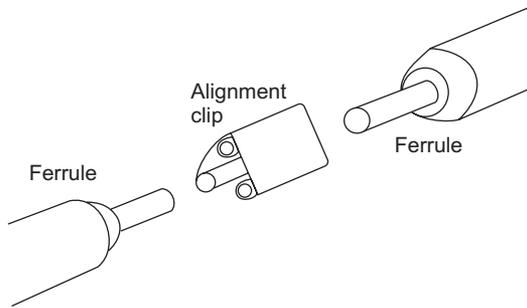
Fig. 8.37   Rotary Mechanical Splice

Losses that occur due to misalignment of two fibres are called *extrinsic losses.* Losses that occur due to irregularity in fibre construction while connecting the fibres are called *intrinsic losses.* Good mechanical splices produce losses less than 0.1–1 dB when identical fibres are connected.

High coupling efficiency, low loss, repeatability, predictability, long life, high strength, compatibility with environment, ease of assembly, ease of use and economy are the requirements of a good connector.

8.10   ENDOSCOPE

Optical fibres are very much useful in the medical field. An endoscope is a tubular optical instrument used to view the internal organs of the human body that are not visible to the human eye.

Figure 8.38 shows a flexible endoscope. It consists of two fibre bundles. The outer fibres consist of a bundle of fibres without any particular order of arrangement and are used to illuminate the inner parts of the body. The inner fibres also consists of a bundle of fibres, with a perfect order of arrangement and are used to collect the reflected light from the inner parts of the body. Each fibre picks up a part from the picture of the body. Hence the picture will be collected bit by bit and is transmitted in an order by the array of fibres.
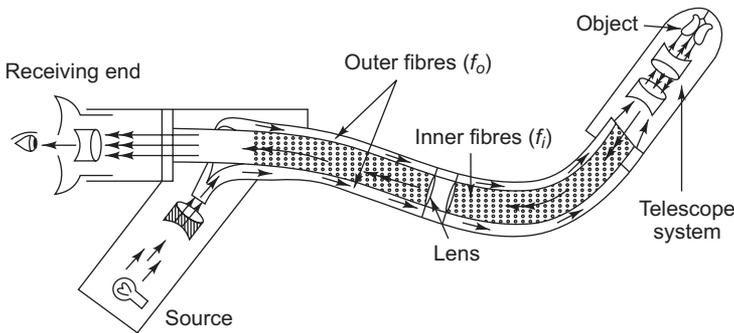
Fig. 8.38 Fibre Optic Endoscope

As the result, the whole picture is reproduced at the other end of the receiving fibre. The output is properly amplified and is viewed through the eyepiece at the receiving end.

SUMMARY

▶▶ Fibre optics is the science and technology of information transfer by means of light guided through dielectric waveguides.

▶▶ Total internal reflection is the principle of transmission of light waves in optical fibre.

▶▶ Light should travel from a denser medium to rarer medium and the angle of incidence at the material boundary should be greater than the critical angle. These are the two essential conditions required for total internal reflection to take place.

▶▶ Core and cladding are the typical configuration of an optical fibre.

▶▶ Acceptance angle is defined as the angle of launching a light ray into the fibre core so that the light ray is transmitted along the core cladding boundary and gets delivered at the output end of the fibre.

▶▶ Numerical aperture of an optical fibre is a measure of its light collecting ability.

▶▶ Based on the refractive index profile, optical fibres are classified as step index and graded index fibres.

▶▶ Based on number of modes, optical fibres are classified as single-mode and multimode fibres.

▶▶ Based on the material, optical fibres are classified as glass and plastic fibres.

▶▶ Double crucible technique is used to produce step index optical fibre.

▶▶ Splicing is the technique used to connect the fibres permanently.

▶▶ The light signal transmitting through the fibre is degraded by two mechanisms, namely, attenuation and dispersion.

▶▶ Transmitter, optical fibre and receiver are the main components of a fibre optic communication link.

▶▶ LED and laser diodes are commonly used light sources for fibre optics.

▶▶ Photodetector is a device used to convert light signals into electrical signals.

▶▶ Fibre optic sensors have high accuracy, high sensitivity and small size.

▶▶ Fibre endoscope is used to study the interior parts of the human body.

SOLVED PROBLEMS

8.1 A step index optical fibre has a core refractive index of 1.50 and a cladding refractive index of 1.48. The refractive index of the surrounding medium is 1.33. Determine the critical angle at the core-cladding boundary, the numerical aperture and the acceptance angle in water for the fibre.

Solution Given Data

$n_1 = 1.50$

$n_2 = 1.48$

$n_0 = 1.33$

$\theta_c = ?; \ NA = ?; \ \theta_a = ?$

(i) The critical angle at the core-cladding boundary is given by

$$\theta_c = \sin^{-1}(n_2/n_1) \tag{1}$$

Substituting the given data into Eq. (1), we get

$$\theta_c = \sin^{-1}\left(\frac{1.48}{1.5}\right)$$

$$\theta_c = \textbf{80.6°}$$

(ii) The numerical aperture of an SI fibre is given by

$$NA = \sqrt{n_1^2 - n_2^2} \tag{2}$$

Substituting the given data into Eq. (2), we get

$$NA = \sqrt{(1.5)^2 - (1.48)^2}$$

$$\therefore \quad NA = \textbf{0.244}$$

(iii) The acceptance angle of an SI fibre is given by

$$\theta_a = \sin^{-1}\left(\frac{NA}{n_0}\right) \tag{3}$$

Substituting the given data into Eq. (3), we get

$$\theta_a = \sin^{-1}\left(\frac{0.244}{1.33}\right)$$

i.e., $\quad \theta_a = \textbf{10.57°}$

8.2 A step index optical fibre has a core refractive index of 1.5 and a relative refractive index difference of 0.02. Determine the numerical aperture and the critical angle at the core-cladding boundary of the fibre.

