INTRODUCTION TO FLIGHT TESTING AND APPLIED AERODYNAMICS

Barnes W. McCormick

American Institute of
Aeronautics and Astronautics

AIAA EDUCATION SERIES
JOSEPH A. SCHETZ
EDITOR-IN-CHIEF

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Published by the
American Institute of Aeronautics and Astronautics, Inc.
1801 Alexander Bell Drive, Reston, Virginia 20191-4344

American Institute of Aeronautics and Astronautics, Inc., Reston, Virginia

1 2 3 4 5

Library of Congress Cataloging-in-Publication Data

McCormick, Barnes Warnock, 1926-
Introduction to flight testing and applied aerodynamics / Barnes W. McCormick;
Joseph A. Schetz, editor-in-chief. – 1st ed.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-60086-827-6
1. Airplanes–Flight testing. 2. Aerodynamics. I. Schetz, Joseph A. II. Title.
TL671.7.M35 2011
629.132′3 – dc23
2011020552

ISBN 978-1-60086-827-6

I dedicate this book to a great teacher of young children,
my wife, Emily.

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PREFACE

I have taught a course titled "Techniques of Flight Testing" at Penn State for approximately 45 years. For 30 of them, I did the flying using either a University-owned Cherokee 180 or a Cherokee Arrow (thanks to the generosity of the Piper Airplane Co.). Now a plane and pilot have been leased to do the flying. It was a lot easier and more fun when I or another professor did the flying, but the course is still a valuable and unique one. The course title should probably be changed to "An Introduction to Flight Testing and Applied Aerodynamics" as the students are required to predict everything they measure. My students really seem to enjoy the course—not just the flying—because it ties together most of what they have learned prior to the course.

The aim of that course and this book is not to obtain precise and accurate data but to introduce students to the real world of measuring and predicting airplane performance. You will learn that theory and experiment do not always agree. You will learn how to collaborate on collecting and making the most of data and discover that a "standard" atmosphere rarely exists. You will find that data must be reduced to a standard atmosphere and weight. The notes that I have collected over the years while teaching "Techniques of Flight Testing" form the basis for this book.

The experiments described in this book do not require any elaborate equipment. The airplane's instruments, a couple of handheld voice recorders, a bubble protractor, and a tape measure will suffice. Typically, students are divided into groups of three to perform the experiments described in the appendix. Each group turns in one report on each experiment. Within each experiment each student is responsible for tasks, such as formulating of computer programs or performing drag estimates for which. The course grade is based on the three group reports, two tests during the semester, and their homework.

Now as Professor Emeritus teaching voluntarily, the university provides me with a grant that I have used to take the class on an overnight trip to the U.S. Navy's Flight Test Center at Patuxent River, Maryland as well as to the Udvar-Hazy Museum. Many of the engineers at "Pax River" have taken this course and two members of the present class have recently had job interviews there.

Barnes W. McCormick
August 2011

Chapter 1 Some Basic Considerations

- Find Properties of a Standard Atmosphere
- Predict Velocity Field Associated with a Prescribed Shape
- Determine Strength of Vortices

1.1 Introduction

An airplane takes off, climbs, cruises, descends, and lands. Associated with each phase of its flight are items of specific importance. There are several questions to ask regarding the phases:

- How accurate is the airspeed measurement?
- How much distance does it take during the takeoff roll to reach a given airspeed?
- What airspeed must be attained before taking off?
- How fast does it climb as a function of altitude?
- What is the power required to maintain level flight for a given altitude and airspeed?
- How do you assure good flying qualities during any phase of flight?
- How slow can you fly during the approach?
- How short can you land?

This text will illustrate progressively how each of these questions can be answered both experimentally and analytically. Advanced mathematics will be avoided if possible, but some basic algebra, trigonometry, and calculus will be used when absolutely necessary. Although one's understanding will be easier if he or she has had some formal training in aeronautics, an attempt has been made to develop all relationships from fundamental relationships. Throughout the book, specific airplanes and engines will be used as examples. Before we get into each topic of airplane performance, we must first cover a few basics including some fluid mechanics and properties of the atmosphere.

1.2 Mass

Most analyses in solid mechanics can be traced back to one classical equation first stated by Sir Isaac Newton that says simply

$$F = ma$$

In words, the equation says that "force equals mass times acceleration." When a person steps on a scale he or she measures the gravitational force pulling him or her toward the center of the earth. We call this force *weight*, or W. On the surface of the earth, if an object is dropped, for every second that follows the object will increase its downward velocity by 32.2 ft per second (fps), using the English system of units. In other words, the object is *accelerating downward at a rate of 32.2 fps/s*. It follows from Newton's equation that

$$W = m(32.2)$$

Thus, the mass of a body is found on the earth by simply dividing its weight in pounds by 32.2. In our ordinary life we tend to think of weight as the fundamental property, but mass is really more basic. The mass property of a body is constant and does not change. The weight, however, depends on gravitational pull and thus will decrease as an object moves away from the center of the earth. In aeronautics, as opposed to astronautics, the acceleration of gravity is assumed to be constant so that changes in weight can be neglected over small changes in altitude. Of course, during a flight fuel is burned, and so the change in weight may have to be considered if it is appreciable in comparison to the gross weight of the airplane. In the English system, the unit of mass is the *slug*. The symbol ρ will be used to stand for the mass density of the air. This means that ρ is equal to the mass of the air per cubic foot.

1.3 Standard Atmosphere

From a large collection of measurements, an average set of properties for the atmosphere has been selected as the basis for aeronautical and other applications. This set is known as the *standard atmosphere* and defines pressure, mass density, temperature, and kinematic viscosity together as a function of altitude. At this point you may not understand the meaning or significance of some of these quantities, but they will be explained as needed.

Figures 1.1 and 1.2 present the ratio of these properties to their sea-level values as a function of altitude. Notice that the temperature decreases linearly with altitude at the rate of 3.57 degrees Rankine (°R) per 1000 ft of altitude. This decrease is known as the *standard lapse rate*. The sea-level

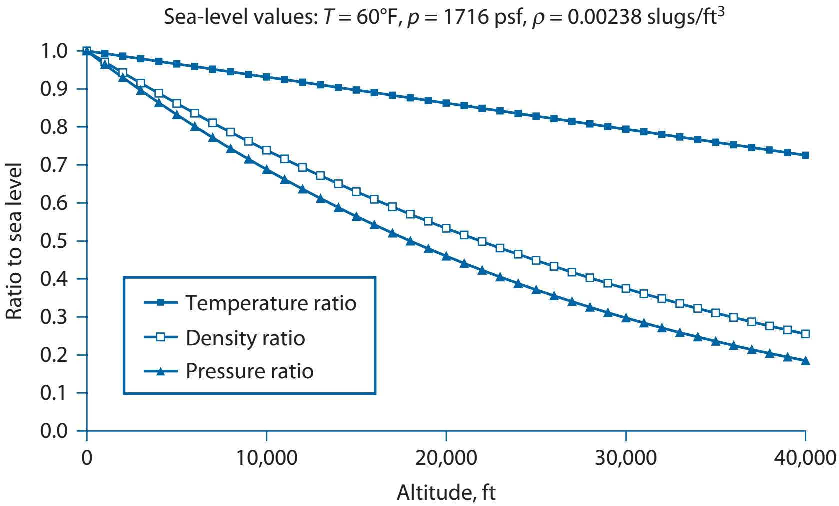

Fig. 1.1 Properties of standard atmosphere.

values are shown on Fig. 1.2 for the English system of units. The English system of units will be used throughout this book, but conversion factors to SI units, together with a brief explanation, are presented in Appendix A. Also found in Appendix A are standard atmosphere properties tabulated for a several altitudes.

For readers versed in calculus, the properties of the standard atmosphere can be derived using the standard lapse rate, the static equilibrium of the atmosphere, and the equation of state for a perfect gas. (If you are

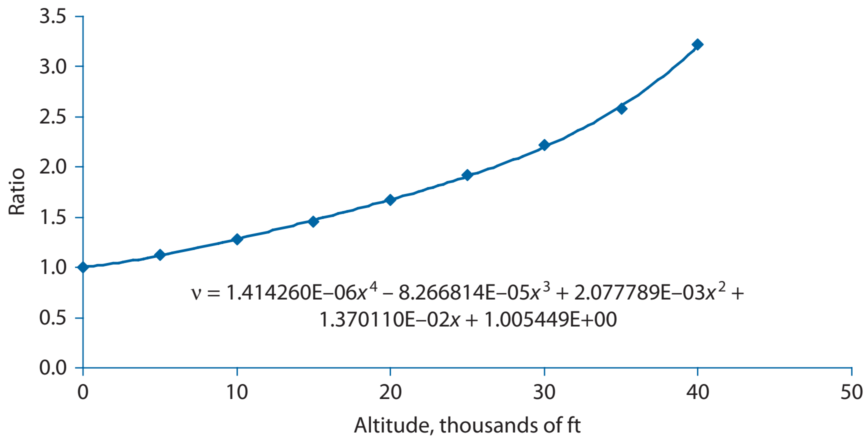

Fig. 1.2 Kinematic viscosity ratio for standard atmosphere.

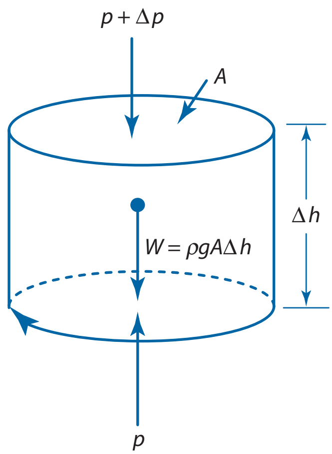

Fig. 1.3 Small cylinder of air.

not interested in determining the standard atmosphere, skip to Sec. 1.4.) Consider Fig. 1.3, which shows a small cylinder of air with a cross-sectional area of A and a small height of Δh.

The cylinder contains a volume of air weighing W. The pressure acting over the bottom is p. Over the top, $p + \Delta p$ (p plus an increment in p) is acting. The volume of the cylinder will be $A\Delta h$ so that the weight of the air, W, contained within the cylinder will be $A\Delta h\rho g$. The pressure force acting downward on the cylinder will be $A\Delta p$. If the cylinder of air is in equilibrium, the sum of the internal and external forces on it will equate to zero.

$$A\Delta p + A\Delta h\rho g = 0.0$$

In the limit as Δh approaches zero, the equation approaches the differential relationship

$$\frac{\mathrm{d}p}{\mathrm{d}h} = -\rho g \tag{1.1}$$

The equation of state for a perfect gas is $p = \rho RT$. See Eq. (2.2) for more on this equation. Inserting the equation of state into Eq. (1.1) and integrating from 0 to h will result in the density and pressure ratios.

$$\frac{\rho}{\rho_0} = \sigma = \theta^{4.256} \tag{1.2}$$

$$\frac{p}{p_0} = \delta = \theta^{5.256} \tag{1.3}$$

θ is the temperature ratio found from the lapse rate.

$$T = T_0 - 3.57\frac{h}{1000} \tag{1.4}$$

In calculating θ, remember to use the temperature in degrees Rankine. This means that 460 degrees must be added to the temperature in degrees Fahrenheit. The preceding equations are graphed in Fig. 1.1. The kinematic viscosity, ν, found in Fig. 1.2 cannot be derived; however, the curve-fit to measured data found on the figure can be used to calculate ν.

1.4 Fluid Mechanics

1.4.1 Bernoulli's Equation

It is not the intent of this book to delve deeply into the science of fluid mechanics. Therefore let it suffice simply to state the well-known Bernoulli's equation that holds along a streamline for the flow about a given shape.

$$p + \frac{1}{2}\rho V^2 = \text{constant} = \text{total pressure} = p_T \tag{1.5}$$

Again, the symbol ρ (the Greek letter rho) stands for the mass density of air, in slugs/ft^3. V is the local airspeed, or velocity, in feet per second at a point in the flow; p is the static pressure at that point in pounds per square foot (psf); and the quantity $1/2\,\rho V^2$ is called the dynamic pressure, again in pounds per square foot.

Figure 1.4 illustrates the difference between the static pressure and the dynamic pressure. Along the top, the flow along the wall is unperturbed by the hole in the wall. As a result, the tube leading from this hole senses only static pressure uninfluenced by the velocity. If the velocity, V, doubles, the

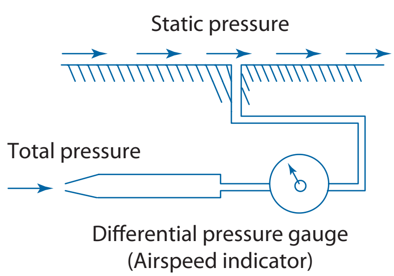

Fig. 1.4 Static and dynamic pressure.

pressure at this point remains unchanged. Conversely, when the flow enters the total head tube at the bottom of the figure, the flow comes to rest and the tube attached to the total head senses the total pressure. If the static pressure and total pressure tubes are connected across a differential pressure gauge as shown, the gauge will read the dynamic pressure denoted by q. Knowing the dynamic pressure we can then calculate the free stream velocity, V, from Eq. (1.5).

$$q = \frac{1}{2}\rho V^2$$

or

$$V = \sqrt{\frac{2q}{\rho}} \tag{1.6}$$

Let us work an example of the preceding equation using the properties for the *standard* atmosphere. Suppose the altimeter on our airplane shows that we are operating at an altitude of 10,000 ft. From Fig. 1.1—or Eqs. (1.2) and (1.4) or Appendix A—the density ratio equals 0.74 so that the density at this height is 0.00176 slugs/ft^3. Now assume that the difference in pressure from two tubes, one connected to total head tube and the other to a hole flush with the surface near the rear of the fuselage, equals 25 psf. The airspeed is found to equal

$$q = \frac{1}{2}\rho V^2$$

or

$$V = \sqrt{\frac{2q}{\rho}} = 168.5\,\text{fps} = 114.9\,\text{mph}$$

Note that using consistent units is very important. Using pressure in pounds per square foot and ρ in slugs, the true airspeed (TAS) is determined in feet per second. This can then be translated to meters per second, miles per hour (mph), or knots if desired.