Solution Given Data

$$n_1 = 1.5$$
$$\Delta = 0.02$$

$$\overline{NA = ?; \quad \theta_c = ?}$$

The relative refractive index difference is given by

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

i.e., $\qquad n_2 = n_1 \sqrt{1 - 2\Delta}$ $\qquad\qquad$ (1)

Substituting the given data into Eq. (1), we get the cladding refractive index as

$$n_2 = 1.5 \times \sqrt{1 - (2 \times 0.02)}$$

i.e., $\quad n_2 = 1.47$

(i) The numerical aperture of the SI fibre is given by

$$NA = \sqrt{n_1^2 - n_2^2} \qquad\qquad (2)$$

Substituting the given data into Eq. (2), we get

$$NA = \sqrt{(1.5)^2 - (1.47)^2}$$

i.e., $\quad$ **NA = 0.299**

(ii) The critical angle at the core-cladding boundary is given by

$$\theta_c = \sin^{-1} (n_2/n_1) \qquad\qquad (3)$$

Substituting the given data into Eq. (3), we get

$$\theta_c = \sin^{-1}\left(\frac{1.47}{1.5}\right)$$

i.e., $\quad \theta_c = \mathbf{78.52°}$

8.3 A step index fibre has a numerical aperture of 0.26 core, refractive index of 1.5 and a core diameter of 100 mm. Calculate the refractive index of the cladding. $\qquad$ *(A.U, B.E/B.Tech, May/June 2005)*

Solution Given Data

$\qquad NA = 0.26$

$\qquad n_1 = 1.5$

$\qquad d = 100 \ \mu m = 100 \times 10^{-6} \ m$

$\therefore \quad a = 50 \times 10^{-6} \ m$

$\qquad \overline{}$

$\qquad n_2 = ?$

Numerical aperture of the step index fibre is

$$NA = \sqrt{n_1^2 - n_2^2} \qquad\qquad (1)$$

From Eq. (1), we get

$$n_2 = \sqrt{n_1^2 - NA^2} \qquad (2)$$

Substituting the values of n_1 and NA into Eq. (2), we get

$$n_2 = \sqrt{(1.5)^2 - (0.26)^2}$$

$$n_2 = 1.48$$

8.4 Calculate the numerical aperture and the acceptance angle of an optical fibre from the following data: refractive index of core is 1.55, refractive index of the cladding is 1.50.

(A.U, B.E/B.Tech, May/June 2005)

Solution Given Data

$$n_1 = 1.55$$
$$n_2 = 1.50$$

$$NA = ?; \quad \theta_a = ?$$

Assume that the given fibre is a step index fibre. Let air be the surrounding medium (i.e., $n_0 = 1$). Then, the numerical aperture of the fibre is

$$NA = \sqrt{n_1^2 - n_2^2} \qquad (1)$$

Substitution of the given values for n_1 and n_2 into Eq. (1) gives

$$n_2 = \sqrt{(1.55)^2 - (1.50)^2}$$

$$NA = 0.39$$

Acceptance angle of the optical fibre is

$$\theta_a = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0} \right) \qquad (2)$$

Substituting the values of n_1, n_2 and n_0 into Eq. (2), we get

$$\theta_a = \sin^{-1}\left(\frac{\sqrt{(1.55)^2 - (1.50)^2}}{1} \right)$$

$$\theta_a = 22.96°$$

8.5 The refractive index of core and cladding materials of an optical fibre are 1.54 and 1.5, respectively. Calculate the numerical aperture of the optical fibre.   *(A.U, B.E/B.Tech, Jan. 2006)*

Solution   Given Data

$n_1 = 1.54$

$n_2 = 1.5$

$NA = ?$

Assume that the given optical fibre is a step index fibre. Then, its numerical aperture is

$$NA = \sqrt{n_1^2 - n_2^2} \qquad (1)$$

Substitution of the given values for n_1 and n_2 into Eq. (1) gives

$$n_2 = \sqrt{(1.54)^2 - (1.5)^2}$$

$$NA = 0.35$$

8.6 A multimode step index fibre has a core diameter of 70 μm, a core refractive index of 1.5 and a cladding refractive index of 1.47. Determine the number of modes supported by the fibre when it transmits a light of wavelength 8380 Å.

Solution   Given Data

$n_1 = 1.5$

$n_2 = 1.47$

$d = 70\ \mu m = 70 \times 10^{-6}\ m$

$a = 35\ \mu m = 35 \times 10^{-6}\ m$

$\lambda = 8380\ \text{Å} = 8380 \times 10^{-10}\ m$

$N = ?$

The normalised frequency of the fibre system is given by

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} \qquad (1)$$

Substituting the given data into Eq. (1), we get

$$V = \frac{2 \times 3.14 \times 35 \times 10^{-6}}{8380 \times 10^{-10}}\sqrt{(1.5)^2 - (1.47)^2}$$

i.e., $V = 78.29$

The number of modes supported by a step index fibre is given by

$$N = \frac{V^2}{2} \tag{2}$$

Hence, for the given fibre system, we get

$$N = \frac{(78.29)^2}{2}$$

i.e., $N = 3065$

8.7 A multimode graded index fibre has a core diameter of 50 μm, a cladding refractive index of 1.48 and a core refractive index of 1.5 at the fibre axis. Determine the number of modes supported by the fibre when it transmits a light of wavelength 8390 Å.