1.4.2 Dimensionless Coefficients

Any aerodynamic force can be expressed in terms of a dimensionless coefficient. The advantage of this is that, within normal operating values of airspeed and size, the coefficient, unlike the actual force, is dependent only on geometry and not on size or airspeed. The lift coefficient, C_L, of a wing is simply a constant of proportionality between the lift, L, and the product of the wing *planform* area, S, and the *dynamic pressure*.

$$C_L = \frac{L}{qS} \tag{1.7}$$

The drag coefficient is defined in a similar way with L replaced by D, which represents the drag. These coefficients will be covered in more detail throughout this book.

1.4.3 Modeling of Aerodynamic Shapes

Some of the material in this section will be covered in more detail and applied to specific applications. (Chapter 6 introduces the problem of predicting the velocity field associated with a prescribed shape.)

To begin, two-dimensional and three-dimensional flows will be defined. To illustrate, imagine a wing with a span of 2 miles and a chord of 2 ft. Near the middle of this ridiculous wing shape it is obvious that the flow around the exact middle of the wing looks the same as the flow 1 ft away from the midspan. The tips are far away and not a factor until we get much farther out on the span. If we define x as the forward direction, y rightward, and z downward, the flow for a large portion of the wing near the center is a function only of the x and z directions. Thus the flow is said to be two-dimensional in nature. As we get near the tips the lift will drop off and flow around the tips becomes a factor, and so in this region the flow is a function of x, y, and z. Thus here the flow is three-dimensional in nature.

We will now distinguish between a wing and an airfoil. An airfoil is a section of a wing and is treated as being in a two-dimensional flow resulting from the velocity of the airplane and perturbations in the velocity produced by the three-dimensional flow around the tips. Two-dimensional measurements are obtained in a wind tunnel by testing a constant chord wing that extends from one wall to another so that there can be no flow around the tips. To predict the performance of a wing, one usually obtains the performance of the airfoil from wind tunnel tests or theory and then corrects the results for the three-dimensional flow effects. For example, consider a more realistic wing having a span of 30 ft and a constant chord of 5 ft. Because of the three-dimensional nature of flow generally about this wing, a downward velocity, w, is experienced along the wing equal approximately to $w = VC_L/(\pi A)$. The variable A is the wing's aspect ratio given by the ratio of the span to the chord, b/c. This downwash causes a decrease in the angle of attack of each section along the wing. Thus the airfoil section lift coefficient would be determined by the wing angle of attack

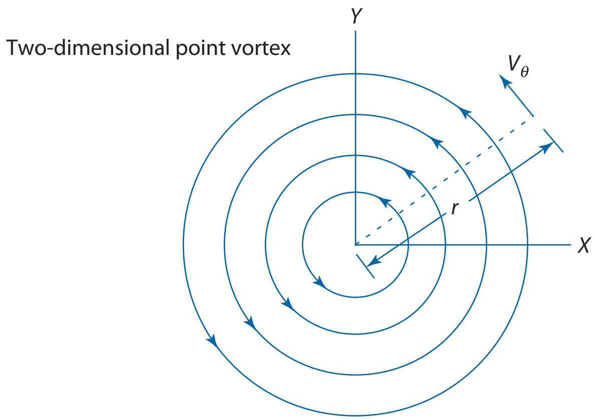

Fig. 1.5 Flow field for a two-dimensional vortex.

minus the *induced* angle given in radians by

$$\alpha_i = \frac{w}{V} = \frac{C_L}{\pi A}$$

Any solution for the flow around a body must satisfy three conditions. First, the solution must satisfy the equations of fluid motion. Second, continuity, or mass conservation, must be satisfied. Finally, all boundary conditions must be satisfied—that is, the velocity normal to any solid boundary must be zero. In the case of an airfoil, an additional boundary condition is imposed based on experimental observations. This condition, known as the *Kutta condition*, specifies that the flow must leave tangent to the trailing edge. The first two conditions can be satisfied automatically by using elementary flow functions to model the flow. There are three basic flow functions: uniform flow, vortices, and sources. For this text, we are only concerned with uniform flow and vortices. Uniform flow is easily handled and needs no explanation.

1.4.4 Vortices

The flow field for a two-dimensional vortex is illustrated in Fig. 1.5. The velocity direction is purely tangential, and the magnitude of the velocity varies so that the strength, Γ, of the vortex is the same for all radii. The strength is called circulation, or circulatory strength, and is equal to the product of the tangential velocity and the distance around the circle enclosing the vortex center. Thus

$$\Gamma = 2\pi r V_\theta$$

or

$$V_\theta = \frac{\Gamma}{2\pi r} \tag{1.8}$$

A three-dimensional vortex lies along a line and is known as a vortex filament. The velocity induced at point P by a straight line vortex filament is obtained by reference to Fig. 1.6 and the magnitude calculated from

$$V_i = \frac{\Gamma}{4\pi h}(\cos\ \alpha + \cos\ \beta) \tag{1.9}$$

Equation (1.9) is called the Biot–Savart law. The direction of the circulation around the vortex filament is shown according to the *right-hand rule*. Point the thumb of your right hand in the direction of the circulation vector and your fingers curl in the direction of the rotation around the filament. In Fig. 1.6, the right-hand rule shows that the velocity is directed into, and normal to, the paper.

At this point, you may think that this is just an exercise in mathematics, but let me assure you that potential flow and elementary flow functions have many applications in real life. Consider Fig. 1.7, which shows the wake trailing from a wing in flight. The pressure is higher underneath the wing, causing the air to flow around the tips to the lower pressure on top. This results in a thin layer at the trailing edge where the flow is generally inward toward the center of the wing on top of the layer and outward from the center on the bottom of the layer. This shearing action of the air in a thin layer is a vortex sheet pictured in Fig. 1.7 by vortex filaments trailing from the wing. These filaments induce motion on each other resulting in a rapid rolling-up of the vortex sheet into two vortices as pictured. Assigning a direction to the filaments according to the direction of the circulation, the vortex system can be modeled as vortex filaments feeding into the wing on the left side, crossing the wing from left to right, and trailing from the wing

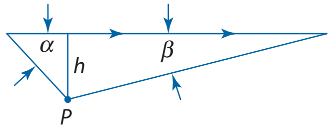

Fig. 1.6 Biot–Savart law. See Eq. (1.9).

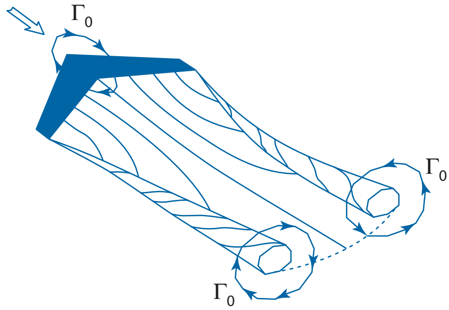

Fig. 1.7 Rolling-up of a vortex sheet trailing from a wing.

on the right. Thus the strength of each of the trailing vortices equals the total *bound* (to the wing) circulation at the wing's midspan point.

As the sheet rolls, the tip vortices get closer together. If b' is the span between the two trailing vortices when completely rolled up, and if b is the wingspan, it can be shown approximately that $b'/b = \pi/4$. The total lift on the wings produces the final pair of trailing vortices. But this same pair could have been produced by a vortex line of constant circulation Γ_0 with a length of b'. There is a theorem in aerodynamics known as the Kutta–Joukowski theorem that states that the lift *per unit length* required to hold a vortex filament in place is given by

$$L = \rho V \Gamma \tag{1.10}$$

Because, in trimmed flight, the lift of the wing is equal approximately (minus some trim tail force) to the gross weight, W, it follows that the strength of a trailing vortex is given closely by

$$\Gamma_0 = \frac{4W/b}{\pi \rho V} \tag{1.11}$$

This is an important equation related to the hazard of wake turbulence of which every pilot should be aware. It is the first step in determining the velocity field that a small plane will experience when encountering the wake from a large airplane ahead of it.

In this introductory chapter, this is as far as this text will delve into vortices. However, they will be used again in Chapter 6 in modeling a wing to predict the stalling speed.

1.5 Summary

This book will cover fundamentals of flight testing and aerodynamics including airplane performance and stability. While aerodynamic forces and moments are the "bottom lines," dimensionless coefficients are utilized to predict and analyze these aerodynamic quantities. Analytical modeling of the aerodynamic characteristics of an airplane is important.

Problems

Answers to the following problems can be found in Appendix C.

1.1 An airplane has a wing loading of 100 psf and is flying at a TAS of 200 kt at 10,000 ft standard. What is the airplane's lift coefficient?

1.2 The airplane in Problem 1.1 is flying at a knots indicated airspeed (KIAS) of 200 with an outside air temperature (OAT) of 10°F. What is the airplane's C_L?

1.3 An airplane weighs 40,000 N and is flying at 2000 m standard. If the airplane C_L equals 0.3 and the TAS equals 150 kt, what is the wing planform area?

1.4 A Cessna 172 with a wingspan of 36 ft is making an approach at 90 KTAS when it becomes aligned with the center of one vortex trailing from a B-767 having a span of 156 ft and a gross weight of 387,000 lb and flying at 135 KIAS. How much would the angle of attack of the Cessna's wing be increased at the left tip by the B-767's vortex trailing from its left wing? The pressure altitude is 2000 ft and the OAT equals 60°F.

1.5 Each vortex of a pair of trailing vortices induce a downward velocity on the other given by Eq. (1.8). How rapidly would you predict the pair of vortices from the B-767 in Problem 1.4 to descend?

Chapter 2 Airspeed Calibration

- Calibrate Airspeed Indicator

2.1 Introduction

The airspeed indicator (ASI) is simply a differential pressure gauge. Instead of indicating pressure, however, the scale on the gauge indicates airspeed in accordance with Eq. (1.6) using standard sea-level (SSL) density.

Like any mechanical instrument, the ASI may not be precisely accurate and may have a mechanical error. Such an error is referred to as *instrument error*. For our purposes, we will assume that the ASI has been calibrated and that there is no instrument error. The other type of error is referred to as *position error*, which results from the measured static pressure being unequal to the undisturbed air stream static pressure. The manufacturer locates the static pressure to avoid this error, but position error generally cannot be avoided over the entire airspeed range of an airplane. General aviation airplanes are certified under Federal Aviation Regulations (FAR) Part 23 (see www.faa.gov). These regulations state that, excluding instrument error, the position error may not exceed 3 percent of the calibrated airspeed (CAS) or 5 kt, whichever is higher, for speeds 30 percent above the stalling speed. CAS is the speed shown on the dial that is inscribed according to Eq. (1.6), letting ρ equal the sea-level value of 0.00238 slugs/ft^3. The dynamic pressure q for a given V is calculated using the sea-level mass density. The value of V is then inscribed on the gauge face corresponding to a pressure equal to the calculated dynamic pressure.

The pilot's operating handbook (POH) will contain a graph showing the airspeed calibration for that particular airplane. However, let us do our first flight test to calibrate the ASI.

2.2 Flight Test No. 1: Calibration of Airspeed System

We will calibrate the ASI by recording the total time to fly a known distance in opposite directions. By flying in opposite directions, the effect of the wind will cancel out.

Referring to Fig. 2.1, suppose we fly a measured distance, s, either by reference to a ground reference or global positioning system (GPS) while holding a constant heading and indicated airspeed (IAS). *Indicated airspeed* is the speed shown on the ASI before any corrections. In so doing we will be flying at some TAS, V. Now do a 180-deg turn and fly the constant reciprocal heading for the same measured distance. Assuming that the wind does not change during the two flights, the ground speeds for each segment will be V_1 and V_2 with corresponding times of t_1 and t_2 to cover the distance s parallel to our heading. We can now write

$$s = t_1(V - V_w \cos\theta)$$
$$s = t_2(V + V_w \cos\theta)$$

Adding these results for the IAS that we were using results in a TAS for this IAS.

$$V = \frac{2s}{(t_1 + t_2)} \tag{2.1}$$

Now the question is: What is the IAS corresponding to this TAS that we measured? Is it the same as the IAS that we were holding? To answer these questions, we need to know the density of the air for our tests. This means that before starting our test, we should set the altimeter to 29.92 inHg corresponding to SSL static pressure. The altimeter will then read the *pressure altitude*, or the altitude corresponding to the standard pressure in Fig. 1.1. Thus, reading the altimeter will determine the atmospheric pressure. Then, during the testing we must read the outside air temperature (OAT) and convert it to degrees R. If this temperature is the same as the standard

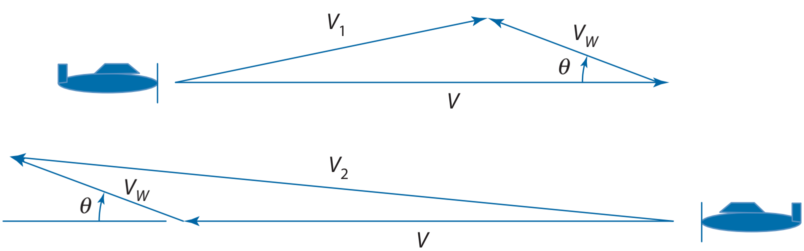

Fig. 2.1 Flying a measured course for airspeed calibration.

temperature, then we know that the density is equal to the standard density. In general, however, the OAT will not be standard, in which case we must calculate the density from a well-known equation used in gas dynamics called the equation of state.

$$p = \rho RT \tag{2.2}$$

Here, R is a universal gas constant and is equal to 1716 $ft^2/s^2/^\circ\mathrm{R}$ in the English system.

For example, suppose we flew our test at a pressure altitude of 4000 ft. At this altitude, from Fig. 1.1, the pressure ratio equals 0.86. Multiplying this by 2116 results in an outside pressure of 1820 psf. Now suppose the OAT reads 65°F. Adding 460° to this gives 525°R. This is higher than the standard temperature that would be calculated from Fig. 1.1. Thus we must calculate the mass density from Eq. (2.2).

$$\rho = \frac{p}{Rt} = \frac{2116}{(1716)(525)} = 0.00235\ \mathrm{slugs/ft^3}$$

We measured an airspeed of V. For this speed and the above density, the dynamic pressure would be $q = \frac{1}{2}(0.00235)\ V^2$. But for this same dynamic pressure

$$\frac{1}{2}(0.00235)\ V^2 = \frac{1}{2}(0.00238)\ V_i^2$$

or

$$V_i = V\sqrt{\frac{0.00235}{0.00238}} = V\sqrt{\sigma} \tag{2.3}$$

The V_i in Eq. (2.3) is the true IAS that the ASI should have read. This will be renamed and called the CAS, V_c. The CAS is simply the IAS corrected for any errors in the ASI. At this point it is important to reflect on Eq. (2.3). The TAS is obtained from the IAS after correcting for errors by dividing the IAS by the square root of the density ratio. Frequently pilots will use IAS and CAS to mean the same quantity because, barring errors in the ASI, they are the same.

Having flown the test as described in the preceding paragraph we now have one CAS for one IAS. The test is now repeated over a range of IASs to obtain a graph of V_c vs V_i. Such a graph is shown in Fig. 2.2 for the Cessna 172R. The Cessna 172R will be used as the example aircraft throughout this book, as it was one of the airplanes I used for instructing students in techniques of flight testing at the Pennsylvania State University.

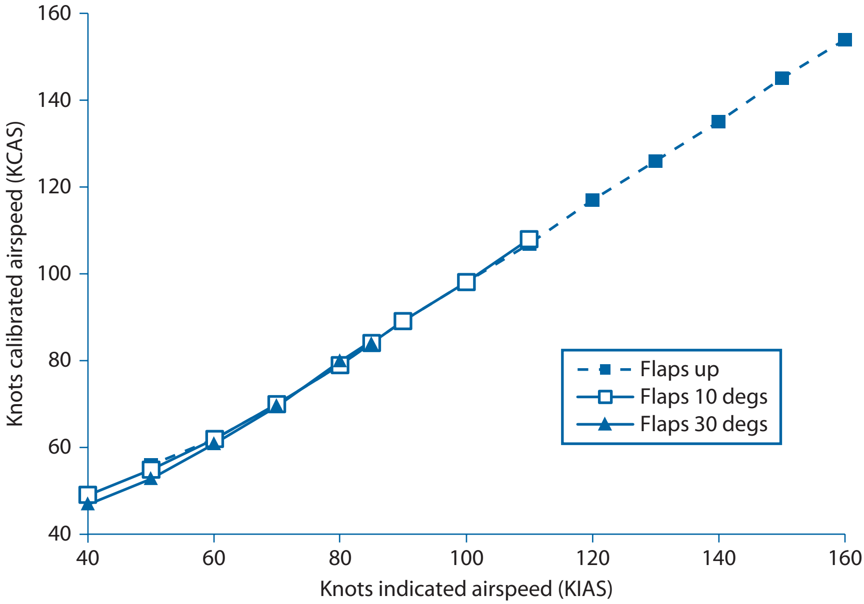

Fig. 2.2 Airspeed calibration for Cessna 172R (reprinted with permission from Cessna Aircraft Company).

A pressure coefficient at any point on an aerodynamic surface is defined as

$$C_p = \frac{p - p_0}{q}$$

From Bernoulli's equation, the pressure coefficient can be related to the free stream and local velocity by

$$\frac{V}{V_0} = \sqrt{1 - C_p}$$

With no instrument error, the equation is equal to the ratio of the IAS to the CAS. Of course, C_p should equal zero at the location of the static pressure source for no position error. However, C_p is a function of the airplane's lift coefficient so that, strictly speaking, the graph of V_c vs V_i should be available for a range of gross weights. However, the dependence of V_c vs V_i on weight is so slight that a graph for only an airplane's standard gross weight is given in the POH.

At this point we are ready to hop in the airplane and do some flight testing. In the interest of time and money, we will combine several tests in one flight. The first flight will encompass four experiments—namely, the ASI calibration (one speed point per student group), ground roll measurement, steady climb, and determination of rate-of-climb (R/C). Of course, you can break up these topics into separate flights if desired. At this point we have covered only the analysis of ASI calibration. However, the details of the first flight-test experiment are given in Appendix B, and from those details one should be able to prepare adequate data sheets for each segment of the test.

Do Problem 2.1 and then check a possible format for the data sheet in Appendix C. Separate data sheets should be prepared in advance of the flight test for each segment of the test. After the test, the data for the ASI segment can be reduced and a report prepared that compares the CAS with the IAS. It will be necessary for each student group to fly along the runway at a different IAS and to exchange its data with other groups so that a range of IAS values can be obtained.

2.3 Summary

The IAS is not the true airspeed (TAS). It must be corrected first for instrument and position errors and then for atmospheric density to obtain the TAS.

Problems

2.1 Prepare a data sheet for the ASI calibration flight test. The sheet should allow for input on airplane ID, payload, weights, distances, headings, times, and temperatures.

2.2 An ASI flight test is performed and the following data are obtained:

measured distance	2 n miles
pressure altitude	10,000 ft
OAT	50°F
headings	060° and 240°
time to fly first heading	61.2 s
time to fly reciprocal heading	53.2 s

What is the IAS in knots if the ratio of CAS to IAS is 1.05?

2.3 In Problem 2.2, assume that the difference between CAS and IAS is due only to position error. If the pilot reads a pressure of 10,000 ft, what is his or her actual pressure altitude? (Remember: The altimeter and ASI are connected to the same static pressure source.)

Chapter 3 Takeoff

- Measure Takeoff Roll
- Determine Forces of Interest on an Airplane
- Determine Engine Power
- Estimate Ground Roll Distance

3.1 Introduction

Now that we have confidence in the ASI, let us begin the takeoff. Fig. 3.1 shows an airplane rolling along a runway. As shown, there are five forces of interest acting on the airplane—namely, lift, drag, weight, thrust, and rolling friction. We must determine these forces in order to predict the instantaneous acceleration along the runway. However, before predicting the takeoff roll, let us consider the experimental procedure for measuring it. Ultimately we want to know how long of a ground roll it takes to attain the liftoff airspeed.

3.2 Federal Aviation Regulations

FAR Part 23 specifies the takeoff performance for general aviation airplanes. McCormick [1, p. 358] interprets part of this FAR as follows:

> FAR Part 23 is simpler in specifying the takeoff procedure. For airplanes over 6000 lb (26,700 N), maximum weight in the normal, utility and acrobatic categories, it is stated simply that the airplane must attain a speed at least 30% greater than the stalling speed with one engine out, V_{S1}. For an airplane weighing less than 6000 lb, the regulations state simply that the takeoff should not require any exceptional piloting skill. In addition the elevator power must be sufficient to lift the tail (for a tail-dragger) at 0.8 V_{S1} or to raise the nose for a nose-wheel configuration at 0.85 V_{S1}.

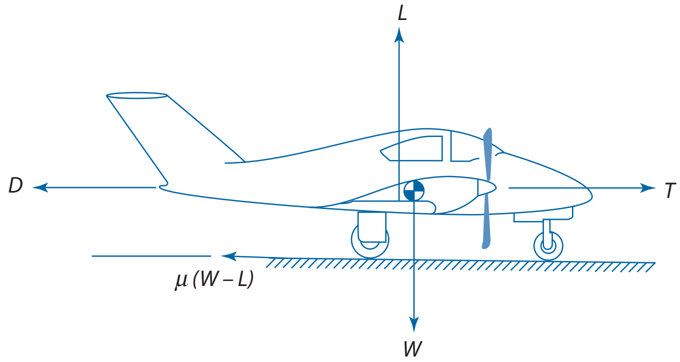

Fig. 3.1 Forces acting on an airplane rolling along a runway.

3.3 Experimental Procedure for Measurement of Ground Roll

Without the use of sophisticated, expensive equipment, the most direct way to measure the ground roll is to brake the airplane at a first marker along the runway, open the throttle wide, release the brake, and start the clock and note the time, revolutions per minute (rpm), and airspeed as the airplane passes markers along the runway. These markers may be purposely placed along the runway or one may make use of existing markers such as runway lights. As an alternative to this method, a GPS may be used to record position, and possibly velocity, as a function of time. Video equipment with a clock can also be used with the tape replayed to measure position vs time. The first method can be readily accomplished with the use of a solid-state, handheld recorder with built-in clocks. Before rolling, it is important to note the gross weight and headwind. Also, as usual before beginning the experiment, the altimeter should be set to 29.92 inHg and the OAT noted.

Airplane takeoff performance in the POH is given for a particular gross weight. To compare your data with the POH, a correction must be made for both weight and headwind. To correct for the weight consider the simple case of a body undergoing a constant acceleration due to a constant force, F. If a is the acceleration in f/s^2 then the acceleration, a, velocity, V, and distance, s, are related by

$$
\begin{aligned}
V &= at = \frac{Fg}{W}t \\
s &= \frac{at^2}{2} = \frac{Fg}{2W}t^2
\end{aligned}
\tag{3.1}
$$

Thus, as a first-order correction, simply assume that the velocity and distance vary inversely with the weight. To be more precise, use the predictive methods to be described in Chapter 4 to correct for both weight and altitude.

3.3.1 Rolling Friction, *F*

The frictional force on an airplane is given by the product of a constant coefficient of friction, μ, and the weight on the wheels. If L denotes the lift and W the weight, then the downward force on the wheels is the weight minus the lift. Thus the retarding friction force is given by

$$F = \mu(W - L) \tag{3.2}$$

From experience typical values for the coefficient μ range from 0.02 to 0.1 for surfaces from a hard runway to moderately tall grass.

3.3.2 Weight, *W*, and Balance

The weight of an airplane is composed of the weight-empty, the fuel weight, and the payload (passengers plus crew and luggage). A figure of 6 lb/gal is used for the fuel weight while the weight-empty can be found in the POH for the airplane.

The location of the center of gravity (c.g.) of an airplane is important to the trim of the airplane. If the c.g. is too far aft, it adversely affects the stability and control of the plane, and if the c.g. is too far forward, it can be difficult to raise the nose during takeoff. The c.g. is the point at which the total weight is taken to act. This means that the moment of the weight about the c.g. is zero. If i denotes an item with a weight of W_i located a distance of x_i aft of a datum buttline, then the total moment about any distance, x, from a datum will be given by

$$M = \sum W_i(x_i - x)$$

The symbol $\sum$ means to sum over all i items. This sum will be zero at $x = x_{cg}$. Hence, the distance of the c.g. aft of the datum will be found from

$$0 = \sum W_i x_i - x_{cg} \sum W_i$$

or

$$x_{cg} = \frac{\sum W_i x_i}{W}$$

Table 3.1 presents x_i values for the major weight components for the Cessna 172R.

Table 3.1 Moment Arms for Cessna 172 Weight Items

Item	Weight (lb)	Moment arm (in.)
Empty weight	1639	39.3
Usable fuel (53 gal max)	Variable	48.1
Front seats	Variable	37.1
Rear seats	Variable	72.7
Baggage (120 lb max)	Variable	708.0
Fuel for warm-up and taxi	−7.0	−39.3

3.3.3 Lift, *L*

The calculation of any aerodynamic force is generally a challenge and requires a mixture of empiricism and theory. Let us start with the simplest case of an isolated airfoil. As stated earlier, an airfoil can be thought of as a constant-chord wing with an infinitely long span. Any section of such a wing has the same flow pattern and thus the flow is said to be two-dimensional. The velocity field about the airfoil is only a function of the x and z coordinates where x is directed forward and z downward. However, if that airfoil shape is employed in a wing, the wing has tips outside of which there can be no lift or pressure difference between the top and bottom surface. The air then flows from the region of higher pressure on the bottom around the tips to the top where the pressure is lower. This sets up a swirling motion of the air behind the wing in its wake. A shearing action occurs along the trailing edge of the wing where the flow is outward underneath and inward on top. This flow pattern is referred to as a vortex sheet. This pattern is unstable, however, and rolls up rapidly into two trailing vortices, one from each tip. Tip vortices are like horizontal tornadoes, except on a smaller scale. When shed from a large airplane, these vortices produce high *induced velocities* that can be hazardous to a smaller, following airplane that may intersect the vortices. This phenomenon is referred to as wake turbulence.