Solution   Given Data

$n_1 = 1.5$

$n_2 = 1.48$

$d = 50 \ \mu m = 50 \times 10^{-6}$ m

$a = 25 \ \mu m = 25 \times 10^{-6}$ m

$\lambda = 8390 \ \text{Å} = 8390 \times 10^{-10}$ m

$N = ?$

The normalised frequency of the fibre system is given by

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} \tag{1}$$

Substituting the given data into Eq. (1), we get

$$V = \frac{2 \times 3.14 \times 25 \times 10^{-6}}{8390 \times 10^{-10}} \sqrt{(1.5)^2 - (1.48)^2}$$

i.e., $V = 45.68$

The number of modes supported by a GRIN fibre is given by

$$N = \frac{V^2}{4}$$

Hence, for the given fibre system, we get

$$N = \frac{(45.68)^2}{4}$$

i.e., $N = 522$

8.8 Calculate the numerical aperture and the critical angle, when the case refractive index is 1.48 and the relative refractive index is 3%.

Solution   Given Data

$\Delta = 3\% = 0.03$

$n_1 = 1.48$

$NA = ?$

$Q_c = ?$

Numerical aperture is given by

$NA = n_1(2\Delta)^{\frac{1}{2}}$

$= 1.48\,(2 \times 0.03)^{\frac{1}{2}}$

$NA = 0.3625$

The relative refractive index is given by

$$\Delta = \frac{n_1^2 - n_2^2}{2\,n_1^2}$$

$$\frac{n_2}{n_1} = (1 - 2\,\Delta)^{\frac{1}{2}}$$

$$\frac{n_2}{n_1} = (1 - 2 \times 0.03)^{\frac{1}{2}} = 0.969$$

The critical angle is

$$Q_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$= \sin{-1}\,(0.969)$$

$$\mathbf{Q_c = 75.69°}$$

8.9 A fibre has a diameter of 5 µm and its core refractive index is 1.4.7; and for cladding it is 1.4.3. How many modes can propagate into the fibre, if the wavelength of the laser is 1.5 µm?

Solution   Given Data

$n_1 = 1.47$

$n_2 = 1.43$

$\lambda = 1.5\ \mu m = 1.5 \times 10^{-6}\ m$

$d = 5\ \mu m = 5 \times 10^{-6}$ m

$N_{step} = ?$

The number of modes of propagation is given by

$$N_{step} = \frac{V^2}{2} = 4.9 \left(\frac{d \times NA}{\lambda} \right)^2$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$NA = \sqrt{1.47^2 - 1.43^2}$$

$$NA = 0.34$$

$$N_{step} = 4.9 \left(\frac{5 \times 10^{-6} \times 0.34}{1.5 \times 10^{-6}} \right)^2$$

$$= 6.29$$

$$\boldsymbol{N_{step} = 6\ \textbf{modes}}$$

8.10   A laser light (signal) is transmitted through a fibre. The power of the incoming laser is 260 mW and that of the outgoing laser is 210 mW. Calculate the power loss in dB.

Solution   Given Data

$$P_{in} = 260\ mW = 260 \times 10^{-3}\ W$$

$$P_{out} = 210\ mW = 210 \times 10^{-3}\ W$$

$$\alpha = ?$$

The power loss 'α' in dB is given by

$$\alpha = 10 \log \frac{P_{in}}{P_{out}}$$

$$= 10 \log \left(\frac{260 \times 10^{-3}}{210 \times 10^{-3}} \right)$$

$$\boldsymbol{\alpha = 0.927\ \textbf{dB}}$$

8.11   A step index fibre has a numerical aperture of 0.3905 and a core refractive index of 1.55. Calculate the refractive index of the cladding and acceptance angle.   *(A.U, B.Tech, Jan. 2011)*

Solution Given Data

$$NA = 0.3905$$

$$\underline{\begin{aligned} n_1 &= 1.55 \end{aligned}}$$

$$n_2 = ?$$

$$\theta_a = ?$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$n_2 = \sqrt{n_1^2 - NA^2}$$

$$n_2 = \sqrt{(1.55)^2 - (0.3905)^2}$$

$$n_2 = 1.50$$

$$\theta_a = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)$$

$$= \sin^{-1}\frac{\sqrt{2.4025 - 2.25}}{1}$$

$$\theta_a = \mathbf{22.98°}$$

8.12 Calculate numerical aperture and angle of acceptance of a fibre if the refraction index of the core is 1.55 and that of cladding is 1.5.

(A.U, B.E/B.Tech, Jan. 2011)

Solution Given Data

$$n_1 = 1.55$$

$$\underline{\begin{aligned} n_1 &= 1.5 \end{aligned}}$$

$$NA = ? \quad \theta_a = ?$$

(i) Numerical aperture,

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{(1.55)^2 - (1.5)^2}$$

$$NA = 0.3905$$

(ii) Acceptance angle,

$$\theta_a = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)$$

$$= \sin^{-1}(0.3905)$$
$$\theta_a = \textbf{22.98°}$$

✓ QUESTIONS AND ANSWERS

8.1   Define refractive index of a material.

The refractive index of a material is defined as

$$n = c/v$$

where c and v are the speeds of light in vacuum and in the given material, respectively.

8.2   How do we distinguish the denser medium and rarer medium with respect to light propagation?

A denser medium is characterised by a higher value of refractive index, while a rarer medium is characterised by a lower value of refractive index.