The trailing vortex system also induces velocities along the wing itself, producing local changes in the angle of attack of the wing sections. Thus the flow around the wing is no longer two-dimensional and the lift vanishes at the tips. We will delve into this problem in further detail in Chapter 6.

As a simpler approach, consider the following. The incidence angle of a wing relative to the fuselage is chosen so that in cruising flight the fuselage is nearly horizontal. When rolling along the ground, the fuselage is also nearly horizontal. Thus, for the want of something better (which is proposed later in Chapter 4), we will assume for the ground roll that the lift coefficient rolling along the ground is the same as that for cruise. Actually, the lift generated by the wing during the ground roll has little effect on the ground roll distance, and so errors in C_L are not too significant.

3.4 Engine Power

In this section of this book the Cessna 172R will be used as an example. This airplane is pictured in Fig. 3.2.

Ideally, when determining the performance of an airplane, one would have installed instrumentation that directly measures thrust and power from the engine-thruster configuration. In lieu of such sophisticated instrumentation we will use—in the case of a propeller-driven airplane—our best estimate of propeller thrust and power from a chart of propeller characteristics and the engine power from an engine chart. Consider Fig. 3.3, the engine chart for the Cessna 172R. The left side of the chart is for SSL performance while the right side is for performance at altitude. The insert along the top explains the correction to be made for nonstandard temperature. We will use this chart to estimate the power being delivered to the propeller but with certain reservations and shortcomings kept in mind. First, our engine is not *calibrated*, and thus the chart may not be strictly applicable. Second, when the engine is installed in the airplane, certain losses will occur, and thus the power will not equal exactly the power measured on a dynamometer in the factory.

If one is flying an airplane with an adjustable-pitch propeller, such as a constant speed propeller governed to change the pitch and maintain a set

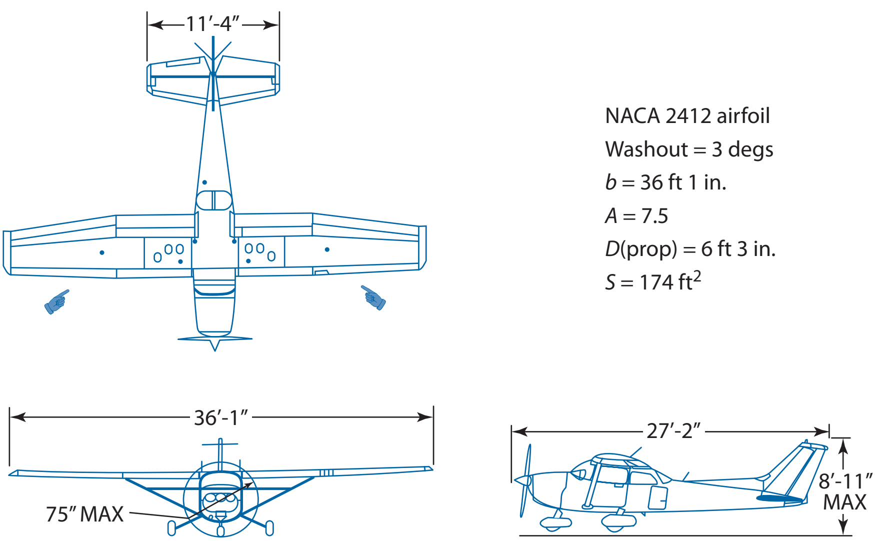

Fig. 3.2 Cessna 172R (reprinted with permission from Cessna Aircraft Company).

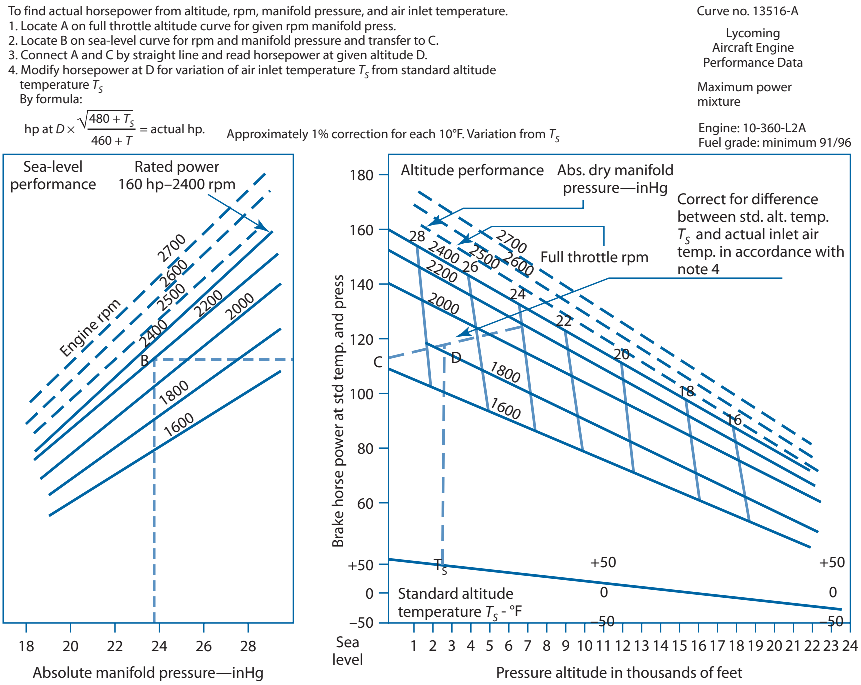

Fig. 3.3 Engine chart for Cessna 172R (reprinted with permission from Cessna Aircraft Company).

rpm, the pilot will have a manifold pressure gauge from which the manifold air pressure (MAP) can be read. In the case of a fixed-pitch propeller, however, such as the Cessna 172R, a MAP gauge is not usually installed, and so only the wide-open-throttle (WOT) power can be estimated from the chart. For part-throttle operation one follows the directions included on the chart. For the WOT operation, one reads the engine rpm and then enters the right side chart at this rpm and pressure altitude to read a power. This power is then corrected for deviation from the standard temperature according to the directions on the chart.

3.5 Propeller Thrust

Figure 3.4 shows a section of a propeller looking in toward the hub. This is the propeller section that would be projected on a cylindrical surface with a radius of r and concentric with the propeller axis of rotation. Relative to the section, the resultant velocity will be the vector sum of the free stream velocity, V, and the velocity due to rotation, ωr. The variable ω is the angular velocity of the propeller in radians per second given by $\omega = 2\pi n$ where n equals revolutions per second (rps). Thus, referring to Fig. 3.4, the angle of attack of the blade section can be found from

$$\alpha = \beta - \tan^{-1}\left(\frac{V}{\omega r}\right)$$
$$= \beta - \tan^{-1}\left(\frac{V}{2\pi n r}\right) \tag{3.3}$$

or

$$= \beta - \tan^{-1}\left(\frac{J}{\pi x}\right)$$

where $J = V/nD$.

J is called the advance ratio and x is the relative radial distance from the propeller axis, r/R, where R is the propeller radius and D the propeller diameter. Since the angles of attack of the blade sections are a function of J, it follows that any dimensionless coefficient defining propeller performance is a function of J. Two such coefficients are defined, a thrust coefficient, C_T, and a power coefficient, C_P.

$$C_T = \frac{T}{\rho n^2 D^4}$$
$$C_P = \frac{P}{\rho n^3 D^5} \tag{3.4}$$

P is the power to the propeller shaft in ft · lb/s or in units consistent with the other variables. The coefficients are dimensionless, and so it does not matter whether you are using the English or the SI system.

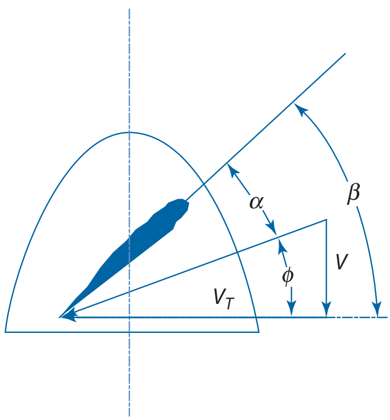

Fig. 3.4 Propeller blade section.

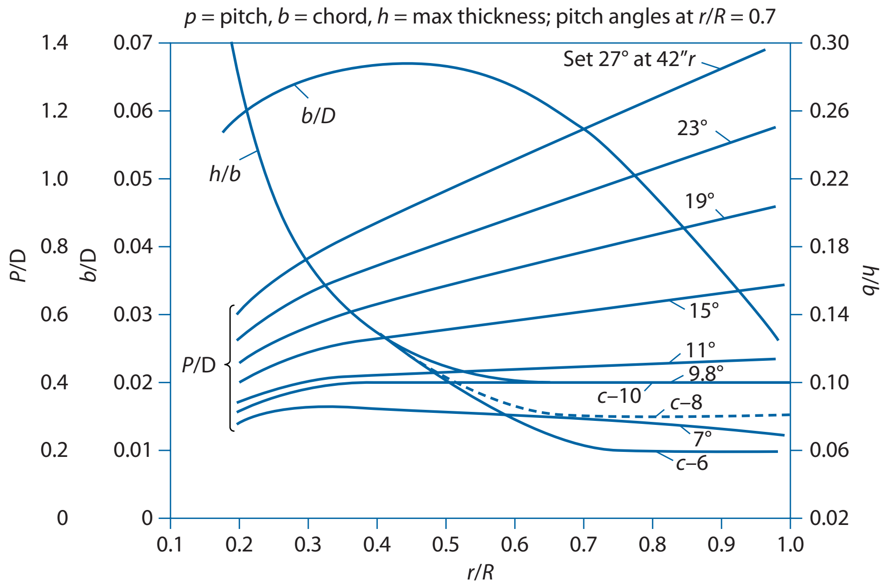

Fig. 3.5 Geometry for propellers presented in NACA Technical Report 421.

The useful power done by a propeller is defined as $P_{\text{use}} = TV$. Dividing this by the power to the shaft results in the propeller efficiency

$$\eta = \frac{TV}{P} = \frac{C_T}{C_P} J \tag{3.5}$$

A propeller chart presents C_T and C_P as a function of J. However, it is difficult to obtain propeller charts applicable to a particular airplane-propeller combination. However, a large variety of propellers have been tested by the National Advisory Committee for Aeronautics (NACA), and from these one can estimate the performance. One such report is NACA Technical Report 421 [2]. Here, a series of propellers typical of general aviation propellers having a range of pitch–diameter ratios (p/D) were tested over a range of advance ratios.

Pitch has not been defined yet. For propellers, it is entirely analogous to the pitch of an ordinary screw. Referring again to Fig. 3.4, if the propeller "screws" itself through the air, then in one revolution, a section of the propeller will advance forward a distance equal to the pitch, p. From the figure,

$$p = 2\pi r \tan\beta \tag{3.6}$$

In terms of the diameter and advance ratio, this can be written as

$$\frac{p}{D} = \pi x \tan\beta \tag{3.7}$$

where $x = r/R$.

Note, in Fig. 3.4, if ϕ equals β, the angle of attack is zero. Therefore the advance ratio at which the thrust of any propeller section vanishes is given approximately by

$$\tan\beta = \tan\phi$$

or

$$\frac{J}{\pi x} = \frac{p}{D} \tag{3.8}$$

Generally, the pitch at any radius is related to the pitch angle, β, by $p = 2\pi r \tan\beta$. Thus, if the pitch is constant along the radius of a propeller, the pitch angle is found from

$$\beta = \tan^{-1}\left(\frac{p/D}{\pi x}\right) \tag{3.9}$$

A propeller having the above distribution of pitch angles is said to be a “constant pitch” propeller. Do not confuse this with a “fixed-pitch” propeller where the blades are rigidly attached to the hub and cannot rotate to change their pitch angles.

Having the above explanations for pitch, advance ratio, and thrust and power coefficients, consider Figs 3.5, 3.6, 3.7, and 3.8, all taken from NACA Technical Report 421 [2].

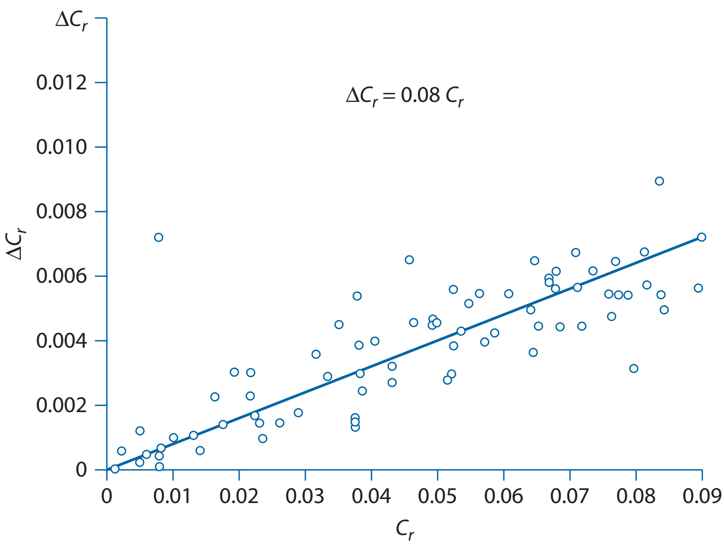

Fig. 3.6 Experimental value for thrust deduction for typical small airplane.