8.3   State Snell's law for light propagation through a material boundary.

According to Snell's law, we have

$$n_1 \sin \theta_i = n_2 \sin \theta_r$$

where n_1 and n_2 are the refractive indices of the first and the second medium, respectively; θ_i is the angle of incidence; θ_r is the angle of refraction.

8.4   Define critical angle.

It is the angle of incidence of a light ray travelling from a denser medium to a rarer medium, for which, the angle of refraction is 90°.

8.5   What is the principle behind light transmission through optical fibres?

Total internal reflection is the principle behind the transmission of light in an optical fibre. When a light ray travelling from a denser medium to a rarer medium is incident on the boundary at an angle greater than the critical angle, the light ray is reflected back into the originating medium. This phenomeon is known as total internal reflection.

8.6   Define total internal reflection of light.

When a light ray travelling from a denser medium to a rarer medium is incident on the boundary at an angle greater than the critical angle,

the light ray is reflected back into the originating medium. This phenomenon is known as total internal reflection.

8.7 What is the typical configuration of an optical fibre?

Typically, an optical fibre consists of a central solid cylinder (called the core) made of a high-refractive-index material. The core is surrounded by a cylindrical tube (called the cladding) made of a low-refractive-index material.

8.8 Define acceptance angle of an optical fibre system.

It is defined as the angle of launching of a light ray into the fibre core so that the light ray is transmitted along the core-cladding boundary and gets delivered at the output end of the fibre.

8.9 Define numerical aperture of an optical fibre.

The numerical aperture (NA) of an optical fibre is a measure of its light collecting ability. It is defined as

$$NA = n_0 \sin \theta_a$$

where n_0 is the refractive index of the surrounding medium and θ_a is the acceptance angle of the fibre.

8.10 What is a step index fibre?

It consists of a core of constant refractive index surrounded by a cladding whose refractive index is less than that of the core.

8.11 What are the merits and demerits of step index fibres?

Merits	Demerits
Low cost	Small bandwidth
Simple fabrication	Large intermodal dispersion
Constant numerical aperture	
Higher coupling efficiency	

8.12 What is a graded index fibre?

A.U, B.Sc (C.T & I.T), Jan. 2006); (A.U, B.E / B.Tech, Jan. 2005)

It consists of a core whose refractive index is not constant. It has the maximum value at the core axis but decreases radially towards the outer edge of the core.

8.13   What are the merits and demerits of graded index fibres?

Merits	Demerits
Small intermodal dispersion	High cost
Large bandwidth	Variable numerical aperture
Smaller coupling efficiency	

8.14   What is meant by mode of light propagation in an optical fibre?
Modes of propagation refer to the various paths that light can take in the fibre core.

8.15   Define single-mode fibre.
It is a fibre allowing only one mode of light propagation through its core.

8.16   Define multimode fibre.
It is a fibre allowing many modes of light propagation through its core.

8.17   State the performance characteristics of a multimode SI fibre.
Low information rate, High source-coupling efficiency, High pulse distortion , Short-range applications

8.18   State the performance characteristics of a multimode GRIN fibre.
High information rate, Low source-coupling efficiency, Low pulse distortion, Long-range applications

8.19   Write the expressions for the number of modes of light propagation supported by the step index and graded index fibres.

$$N = V^2/2 \ (\text{SI fibre}); \qquad N = V^2/4 \ (\text{GRIN fibre})$$

where N is the number of modes supported by the fibre and V is the normalised frequency of the fibre system.

8.20   State the single-mode condition for a SI fibre.

$$V \equiv \frac{2\pi a \sqrt{n_1^2 - n_2^2}}{\lambda} \le 2.405$$

where V is the normalised frequency of the fibre system, a is the core radius, n_1 is the core refractive index, n_2 is the cladding refractive index and λ is the wavelength of light.

8.21 State the single-mode condition for a GRIN fibre.

$$\frac{2\pi a \sqrt{n_1(n_1 - n_2)}}{\lambda} \leq 2.405$$

where a is the core radius, n_1 is the refractive index at the core axis, n_2 is the cladding refractive index and λ is the wavelength of light.

8.22 What are the merits and demerits of a plastic fibre?
It has greater mechanical strength compared to a glass fibre. But, the plastic fibre has higher attenuation compared to a glass fibre.

8.23 State the advantages of fibre optic communication system.
Large information carrying capacity, Small size and weight, Electrical isolation, Immunity to interference and cross talk, Singal security, Low transmission loss, Flexibility, System reliability, Easy maintenance and Low cost

8.24 What are the applications of fibre optic sensors?
(A.U, B.E / B.Tech, Jan. 2006)
Optical fibres can be used to acquire information about an environmental parameter of interest such as pressure, temperature, voltage, current, etc., and to transport the information to a convenient location for processing.

8.25 Write a note on fibre optic displacement sensor.
It consists of two bundles of fibres. One bundle of fibres transmits light to the reflecting target whose location is to be determined. The other bundle traps the reflected light and transmits it to the detector. The intensity of the detected light depends on the relative distance between the fibre optic probe and the target.

8.26 Write a note on fibre optic voltage sensor.
It consists of two bundles of fibres. One bundle of fibres transmits light to the reflecting target. The other bundle traps the reflected light and transmits it to the detector. An electro-optic material is placed in the light path. Electro-optic materials exhibit the Pockels effect or the Kerr effect. The intensity of the received light is proportional to the refractive index of the electro-optic material and hence to the voltage.

8.27   Write a note on fibre optic magnetic field sensor.

The fibre optic magnetic field sensor operates on the Faraday rotation effect. A beam of polarised light propagates through the optical fibre in a direction parallel to that of the magnetic field to be determined. It causes the polarisation of the light to rotate in proportion to the strength of the magnetic field. Thus, by detecting the polarisation rotation by means of polarisation-sensitive optical receiver, the magnetic field strength can be determined.

8.28   Write a note on fibre optic temperature sensor.

It consists of two bundles of fibres. The transmitting fibre carries the light to the reflecting mirror. In its path, the light is polarised and it passes through a birefringent crystal. In a birefringent crystal, the refractive index is different for the orthogonally polarised light waves. The light reflected from the mirror passes through the birefringent crystal and the polariser to enter the receiving fibre. The receiving fibre, then, carries the light to the detector. The intensity of the received light is proportional to temperature. This is due to the fact that the refractive index and hence the birefringence is a strong function of temperature.

8.29   How is the attenuation coefficient of an optical fibre defined?

The measure of fibre losses, the attenuation coefficient, is given as

$$a(\text{dB/km}) = -\frac{10}{L}\log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right)$$

where P_{in} is the power launched at the input of a fibre of length L and P_{out} is the output power.

8.30   What is intrinsic material absorption in an optical fibre?

Optical fibres are generally made of fused silica. Intrinsic material absorption corresponds to the loss caused by pure silica.

8.31   What is extrinsic material absorption in an optical fibre?

Optical fibres are generally made of fused silica. Extrinsic absorption is related to the loss caused by impurities.

8.32   What is the Rayleigh scattering in an optical fibre?

Scattering losses in glass arise from microscopic fluctuations in density, from compositional fluctuations and from structural inhomogeneities or defects occurring during manufacturing of the fibre. During fibre

fabrication, silica molecules move randomly in the molten state and then freeze. Density fluctuations lead to fluctuations of the refractive index on a scale smaller than the optical wavelength 'λ'. Light scattering in such a medium is called Rayleigh scattering.

8.33   What is macrobending loss in an optical fibre?
Radiative losses occur whenever an optical fibre undergoes a bend of finite radius of curvature. Normally, a guided ray hits the core-cladding interface at an angle greater than the critical angle to experience total internal reflection. However, near a bend, the angle of ray incidence decreases and may become smaller than the critical angle for light bends. The ray would escape out of the fibre. This is called macrobending loss in an optical fibre.

8.34   What is microbending loss in an optical fibre?
Microbending losses occur when the fibre is pressed against a surface that is not perfectly smooth. Such losses are called microbending losses.

8.35   What is dispersion in an optical fibre?
An optical signal becomes increasingly distorted as it travels along a fibre. This called dispersion in an optical fibre.

8.36   What is intramodal dispersion in an optical fibre?
It is the pulse spreading that occurs within a single mode of an optical fibre.

8.37   What is material dispersion in an optical fibre?
It is the dispersion due to variation of the refractive index of the core material as a function of wavelength.

8.38   What is waveguide dispersion in an optical fibre?
A single-mode fibre confines about 80 per cent of the optical power to the core. Since the 20 per cent of the light propagating in the cladding travels faster than the light confined to the core, dispersion occurs. This is called waveguide dispersion.

8.39   What is intermodal dispersion in an optical fibre?
In a step index fibre, light rays that travel parallel to the axis (fundamental mode) will have a shorter path length than rays that zigzag down the fibre (the highest-order mode). The highest-order mode will take a longer time to reach the output. The fundamental

mode will take a minimum time. This is called intermodal dispersion in an optical fibre.

8.40 What is the basic principle of operation of an LED?

Injection electroluminescence is the principle used in an LED. LED is the semiconductor p-n junction that under proper forward-biased condition emits light due to recombination of electron-hole in the depletion region.

8.41 Write a note on surface emitting LED.

In this structure, the light produced by the electron-hole recombinations in the depletion region radiates in all directions. However, only a transparent window of the upper electrode allows light to escape from the LED structure. All other possible directions are blocked from light by the LED's packages.

8.42 Write a note on edge emitting LED.

In this structure, the light produced by the electron-hole recombinations in the depletion region radiates in all directions. However, only an open edge allows light to escape from the LED structure. The opposite edge is blocked from light by the LED's packages.

8.43 What is the basic principle of operation of a laser diode?

Injection electroluminescence is the principle used in a laser diode. Laser diode is the semiconductor p-n junction that under proper forward-biased condition emits light due to recombination of electron-hole in the depletion region.

8.44 What is a homojunction laser diode?

In this type of laser, the p-n junction is fabricated in a single semiconductor.

8.45 What is a heterojunction laser diode?

In heterojunction lasers, p-n junctions are formed by dissimilar semiconductors having different band-gap energies and different refractive indices.

8.46 State the advantages of a heterojunction light source over a homojunction light source.

Smaller threshold current density and improved confinement of emitted radiation.

8.47   What are photodetectors used in fibre optic system?

Photodetectors convert optical signal into electrical form and recover the data transmitted. The fundamental mechanism behind the photodetection process is optical absorption.

8.48   What is a *p-n* photodiode?

A reverse biased *p-n* semiconductor junction consists of a region, known as the depletion region. When a *p-n* junction is illuminated with light on one side, electron-hole pairs are created through absorption. Because of the large built-in electric field, electrons and holes generated inside the depletion region accelerate in opposite directions and drift to *n* and *p* sides respectively. The resulting flow of current is proportional to the incident optical power.

8.49   What is a *p-i-n* photodiode?

The intrinsic layer sandwiched between the *p* and *n* layers. It has no free charges, so its resistance is high. Most of the voltage appears across it and the large electric field exists in the middle region. Because the width of the depletion region extends throughout the *i* region, most of the incident power is absorbed inside the *i* region rather than in the *p* or *n* regions. The high electric field causes the carriers to separate and be collected across the junction. This causes a current flow proportional to the incident optical power.