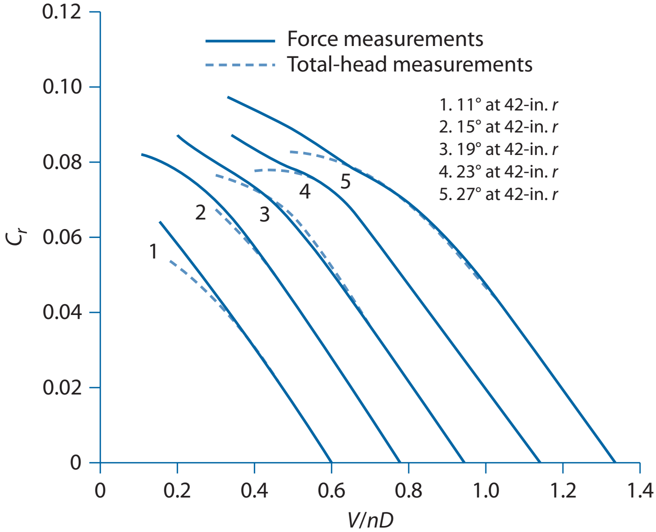

Fig. 3.7 Thrust coefficients from NACA Technical Report 421.

Examine Fig. 3.7 and propeller No. 4 with regard to the relationships between J, β, and ϕ. This propeller is chosen because, as we will see, it is similar to the propeller for the Cessna 172R. The β values are chosen at the 42-in. radius and the propellers tested in Technical Report 421 have a

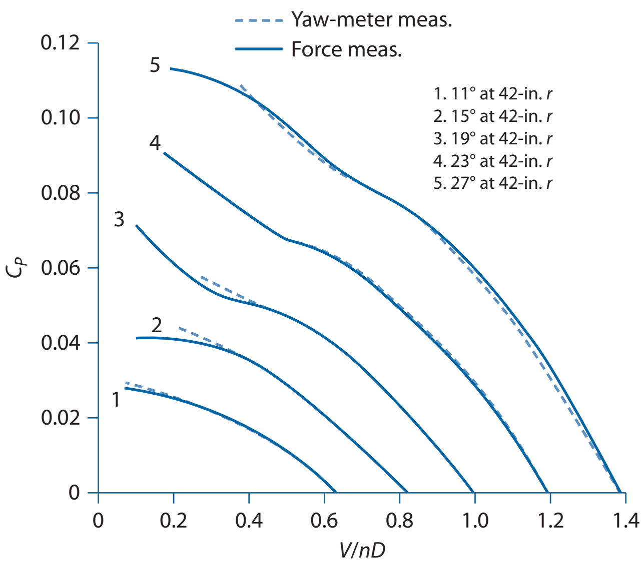

Fig. 3.8 Power coefficients from NACA Technical Report 421.

diameter of 9.5 ft. Thus, $x = 0.737$ for the section where the pitch angles are defined. This radius was probably chosen because the sections around the 75 percent radius tend to represent the whole propeller. Note that C_T vanishes at a J of 1.15. For this x, from Eq. (3.9), this corresponds to a pitch angle of 26.4 deg. But the curve is labeled 23 deg. This leads to the fact that another 3.4 deg must be added to the labeled pitch angle to make it consistent with the experimental C_T curve. This is entirely reasonable, because the pitch is probably measured relative to the flat pressure side of the Clark Y airfoils employed for these propellers, and the zero-lift for the Clark Y airfoils is approximately 3 or 4 deg above the flat side.

The geometry of the Cessna 172R propeller was measured using an ordinary scale and a protractor with the measured pitch shown in Fig. 3.9 and the chord distribution in Fig. 3.10. By comparison to Fig. 3.4, it can be seen that the pitch of the Cessna propeller over a large outer portion of the outer radius is close to that of propeller No. 4, having a pitch angle of 23 deg. The planforms of both propellers are also similar, but the blade is wider for the Cessna 172R propeller. The solidity of a propeller, σ, is defined as the total blade area divided by the disc area πR^2. The solidity for propeller No. 4, from the chord distribution, was found to be approximately 0.0569. For the Cessna 172R propeller, the solidity was estimated from the measured chords, shown in Fig. 3.10, to equal 0.0725. Since the pitches are approximately equal for the two propellers, all flow angles will be the same for the same advance ratio. Thus, both the thrust and power

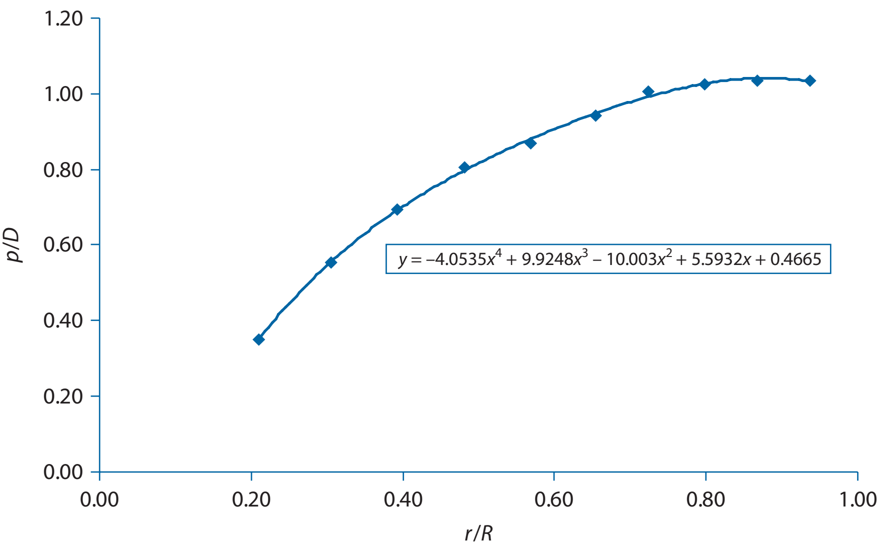

Fig. 3.9 Pitch–diameter ratio for Cessna 172R propeller.

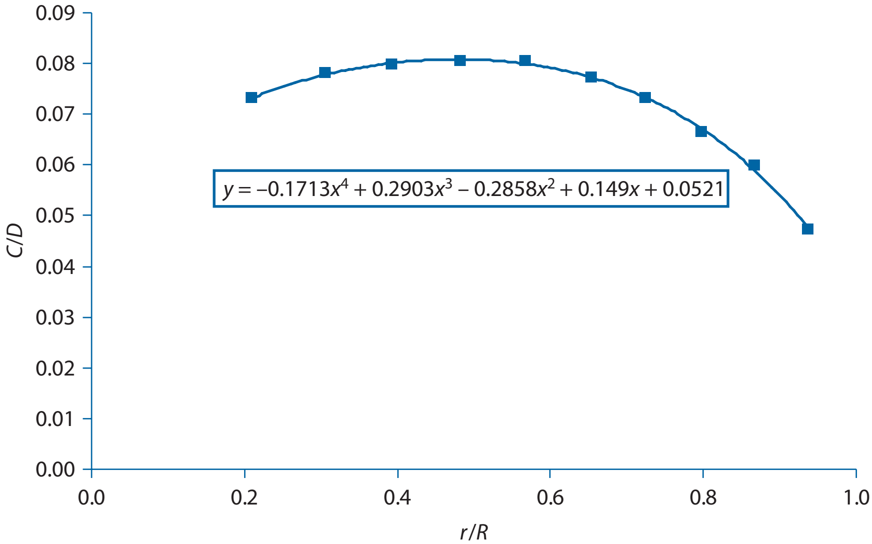

Fig. 3.10 Chord–diameter ratio for Cessna 172R propeller.

coefficients will vary directly with the solidity. Therefore, a propeller chart for the Cessna 172R propeller can be constructed by simply multiplying C_T and C_P by the ratio of the solidities.

Before going further with the analysis, the procedure for measuring the geometry of the Cessna 172R propeller will be described. Making sure that

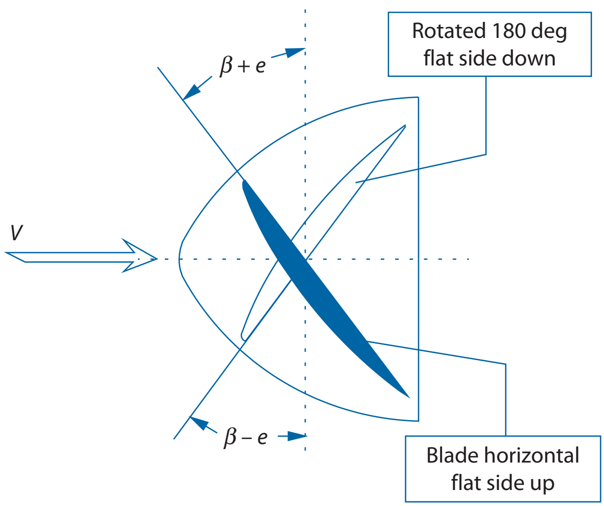

Fig. 3.11 Measuring propeller pitch and chord.

the mag switch is off, the propeller is rotated to a horizontal position as shown in Fig. 3.11. On one side, where the surface is flat, strips of masking tape are placed at a number of radial locations from the hub to the tip. Each strip runs from the leading edge to the trailing of the blade and is placed normal to the radius. After marking stations along the blade in this manner, the chord is then measured at each strip. Next an ordinary protractor with a bubble level is placed along each strip, and the angle between the vertical and the face of the blade surface is measured. If the airplane were sitting exactly level on the ground, the measured angle would equal the pitch angle of the blade at that particular radius. However, the propeller axis is not necessarily parallel to the ground and may be pitched nose-up or nose-down at an angle, *e*. Therefore, after completing the measurements with the propeller on one side, the propeller is rotated 180° and the angular measurements repeated at the same stations on the other side. The two angles for the blade surface with the vertical are then averaged so that the angle of the propeller axis with the ground cancels out.

The C_T and C_P curves for the Cessna 172R propeller shown in Fig. 3.12 were obtained by modifying the corresponding curves for propeller No. 4 in Figs 3.6 and 3.7 as just described. In addition, the thrust coefficients are reduced by 8 percent in agreement with Fig. 3.6. This reduction is called "thrust deduction" and results from the fact that the drag of a fuselage is increased when a propeller is placed in front of it thus decreasing the effective propeller thrust.

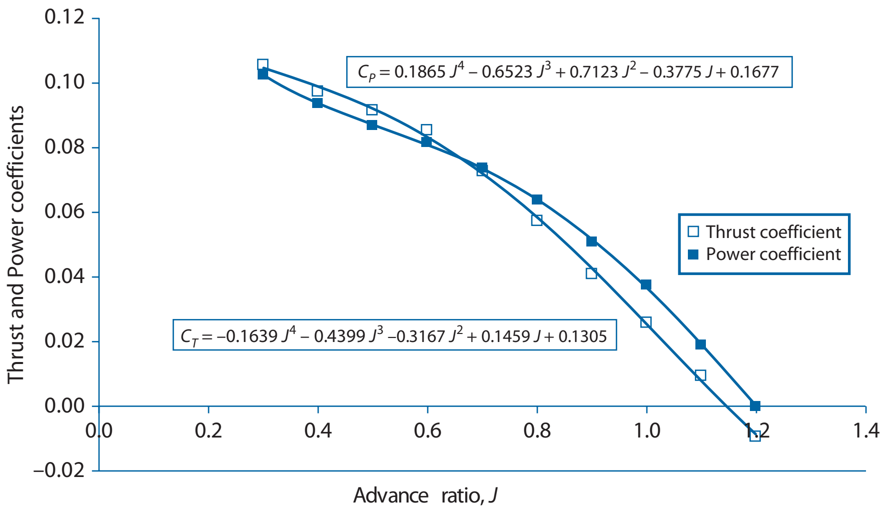

Fig. 3.12 Predicted Cessna thrust and power coefficients including thrust deduction.

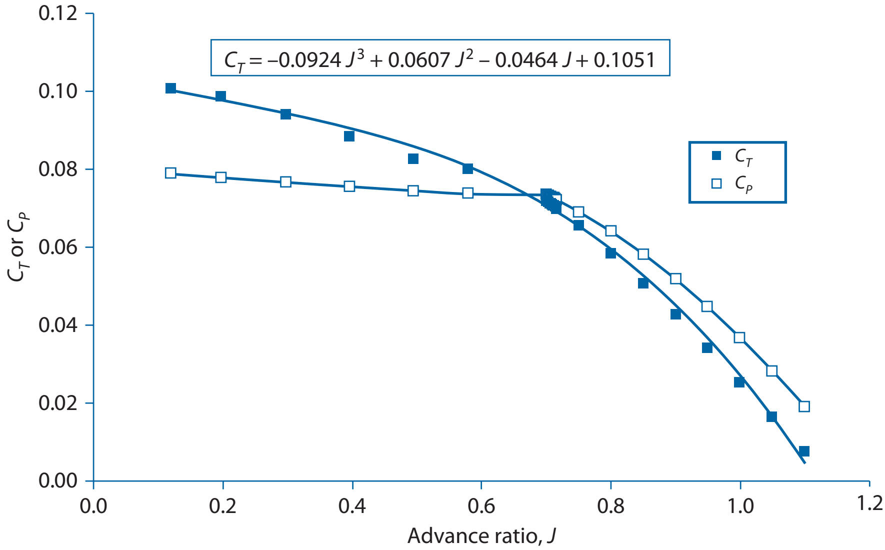

Fig. 3.13 Synthesized thrust and power coefficients for the Cessna 172R propeller.