8.50   What is an avalanche photodiode?

An additional layer is added to a *p-i-n* photodiode. Under reverse bias, a high electric field exists in the *p*-type layer sandwiched between *i*-type and *n*-type layers. This region is known as the gain region or the multiplication region since secondary electron-hole pairs are generated here through impact ionisation. The *i* layer acts as the depletion region in which most of the incident photons are absorbed and primary electron-hole pairs are generated. The generated electron cross the gain region and generate secondary electron-hole pairs responsible for the current gain. As the reverse bias is increased, the high field region continues to deplete and the fields within that region begin to be sufficient to cause multiplication.

8.51   What are splices?

Splices are joints used for permanently joining optical fibres.

8.52   What are connectors?

Connectors are joints used for temporarily joining optical fibres.

8.53   What are the requisites of splices and connectors?
(i)   Coupling losses should be low.
(ii)   The connector joint should not be affected by temperature, dust and moisture.
(iii)   Cost should be low.

EXERCISE PROBLEMS

8.1   The relative refractive index difference of a step index fibre is 1%. The core refractive index of the fibre is 1.46. Determine the numerical aperture of the fibre and its critical angle at the core-cladding boundary.   *[NA = 0.171; θ_c = 83.29°]*

8.2   A multimode step index fibre of core radius 40 μm and of numerical aperture 0.24 is transmitting a light of wavelength 0.85 μm Determine the number of modes supported by the fibre.   *[N = 2515]*

8.3   A multimode graded index fibre has a core radius of 25 μm and a relative refractive index difference of 2%. The refractive index at the core axis is 1.48. Determine the number of guided modes supported by the fibre transmitting a light of wavelength 1 μm.   *[N = 542]*

8.4   Find the maximum core radius possible for single-mode operation at 850 μm in a step index fibre with a core refractive index of 1.48 and a cladding refractive index of 1.47.

[a_{max} = 1.89 mm]

8.5   Find the maximum core radius possible for single-mode operation at 850 μm in a graded index fibre with a refractive index of 1.48 at the core axis and a cladding refractive index of 1.47.   *[a_{max} = 2.68 mm]*

8.6   A refractive index of core and cladding materials of an optical fibre are 0.66 and 0.6 respectively. Calculate the numerical aperture of the optical fibre.   *[NA = 0.27]*

8.7   A fibre has a diameter of 6.3 μm and its love refractive is 2.82 and cladding is 2.79. How many modes can it propagate into the fibre if the wavelength of the laser source is 2.3 μm.

[N_{step} = 5 modes]

8.8 A step index fibre has a numerical aperture of 0.18, core refractive index of 2.3 and a core diameter of 210 mm. Calculate the refractive index of the cladding. *[$n_2 = 2.29$]*

8.9 Calculate the numerical aperture and the critical angle, when the core refractive index of a fibre is 0.21. Relative refractive index is 1%. *[$N_A = 0.029$, $\theta_c = 78.52°$]*

8.10 A multimode graded index fibre has a core diameter of 25 μm, a cladding refractive index of 3.20 and a core refractive index of 3.26 at the fibre axis. Determine the number of modes supported by the fibre when it transmits light of wavelength 7460 A°. *[$N = 3393$]*

✓ QUESTION BANK

PART-A

8.1 What is meant by optical fibre?

8.2 What is the electrical property of optical fibre?

8.3 What type of material is usually used for the production of optical fibres?

8.4 What is the angle of refraction when the angle of incidence of light is the critical angle?

8.5 What is the principle of light propagation through the optical fibre?

8.6 What is meant by refractive index?

8.7 What is the difference between the denser and rarer medium?

8.8 What is the angle of incidence when the refracted light ray travels through the boundary?

8.9 What is meant by acceptance angle?

8.10 What is meant by numerical aperture?

8.11 What is the relation between the numerical aperture and the acceptance angle?

8.12 Write the formula for critical angle.

8.13 Write the formula for numerical aperture of an SI fibre.

8.14 Write the formula for acceptance angle.

8.15 The innermost layer in an optical fibre is called the _____ .

8.16 What is the role of cladding medium in an optical fibre?

8.17 The core and the cladding are covered by _____ .

8.18 Name a way to classify the optical fibres?

8.19 What are the types of optical fibres based on materials used?

8.20 What are the types of optical fibres based on modes of propagation?

8.21 What are the types of optical fibres based on refractive index?

8.22 Which mode is used for long-haul fibre optic communication?

8.23 The power loss in an optical fibre is also called the _____ .

8.24 Which mode is used for short-haul fibre optic communication?

8.25 Why is the single-mode fibre used for long-haul fibre optic communication?

8.26 A compressible outer jacket of an optical fibre reduces _____ bending loss.

8.27 Which mode of propagation is preferable in GRIN fibres?

8.28 The refractive index of core in a step index fibre is _____ .

8.29 Write the formula for the number modes propagated through graded index fibres.

8.30 The spreading of light is called _____ .

8.31 What are the types of absorption in an optical fibre?

8.32 Write the formula for attenuation coefficient of an optical fibre.

8.33 _____ contribute to Rayleigh scattering and Mie scattering in optical fibres.