During the ground roll, the advance ratio drops below the values reported in NACA Technical Report 421. It was therefore necessary to "work the takeoff backwards" to deduce, for a reasonable value of the parameter, *f*, what the propeller performance should be for low advance ratios. These synthesized values for C_T and C_P are presented in Fig. 3.13. Admittedly, this procedure is not very satisfying for the experimenter and can be avoided if one is able to obtain a complete propeller chart. However, the values shown in Fig. 3.12 are adequate for predicting performance in forward flight to be described in the next chapter.

3.6 Drag

The estimation of the drag of an airplane composed of many parts is as much of an art as it is a science. The drag of an isolated part can be done with precision, but when you put several parts together the interference effects of one part on another make the total drag estimate somewhat elusive. If you already know the total drag of a configuration and you wish to add something to it, the incremental drag can be determined fairly accurately. Even here you have to be careful. Hoerner [3] has become somewhat of a bible for drag estimation. (Unfortunately the book is somewhat difficult to obtain because it was self-published.) Rather than attempt a *drag breakdown* where the contributions to the drag of each component (such as wing, tails, fuselage, wheels, antennae, etc.) are calculated and then summed, it is suggested that you use Table 3.2 (taken from [1], reprinted with permission of John Wiley & Sons Inc.), which presents the total drag coefficient of various types of airplanes based on the wetted area of the airplane. The wetted area refers to the total skin surface area exposed to the air (i.e., both sides of the wing, both sides of the tails and the fuselage). In Table 3.2, the coefficient C_f is simply a drag coefficient based on the wetted area of the airplane and, because of that, it is called the *skin friction coefficient*. Another term relating to drag that is used often is the *equivalent parasite* or *flat plate area*, f, which is simply the product of the drag coefficient and the area on which it is based. The advantage of using f is that the f values for several components can be added but the drag coefficients cannot, because the coefficients are based on different areas. Also, after a

Table 3.2 Typical Overall Skin Friction Coefficients for Airplanes Built from 1940 to 1976 (from (1), reprinted with permission from John Wiley & Sons Inc.)

C_f	Airplane designation	Description
0.0100	Cessna 150	Single prop, high wing, fixed gear
0.0095	PA-28	Single prop, low wing, fixed gear
0.0070	B-17	Four props, World War II bomber
0.0067	PA-28R	Single prop, low wing, retractable gear
0.0066	C-47	Twin props, low wing, retractable gear
0.0060	P-40	Single prop, World War II fighter
0.0060	F-4C	Jet fighter, engines internal
0.0059	B-29	Four props, World War II bomber
0.0054	P-38	Twin props, twin-tail booms, World War II fighter
0.0050	Cessna 310	Twin props, low wing, retractable gear
0.0049	Beech V35	Single prop, low wing, retractable gear
0.0046	C-46	Twin props, low wing, retractable gear
0.0046	C-54	Four props, low wing, retractable gear
0.0042	Learjet 25	Twin jets, pod-mounted on fuselage, tip tanks
0.0044	CV 880	Four jets, pod-mounted under wing
0.0041	NT-33A	Training version of P-80 (see below)
0.0038	P-51F	Single prop, World War II fighter
0.0038	C-5A	Four jets, pod-mounted under wing, jumbo jet
0.0037	Jetstar	Four jets, pod-mounted on fuselage
0.0036	747	Four jets, pod-mounted under wing, jumbo jet
0.0033	P-80	Jet fighter, engines internal, tip tanks, low wing
0.0032	F-104	Jet fighter, engines internal, midwing
0.0031	A-7A	Jet fighter, engines internal, high wing

while, one begins to develop a feeling for what the flat plate area should be for a given shape and size. Remember that

$$D = qf = qSC_D$$

or

$$f = SC_D \tag{3.10}$$

If the airplane under consideration has retractable gear, then an increment in the drag obtained from Table 3.2 must be added for takeoff. In this case, drag coefficients for the struts, wheels, and fairing must be estimated based on their respective areas by using data found in [1]. Typically, for a small, single-engine airplane the added flat plate area due to the landing gear is of the order of 2 or 3 ft^2. So far we have only considered the parasite drag (i.e., the drag of a component not attributable to lift). The wing not only has a parasite drag but suffers a drag penalty because it is producing lift. This will be discussed in more detail in Chapter 4. For the present, let us simply assume the fact that the induced drag coefficient due to lift is given by

$$C_{D_i} = \frac{C_L^2}{\pi A e} \tag{3.11}$$

where A is the wing's aspect ratio, the ratio of span to an average chord, and e is an empirical correction known as Oswald's efficiency factor. The factor e is approximately equal to 0.6 for low wings and midwings and 0.8 for high wings. Close to the ground, in what is called "in ground effect," the induced drag is reduced significantly. For a typical small airplane, Eq. (3.11) is reduced by approximately 50 percent.

3.7 Numerical Calculation of Speed and Distance During Ground Roll

The foregoing information will now be used in the calculation of ground roll distance. Measured results from the ground roll experiment described in Appendix B where the rpm, airspeed, and distance were obtained as a function of time will be compared with predictions. Table 3.3 presents some of the information needed to make the predictions. To illustrate this approach, make use of Fig. 3.13, which presents C_T and C_P as a function of J for the Cessna 172R, as discussed earlier. Note that Fig. 3.13 is not endorsed by the airplane or propeller manufacturers.

Let us take a small time increment of 1 s and an initial velocity of zero. At this point, J is zero so that, approximately, $C_T = 0.105$ and $C_P = 0.081$.

Table 3.3 Cessna 172R Geometry and Parameters

Parameter	Value
Wing planform area, S (through fuselage)	174 ft^2
Horizontal tail planform area, S_H	32.4 ft^2
Vertical tail planform area, S_V	11.3 ft^2
Weight (quoted for ground roll distance)	2450 lb
Wing span, b	36.08 ft
Wing aspect ratio	7.48
Propeller diameter	6.25 ft
Propeller solidity	0.0725
Fuel Capacity, useable	53 gal
Weight-empty	1639 lb

Using a density, ρ, of 0.0023 corresponding to 1200-ft altitude at University Park, Pennsylvania, and a of 6.25 – ft diam, the horsepower absorbed by the propeller and its thrust can be written as

$$hp = \frac{1}{550}\rho n^3 D^5 C_P = \frac{(0.0023)(6.25)^5(0.081)}{550}\left[\frac{N}{60}\right]^3 = 14.96\left[\frac{N}{1000}\right]^3 \quad (3.12)$$

$$T = \rho n^2 D^4 C_T = (0.0023)(6.25)^4(0.105)\left[\frac{N}{60}\right]^2 = 102.4\left[\frac{N}{1000}\right]^2 \quad (3.13)$$

Now let us guess at an rpm, N, of 2000. Equation (3.12) gives a horsepower of 119.7, but from Fig. 3.2 a value of 135 hp is obtained. So let us take another guess of 2100 rpm. This results in 138.5 hp for the propeller and 143 hp for the engine. These two powers are fairly close. An extrapolation of these numbers produces an rpm of 2130. Thus, by a tortuous hand method, we have iterated on the rpm to find, for this airspeed and altitude, that an rpm of approximately 2130 predicts that the propeller will absorb the WOT power of the engine. Using this value for N, the thrust is obtained from Eq. (3.13) as 465 lb.

As V equals zero, there is no drag or lift and we calculate the initial acceleration from

$$T - \mu W = \frac{W}{g}a = 465 - (0.02)(2350) = \frac{418}{32.2}a$$

or

$$a = 12.98 \text{ fps/s}$$

If we assume that, approximately, this acceleration is constant for 1 s, then, starting from rest, the velocity at the end of 1 s will equal 12.98 fps or 7.7 kt. The average speed during this time will be the average of the initial speed and the final speed, or 6.49 fps. Thus the distance traveled during this first second is predicted to equal 6.49 ft.

Having the predicted speed and distance after 1 s, we now increment the time for another second and repeat the above iteration for rpm and thrust. By hand, this iterative procedure is extremely laborious and begs for a computer application. This can be accomplished by reducing Fig. 3.2 to a system of equations for the WOT power as a function of rpm and altitude. Equations are also formulated for C_T and C_P from Fig. 3.12. In so doing, note that the power decreases linearly with altitude at a constant rpm. Thus, using a spreadsheet program such as Excel, we can obtain equations for the rate of power decrease with altitude as a function of rpm. Similarly, equations can be obtained for the WOT power at SSL as a function of rpm and equations for C_T and C_P as a function of J. A computer program has been written to perform the iterations just described. The simplest way to determine the operating rpm is to guess initially at a low rpm and then

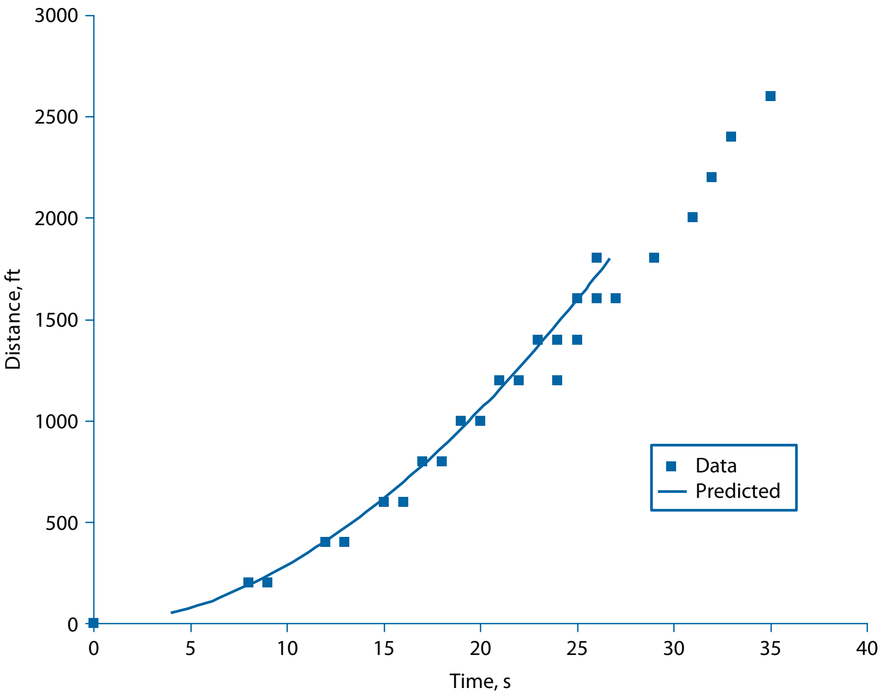

Fig. 3.14 Comparison of ground roll distance with theory: Cessna 172R, 2450-lb, 1200-ft altitude.

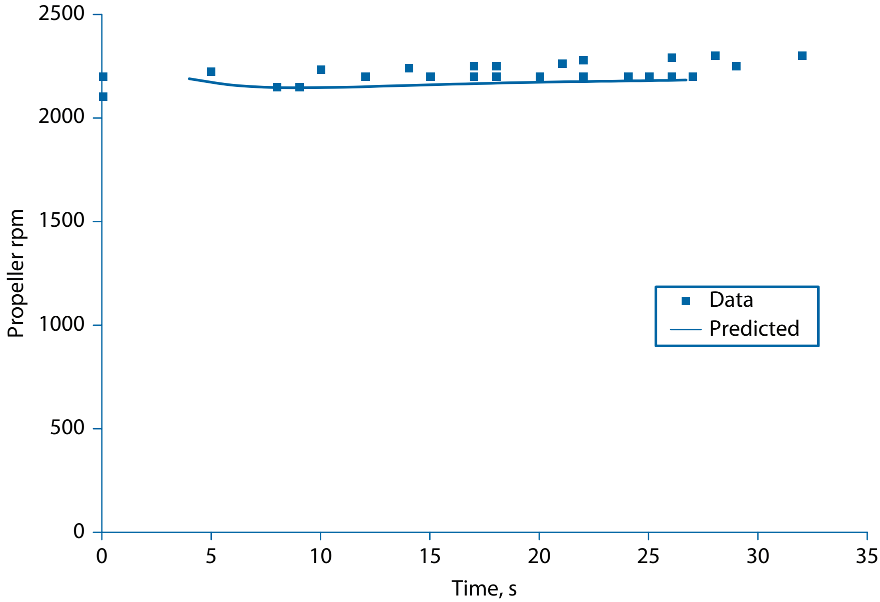

Fig. 3.15 Comparison of measured rpm with theory: Cessna 172R ground roll, 2450-lb, 1200-ft altitude.

continually increase rpm by a very small amount until the propeller power just exceeds the engine power. Figure 3.14 presents some results for ground roll distance from this program as well as data taken by groups of students at Pennsylvania State. Comparisons between predictions and measurements for the velocity and rpm during the ground roll of a Cessna 172R are presented in Figs 3.15 and 3.16. The predictions agree fairly well with the student data, but there is a lot of scatter in the velocity measurements.