8.34 What are the light sources used in optical fibre communication systems?

8.35 State the conditions for total internal reflection of light.

8.36 Give an advantage of heterojunction light source over homojunction light source.

8.37 The scattering of light in an optical fibre is due to _____ .

8.38 What is the role of a photodetector?

8.39 Name the region adjoining the *p-n* junction.

8.40 State the nature of depletion region adjoining the *p-n* junction.

8.41 Why do we increase the depletion region thickness in the photodetector?

8.42 What are the main parts of an optical fibre communication link?

8.43 What is the role of a coupler in an optical fibre communication link?

8.44 What are the basic components in the transmitter of an optical fibre communication link?

8.45 What are the basic components in the receiver of an optical fibre communication link?

8.46 What is the medium present between transmitter and receiver an optical fibre communication link?

8.47 What is the role of modulator in an optical fibre communication link?

8.48 Dispersion leads to _____ .

8.49 What is the role of demodulator in an optical fibre communication link?

8.50 What are the types of fibre optic sensors?

8.51 What are the conditions for the light propagation through the optical fibre?

8.52 Define total internal reflection.

8.53 What is the typical configuration of an optical fibre?

8.54 Define acceptance angle.

8.55 Define numerical aperture.

8.56 A step index optical fibre has a core refractive index of 1.50 and a cladding refractive index of 1.48. Find the critical angle.

8.57 A step index optical fibre has a core refractive index of 1.50 and a cladding refractive index of 1.48. Find the numerical aperture.

8.58 A step index optical fibre has a core refractive index of 1.50 and a cladding refractive index of 1.48. Find the acceptance angle with the air as the surrounding medium.

8.59 An optical fibre has a core refractive index of 1.5 and a relative refractive index difference of 0.02. Find the numerical aperture.

8.60 What are the classifications of optical fibres?

8.61 What are the characteristics of single-mode fibres?

8.62 What are the characteristics of multimode fibres?

8.63 What are the characteristics of step index fibres?

8.64 What are the characteristics of graded index fibres?

8.65 What are the differences between single-mode and multimode fibres?

8.66 What are the differences between step index and graded index fibres?

8.67 A multimode step index fibre has a core diameter of 70 μm and a numerical aperture of 0.25. Find the number of modes supported by the fibre transmitting a light of wave length 5500 A.

8.68 Write a short note on attenuation in optical fibres.

8.69 What is the meaning of attenuation coefficient of an optical fibre?

8.70 What is the Rayleigh scattering in an optical fibre?

8.71 What are bending losses in an optical fibre?

8.72 What is dispersion in an optical fibre?

8.73 What is material dispersion in an optical fibre?

8.74 What is waveguide dispersion in an optical fibre?

8.75 What is the operational principle of LED?

8.76 What is the basic principle of operation of a laser diode?

8.77 What are the basic differences between LED and LASER?

8.78 What are the basic differences between homojunction and heterojunction laser diodes?

8.79 What is p-n photodiode?

8.80 What is pin photodiode?

8.81 What is an avalanche photodiode?

8.82 Draw the block diagram for optical fibre communication link.

8.83 What is electro-optic effect?

8.84 What is magneto-optic effect?

8.85 What it Pockel's effect?

8.86 What is Kerr effect?

8.87 What are the applications of fibre optic sensors?

8.88 Write note on fibre optic displacement sensor.

8.89 Write note on fibre optic voltage sensor.

8.90 Write note on fibre optic magnetic field sensor.

8.91 Write note on fibre optic temperature sensor.

PART-B

8.1 Write, in detail, the various classifications of optical fibres.

8.2 Describe the losses in an optical fibre.

8.3 Describe, in detail, the light sources in fibre optic communication link.

8.4 Describe, in detail, the photodetectors in fibre optic communication link.

8.5 Explain, in detail, the optical fibre communication link.

8.6 Explain, in detail, the fibre optic sensors.

8.7 Discuss the construction and performance characteristics of various types optical fibres.

8.8 Describe the construction and working of a heterojunction laser.

8.9 Describe the construction and working of an avalanche photodiode.

8.10 Describe the construction and working of a homojunction laser.

8.11 Describe the construction and working of the *p-i-n* photodiode.

8.12 Describe the construction and working of a homojunction LED.

8.13 Describe the construction and working of the *p-n* photodiode.

8.14 Compare the construction and performance characteristics of laser diode and LED.

8.15 Compare the construction and performance characteristics of a *p-i-n* photodiode and avalanche photodiode.

8.16 Describe the construction and working of a heterojunction LED.

8.17 Describe the crucible-crucible technique of optical fibre fabrication.

8.18 Describe the construction and working of surface emitting LED.

8.19 Discuss the losses in the optical power launched into an optical fibre.

8.20 Describe the construction and working of edge emitting LED.

8.21 Discuss any two noncommunication applications of optical fibres.

8.22 Derive the expression for optical fibre parameters, namely, the critical angle, the acceptance angle and the numerical aperture.

8.23 Discuss, quantitatively, the modes of propagation supported by optical fibres.

8.24 In a particular application, a weak optical signal is to be transmitted through a multimode fibre. Discuss the construction and working of the appropriate light source that may be coupled to the given fibre.

8.25 In a particular application, a weak optical signal is to be transmitted through a multimode fibre. Discuss the construction and working of an appropriate photodetector that may be coupled to the given fibre.