3.8 Airborne Distance

Refer to FAR Part 23 paragraphs 23.51 – 23.61 for details on the regulations regarding the takeoff. Generally, takeoff distances are specified over a 50-ft obstacle height with a liftoff speed that is 30 percent above the single engine stalling speed. For small airplanes weighing less than 6000 lb, the regulations simply state that the takeoff shall not require any extraordinary piloting skill. This text will not specify any specific technique for determining airborne distances. Generally, a recording theodolite or possibly a recording GPS is required. The airborne distance depends in large part on pilot technique. The pilot may hold the airplane on the ground until it is well above the speed needed for liftoff. Then the pilot

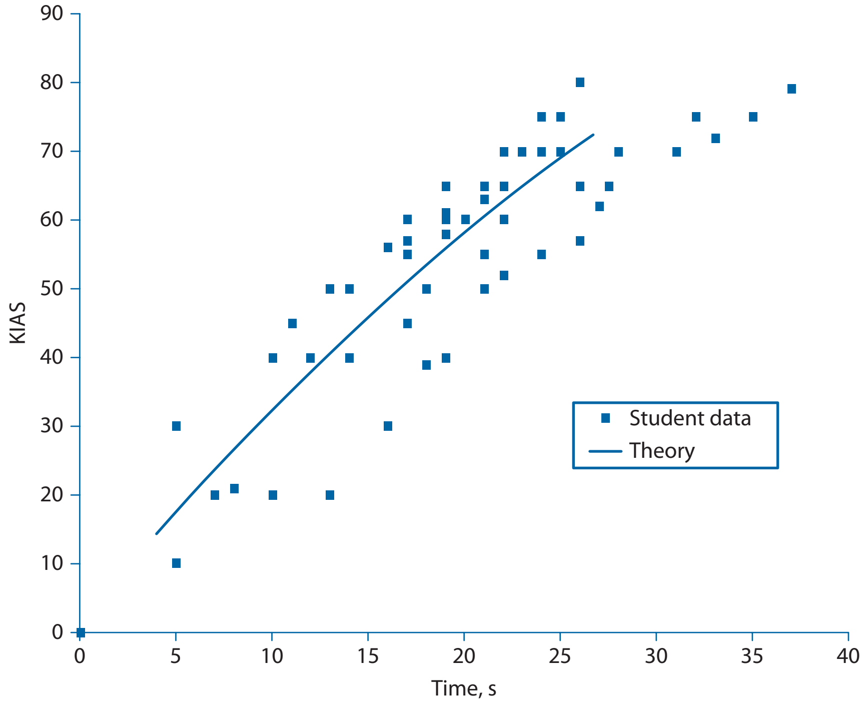

Fig. 3.16 Comparison of ground roll speed with theory: Cessna 172R, 2450-lb, 1200-ft altitude.

can haul back on the wheel hard, pulling a high g and climbing rapidly. Or, the pilot can ease back on the wheel and climb out gradually as the airplane accelerates above the stalling speed. In either case, the total distance from the start of the roll to a point over a 50-ft obstacle will not be significantly different.

Earlier, it was shown that the work done by the propeller on an airplane produces an equal change in its kinetic energy, $mV^2/2$. If the airplane is gaining altitude during the flight, then its potential energy (PE), represented as Wh in Eq. (3.14), is also increasing. In such a case the earlier result can be extended to show that the work done equals the change in the total energy, where total energy is kinetic energy (KE) plus PE. Thus it follows that

$$TV - DV = \frac{\mathrm{d}(Wh)}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{W}{2g}V^2\right) \tag{3.14}$$

In words, TV is the power available and DV is the power required to maintain level flight. Thus the left-hand side of the equation is the power available in excess of that required to maintain level flight. The right-hand side of the equation says that this excess power can be used to increase the PE of the airplane ($W \times \mathrm{R/C}$) or increase the KE ($1/2\, mV^2$). As an example, consider the results of the ground roll calculations and, although we do not know the stalling speed at this point, suppose we lift off at 70 kt and hold the speed constant with WOT power. At 70 kt, the predicted rpm is 2500 with a J of 0.46. This gives a C_T of 0.062 and a corresponding thrust of

$$T = \rho n^2 D^4 C_T = (0.0023)\left(\frac{2500}{60}\right)^2 (6.17)^4 (0.062)$$

$$= 359\ \mathrm{lb}$$

The drag must now consider a change in the induced drag since we are off the ground. The dynamic pressure, q, for 70 kt at 1200 ft is 16.09 psf, and the C_L for the weight of 2350 lb and a wing planform area of 170 ft^2 will be $W/q/S$, or 0.859. Thus the drag, for climb-out, will be

$$D = qf + qSC_{D_i} = qf + qS\frac{C_L^2}{\pi Ae} = (16.09)(5.9) + (16.09)(170)\frac{0.859^2}{\pi(7.2)(0.6)}$$

$$= 243.6\ \mathrm{lb}$$

Thus, from Eq. (3.14) for a climb at constant speed, the R/C, $\mathrm{d}h/\mathrm{d}t$, will equal

$$\mathrm{R/C} = \frac{(T - D)}{W} V = \frac{(359 - 243.6)}{2350}(70)(1.69) = 5.81\ \mathrm{fps} = 349\ \mathrm{fpm}$$

The final value for the R/C is given in feet per minute (fpm) because this is the traditional measure used for R/C. At this R/C, it will take 8.6 s to reach 50 ft. During that time the airplane, at 70 kt, will travel another 1018 ft. Thus the total takeoff distance for the Cessna 172R over a 50-ft obstacle would be 1018 + 1700 (from Figs 3.14 and 3.16), or 2718 ft.

3.9 Approximate Treatment of Propeller Using Momentum Theory

One can also use the momentum theory of propellers to obtain an estimate of the propeller thrust given the shaft power. According to momentum principles (McCormick [1]), a propeller that is in a free stream with a velocity of V will, on the average, induce an added velocity, w, through the propeller that can be obtained from

$$T = \rho A(V + w)2w \tag{3.15}$$

The power, P, required by the propeller will be the product of the thrust, T, and the velocity through the propeller. The symbol A represents the disk area of the propeller, $\pi D^2/4$.

$$P = T(V + w) \tag{3.16}$$

This can be divided into the useful power, TV, and the induced power, Tw.

Equation (3.6) represents an ideal power to produce the thrust and does not include the profile power to overcome the drag of the blades as they rotate. If σ denotes the ratio of the blade area to the disk area (πR^2), known as the solidity, the profile power can be written in coefficient form as

$$C_{PP} = \frac{\sigma \pi^4 \overline{C_d}}{32} \tag{3.17}$$

where $\overline{C_d}$ is an "average" drag coefficient for the blade section. A typical value for $\overline{C_d}$ of 0.01 is recommended. Finally, the power required by a propeller operating at a velocity of V, an rps of n, and an atmospheric density of ρ with a diameter of D and a solidity of σ will have its thrust and power related approximately by

$$P = TV + 1.15Tw + \frac{\sigma \pi^4 (0.01)}{32} \rho n^3 D^5 \tag{3.18}$$

Note that the induced power has been increased, based on experience, by 15 percent to allow for the ideal assumptions in the momentum theory. The induced velocity, w, in the equation is obtained from Eq. (3.15).

Equation (3.8) can be written in coefficient form as

$$C_p = C_T J + 1.15 C_T \frac{w}{nD} + \frac{0.01 \sigma \pi^4}{32} \tag{3.19}$$

3.10 Approximate Calculation

We start with a very simple approximation. The distance and time to attain a takeoff velocity of V_{to} will be determined by assuming a constant average acceleration between the time at rest and the time to reach a velocity of V_{to}. The forces producing the acceleration are generally a function of q. Therefore, the forces will be calculated for a q that is the average between the initial value of zero and $\rho V_{to}^2/2$. This means that the forces will be calculated at $V_{to}/\sqrt{2}$.

As an example, let us predict the ground roll distance for the Cessna 172R to reach an airspeed of 50 kt with no headwind at a pressure altitude of 1200 ft with an OAT of 60°F. The density for this nonstandard atmosphere is found to equal 0.00227 slugs/ft^2. Since 1 kt equals 1.69 fps, the dynamic pressure, q, for 50 kt and this density is

$$q = \frac{\rho V^2}{2} = \frac{(0.00227)(84.5)^2}{2} = 8.104 \text{ psf}$$

The average q and V are thus

$$\overline{q} = 4.05$$
$$\overline{V} = 59.75 \text{ fps}$$

The lift coefficient for cruise for the Cessna 172R is approximately 0.4. At the average q, this gives a lift of qSC_L or 281.9 lb. From Table 3.2, a C_f of 0.010 is selected and from Fig. 3.2 and Table 3.2, the wetted area for the Cessna 172R is estimated to equal 600 ft^2. These numbers give a total flat plate area for the Cessna 172R of 6.0 ft^2. The parasite drag at the average q is therefore qf or 24.3 lb, and the induced drag, using Eq. (3.11), equals 6.0 lb. This will be reduced to 3.0 for ground effect. From the engine chart, the WOT power at a standard altitude of 1200 ft and 2200 rpm is read as 145 hp. This is corrected for the nonstandard atmosphere by multiplying by the square root of the ratio of the standard absolute temperature to the absolute OAT. This correction results in a 1 percent reduction in power, or a corrected power of 143.6 hp.

Using the foregoing average V and rpm results in a J of 0.261 which, from Fig. 3.13, gives $C_P = 0.0765$ and $C_T = 0.096$. The average acceleration can now be determined from

$$T - D_{\text{par}} - D_i - \mu(W - L) = \frac{W}{g} a$$

or

$$\begin{aligned} a &= \frac{32.2}{2450}[447 - 24.3 - 3.0 - 0.02(2450 - 282)] \\ &= 5.86 - 0.449 - 0.039 - 0.568 \\ &= 4.80 \text{ fps/s} \end{aligned} \tag{3.20}$$

Let us go back and reconsider Newton's second law of motion. For those not familiar with calculus, bear with me—the final result makes a lot of sense. We can write

$$F = ma = m\frac{\mathrm{d}v}{\mathrm{d}t} = m\frac{\mathrm{d}v}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} = mV\frac{\mathrm{d}v}{\mathrm{d}s} = \frac{\mathrm{d}(\frac{1}{2}mV^2)}{\mathrm{d}s}$$

or

$$\int F \mathrm{d}s = \int \mathrm{d}(KE) \tag{3.21}$$

But the first integral is equal to the work done by the force in moving a distance of s while the last integral is equal to the change in KE, $mV^2/2$. Thus the work done on the system equals the change in the KE of the system. In this case the work done is the accelerating force, ma, multiplied by the ground roll distance, s.

$$m\,as = \frac{m\rho V^2}{2} \quad \text{or} \quad s = \frac{\rho V^2}{2a}$$

Thus, with the approximations utilized in calculating the thrust and an average q, the ground roll distance needed by the 172R to attain an airspeed of 50 kt is predicted to be $s = 84.5^2/2.0/4.80$, or 744 ft. This prediction compares fairly well with the experimental results conducted by my students at the University Park airport as shown in Figs 3.14 and 3.16.

The foregoing was presented to show that an estimate of ground roll distance can be made without any specific knowledge of the geometry of the propeller except for its diameter. However, some idea of the power being delivered to the propeller is required. A more accurate approach is to use the numerical procedure presented earlier in this chapter. If a propeller chart is available, iterate on the rpm at a known V until the power obtained from C_P matches the WOT power from the engine chart. The thrust is then obtained from C_T at the J for which the power is matched, and then the acceleration can be calculated at that V. It is assumed that this acceleration is constant over a very small time increment. The velocity can then be calculated at the end of the time increment and the process repeated. In carrying out this process from the beginning of the ground roll where V equals zero up to some desired V, we are performing what is referred to as a numerical integration. Similarly, having V as a function of time, we can numerically integrate V to obtain the ground roll distance as a function of time.

3.11 Summary

The ground roll performance can be predicted fairly accurately if the engine power is known. Propeller charts can be used based on NACA data to obtain propeller thrust and power. In the lack of such charts the propeller thrust can be predicted using momentum theory and knowing the operating state of the propeller, its diameter, and the power available. Corrections for measured data to a standard weight and altitude can be made using the methods presented in this chapter. The POH appears to be conservative with regard to ground roll distance.

Problems

3.1 Estimate the total takeoff distance for the Cessna 172R over a 50-ft obstacle in Denver, Colorado, where the standard altitude is 5300 ft.

3.2 Formulate a computer program and validate Figs 3.14, 3.15, and 3.16.

3.3 An airplane has a 5.5-ft diam propeller and is powered by a 300-bhp engine. What is the maximum static thrust that the propeller can produce at SSL conditions?

3.4 Here is an exercise in numerically integrating equations of motion. The drag coefficient of a smooth golf ball is 0.4. When dimpled, C_D reduces to 0.17. A golfer drives a ball and it leaves the ground at an initial angle of 30 deg above the level and at some initial velocity. Under these conditions, the golfer can drive a dimpled ball 200 yd. How much less would the golfer drive it if the ball were smooth? Use SSL conditions. A golf ball has a diameter of 1.68 in. and weighs 1.62 oz.