BE/BTech Degree Examination – December 2013

First Semester

Common to All Branches

11PH101 – APPLIED PHYSICS

(Regulations 2011)

Time: Three hours   **Maximum: 100 marks**

Answer all Questions

Part – A (12 × 2 = 24 marks)

1. State Weber–Fechner law in sound.
2. A hall has a volume 4000 m^3. What should be the total absorption in the hall if the reverberation time of 2.1 seconds is to be maintained?
3. State any four properties of ultrasonic.
4. A quartz crystal of thickness 2×10^{-3} m is vibrating at resonance. Calculate fundamental frequency. Assume Young's modulus of the quartz as $8.69 \times 10^{10}\ Nm^{-2}$ and its density as $2.65 \times 10^3\ kg\ m^{-3}$.
5. Define population inversion.
6. What are the differences between homo junction and hetero junction lasers?
7. What are the losses that occur during optical fiber communication?
8. A fiber cable has an acceptance angle of 30° and core index of refraction of 1.4. Calculate the refractive index of cladding.
9. What is black body radiation?
10. State uncertainty principle.
11. Give the physical significance of wave function ψ.
12. What are the limitations of optical microscope?

Part – B (4 x 15 = 60 marks)

13. a. Derive Sabine's formula for reverberation time.   (15)

(OR)

     b. What is piezoelectric effect? Explain with neat circuit diagram, the generation of ultrasonic waves using piezoelectric oscillator.   (15)

14. a. (i) Describe the principle, construction and working of CO_2 laser with neat energy diagram.   (12)

     (ii) Name any three uses of laser in industry. (3)

<div align="center">(OR)</div>

b. Explain with neat sketch the construction and reconstruction of hologram using laser beam. (15)

15. a. (i) Discuss in detail, with neat diagrams the optical fiber classified under refractive index. (8)

     (ii) Explain double crucible method of fiber manufacturing. (7)

<div align="center">(OR)</div>

b. (i) Explain fiber optical communication system with a neat block diagram. (8)

     (ii) What are the different types of fiber optic sensor? Explain the working of any one sensor. (7)

16. a. (i) Derive the expression for Schrodinger's time independent wave equation. (7)

     (ii) Deduce an expression for the energy of a particle in one dimensional box. (8)

<div align="center">(OR)</div>

b. Explain with neat sketch, the construction and working of scanning electron microscope. (15)

<div align="center">

Part – C (1 × 16 = 16 marks)

</div>

17. a. (i) Discuss the factors affecting acoustics of buildings and their remedies. (12)

     (ii) What is SONAR? How is it used to find depth of a sea? (4)

<div align="center">(OR)</div>

b. What is Compton effect? Show that the wavelength shift of the scattered photons depends only on the scattering angle and not on the incident wavelength. (16)

B.E/B.Tech. DEGREE EXAMINATION, JANUARY 2014.

First Semester

Civil Engineering

PH 6151 — ENGINEERING PHYSICS - I

(Common to all branches)

(Regulation 2013)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is a primitive cell? Give an example.
2. Name few techniques of crystal growth from melt.
3. A copper wire of 3 m length and 1 mm diameter is subjected to a tension of 5N. Calculate the elongation produced in the wire if the Young's modulas of copper is 120 GPa.
4. State Newton's law of cooling.
5. Find the lowest energy of electron confined to move in a one dimensional box of length 1 Å. Given.

   $m_e = 9.1 \times 10^{-31}$ kg

   $\lambda = 6.625 \times 10^{34}$ Js

6. Write the principle of transmission electron microscope.
7. State Weber-Fechner law.
8. Are ultrasonic waves electromagnetic waves in nature? Explain.
9. Can a two level system be used for the production of laser? Why?
10. Write any four major advantages of optical fibre communication over other communication systems.

PART B — (5 × 16 = 80 marks)

11. a. (i) What is packing factor? Prove that the packing factor of HCP is 0.74.   (2 + 10)

    (ii) Copper has fcc structure and its atomic radius is 1.273Å. Find

    (1) Lattice parameter and   (2)

    (2) Density of copper.   (2)

Given
Atomic weight of copper = 63.5
Avagadro's number = 6.026×10^{26} mol^{-1}.

Or

b. (i) Describe Bridgman method of crystal growth. (8)
(ii) Briefly explain the Chemical Vapour Deposition (CVD) method. (8)

12. a. (i) Derive an expression for depression at the free end of cantilever due to load. (12)
(ii) Give an account of I-shape Girders. (4)

Or

b. Describe with theory Lee's disc method of determination of thermal conductivity of a bad conductor. (16)

13. a. (i) Derive Planck's law of radiation. (12)
(ii) In a Compton scattering experiment the incident photons have a wavelength of 3Å. What is the wavelength of the scattered photons if they are viewed at an angle of 60° to the direction of incidence? (4)

Given:
$M_e = 9.1 \times 10^{-3}$ Kg
$\lambda = 6.625 \times 10^{-34}$ Js
$C = 3 \times 10^8$ ms^{-1}

Or

b. Write the principle, working, advantages and disadvantages of scanning electron microscope. (16)

14. a. State and explain Sabine's formula for reverberation time of a hall. Derive Sabine's formula for reverberation time. (16)

Or

b. (i) Explain Piezoelectric effect. Describe the piezoelectric method of producing ultrasonic waves. (2 + 10)
(ii) Calculate the velocity of ultrasonic waves in a liquid in an acoustic grating experiment using the following data. (4)
Wavelength of light used   = 600 nm
Frequency of ultrasonic waves   = 100 MHz
Angle of diffraction   = 5°

15. a. (i) Describe the construction and working of CO_2 laser and their uses. (14)

(ii) For a semiconductor laser, the bandgap is 0.9 eV. What is the wavelength of light emitted from it. Use the following data; (2)

$$C = 3 \times 10^8 \text{ m/s}$$
$$\lambda = 6.625 \times 10^{-34} \text{ Js}$$

Or

b. Explain the construction and working of displacement and temperature fibre optic sensors. (8 + 8)