3.5 Given: Cessna 172R propeller

hp	5000 ft
IAS	60 kt
rpm	2400

Find: The amount of thrust and power required by the propeller.

3.6 Show that the work done on a system equals the change in the total energy of the system.

Chapter 4 Power-Required and Trim

- Determine Power Required
- Determine Relationship Between Wing Lift and Weight Distribution

4.1 Useful Power

The useful power—the power required to maintain trimmed, level flight for an airplane—is referred to as "power-required." In Chapter 3 this was stated to be equal to the product of the thrust and the velocity, TV. In trim the thrust is equal to the drag, and so this becomes DV. To better understand why this is a power, consider the work done in moving the drag force a distance of s at a constant speed. The work done is Ds and the time to do this work is s/V. But power is the rate of doing work, or in this case, the work divided by the time to do the work. Hence it follows that the power, P equals Ds divided by s/V, resulting in $P = DV$.

The drag of an airplane is the sum of the parasite drag and the induced drag:

$$D = q(f + SC_{D_i}) \tag{4.1}$$

But

$$C_{D_i} = \frac{C_L^2}{\pi A e}, \qquad C_L = \frac{W}{qS}$$

where $q = \frac{1}{2}\rho V^2$.

With a little algebraic reduction, the power, DV, becomes

$$P = \frac{1}{2}\rho f V^3 + \frac{2(W/b)^2}{\pi \rho e}\frac{1}{V} \tag{4.2}$$

Remember, this is the power that must be delivered *by* the propeller *not to* the propeller. Divided by 550, this becomes thrust horsepower (thp). As expected, the first term is called the *parasite power* and the second term the *induced power*.

The calculated power breakdown for the Cessna 172R is presented in Fig. 4.1. The parasite power is seen to increase, in accordance with Eq. (4.2), as the cube of the airspeed. The induced power varies inversely with the airspeed. These powers add to produce a minimum total power at a TAS of around 60 kt.

The power-required can be obtained from a flight test fairly easily if a good propeller curve giving C_T as a function of advance ratio is known. Simply fly at a constant airspeed, read the rpm, calculate J, determine C_T, and then calculate T knowing the air density, ρ. Of course we can calculate P if we know the flat plate area, f, and Oswald's efficiency factor, e. The determination of these two parameters should be the objective of the flight testing.

To do this in an efficient and accurate fashion, some manipulation of Eq. (4.1) is performed. Consider the fact that C_D is a function only of C_L (neglecting Mach number and Reynolds number effects). Thus, if the power is found at some density and weight, we know that the total C_D will be the same at SSL at a standard weight, W_S, if we hold C_L constant. Thus,

$$C_L = \frac{W}{\frac{1}{2}\rho V^2} = \frac{W_S}{\frac{1}{2}\rho_0 V_{\mathrm{ew}}^2}$$

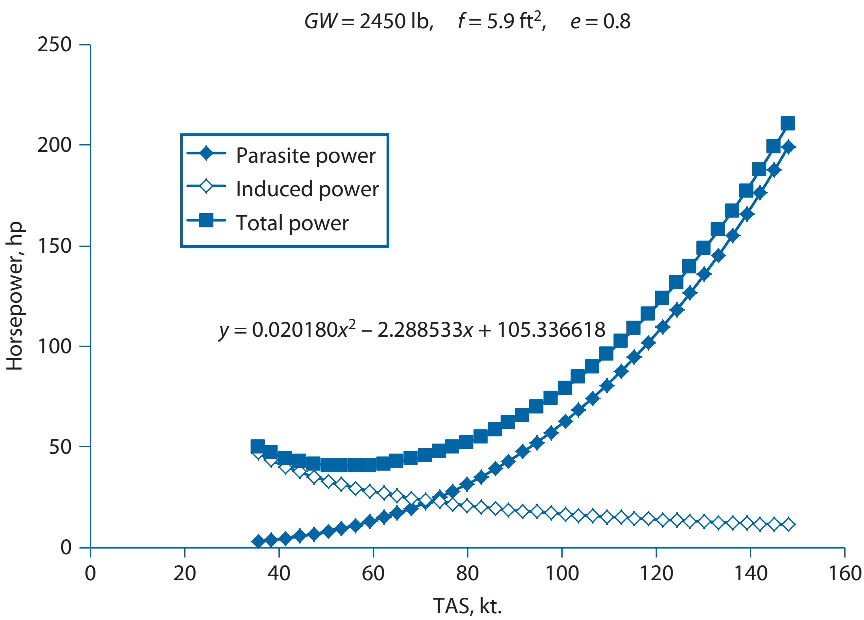

Fig. 4.1 Calculated horsepower for Cessna 172R at standard sea level.

or the equivalent velocity, V_{ew}, at which the power data are plotted at SSL for the standard weight, is

$$V_{ew} = V\sqrt{\sigma}\sqrt{\frac{W_S}{W}} \tag{4.3}$$

Of course the weight, density, and velocity are now different, and so the power must be corrected using Eq. (4.1), remembering that f, S, and C_{D_i} remain constant.

$$\frac{P_{ew}}{P} = \frac{\rho_0 V_{ew}^3 (f + SC_{D_i})}{\rho V^3 (f + SC_{D_i})} = \sqrt{\sigma}\left(\frac{W_S}{W}\right)^{3/2} \tag{4.4}$$

In summary, a power point taken at some density altitude and some gross weight is converted to an equivalent power from Eq. (4.4) and plotted against the equivalent airspeed given by Eq. (4.3). The resulting curve will be the required power at SSL for the standard gross weight.

Like all data, there will be some scatter in the power-required data. However, the error in evaluating these data can be minimized by considering Eq. (4.2) in terms of equivalent speed, power, and standard weight at SSL. This can be written as

$$P_{ew} = \frac{1}{2}\rho f V_{ew}^3 + \frac{2(W_S/b)^2}{\pi\rho_0 e}\frac{1}{V_{ew}}$$

where P_{ew} is the power required at SSL in ft · lb/s for the standard weight.

If this equation is multiplied through by V_{ew}, it becomes a linear equation for $P_{ew}V_{ew}$ in terms of $(V_{ew})^4$.

$$P_{ew}V_{ew} = \frac{\rho_0 f}{2} V_{ew}^4 + \frac{2(W_S/b)^2}{\pi\rho_0}\frac{1}{e} \tag{4.5}$$

The slope of the line $P_{ew}V_{ew}$ plotted against $(V_{ew})^4$ will equal $2\rho_0(V_{ew})^3 f$, and the value of $P_{ew}V_{ew}$ at V_{ew} equal to zero will result in a value of e.

$$(P_{ew}V_{ew})_0 = \frac{2(W_S/b)^2}{\pi\rho_0}\frac{1}{e} \tag{4.6}$$

Using a value for e of 0.8 and a value for f of 6.0 ft^2 for the Cessna 172R, Eq. (4.6) is graphed in Fig. 4.2.

The slope of this graph is 0.0070 and the intercept is 15.43×10^5. Inserting these values into Eqs. (4.5) and (4.6) for the slope and intercept will, of course, return the flat plate area of 6.0 and Oswald's efficiency factor of 0.8.

Typical data, taken by students at Pennsylvania State, are also presented in Fig. 4.3 for the Cessna 172R. Although there is a lot of scatter in the data, many of the data points are in line with the calculated line. Possibly this agreement can be improved in the future with better atmospheric test conditions, with thrust and power curves for the propeller and engine, and with students who are willing to interpolate between tick marks on a instrument scale. Despite the fact that most of the data points lie below the line on Fig. 4.2, we will continue to use the values for the Cessna 172R of

$$f = 6.0 \ \text{ft}^2$$
$$e = 0.8$$

Having decided on values for f and e, let us examine the behavior of the drag and power-required curves and what can be derived from them. The drag curve will be rewritten in the form

$$D = \frac{1}{2}\rho V^2 f + \frac{2(W/b)^2}{\pi\rho e}\frac{1}{V^2} \tag{4.7}$$

The reader proficient in calculus can differentiate Eq. (4.7) with respect to V, equate the result to zero, and find the V for minimum drag. If this

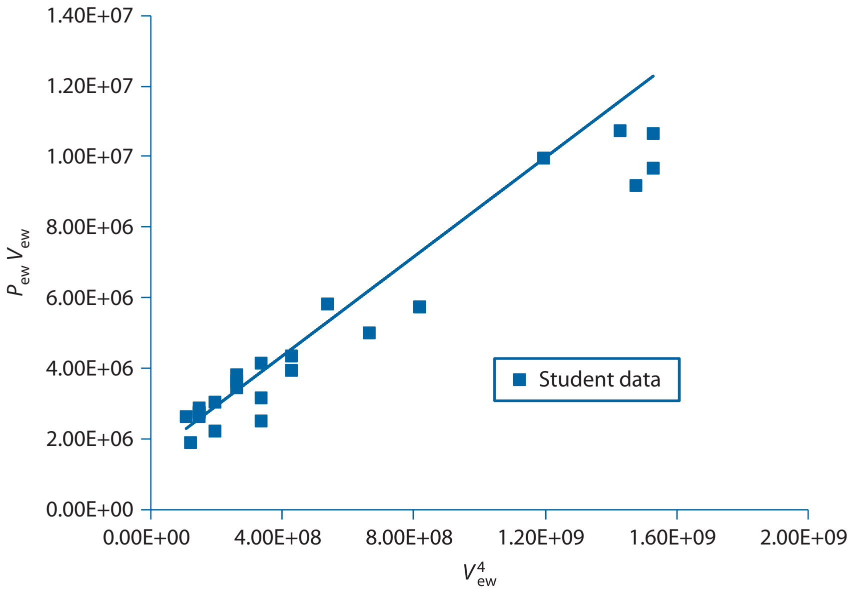

Fig. 4.2 Comparison of experimental and calculated linear equivalent power variation with equivalent airspeed for Cessna 172R.

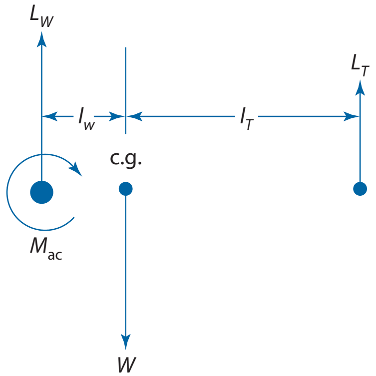

Fig. 4.3 Principal forces on a trimmed airplane.

velocity is substituted back into Eq. (4.7) the minimum drag is found to be

$$D_{\min} = 2\left(\frac{W}{b}\right)\sqrt{\frac{f}{\pi e}} \tag{4.8}$$

Seeing this the first time is surprising because Eq. (4.8) states that the minimum drag of an airplane is the same for all altitudes. It is also fairly easy to show that, at the velocity for minimum drag, the induced drag is equal to twice the parasite drag.

Now let us take at look at the power. In a similar manner, if we differentiate Eq. (4.2) with respect to V and set the result to zero, the velocity for minimum power is found.

$$V\sqrt{\sigma} = \left[\frac{4(W/b)^2}{3\pi\rho_0^2 ef}\right]^{1/4} \tag{4.9}$$

Unlike the minimum drag, the minimum power is found to increase as the altitude increases. Substituting Eq. (4.9) into Eq. (4.7) and multiplying by V to give the thrust power,

$$P_{\min}\sqrt{\sigma} = 2\left(\frac{4}{3\pi}\right)^{3/4}\frac{f^{1/4}(W/b)^{3/2}}{\rho_0^{1/2}e^{3/2}} \tag{4.10}$$

4.2 Trimmed Lift Curve Slope

As one decreases the airspeed of an airplane, its angle of attack and lift coefficient must increase in order to balance the weight of the airplane. However, the wing lift is not necessarily equal to the weight because the horizontal tail produces a vertical aerodynamic force to trim the moments on the airplane. This force may be a lift or a download depending on the c.g. position and aerodynamic pitching moment. Consider Fig. 4.3.

The principal forces on a trimmed airplane, as shown in Fig. 4.3, are the wing lift (L_W), weight (W), tail lift (L_T), and the wing's aerodynamic moment (M_{ac}). There are other forces that should be considered in a more complete analysis, but these contributions from the fuselage and propeller are generally small and will be neglected here.

Summing forces and moments (about the tail) result in

$$L_W + L_T = W \tag{4.11}$$

$$L_W(l_W + l_T) + M_{ac} = Wl_T \tag{4.12}$$

Solving the above equations for the wing lift gives a relationship between the wing lift and the weight for a trimmed airplane.

4.3 Summary

An expression for the power required by an airplane in level flight has been derived in terms of a flat plate area and Oswald's efficiency factor. It is shown that a power data point taken at an arbitrary altitude, airspeed, and gross weight can be transformed into an equivalent power at an equivalent airspeed at standard sea level and standard gross weight. This equivalent power can then be graphed in such a way as to obtain experimentally values for f and e.

Problems

4.1 An airplane is flying at a pressure altitude of 6000 ft, where the OAT is 10°F. It has an operating weight of 5350 lb, but its standard weight is 6000 lb. The airplane is traveling at an airspeed of 180 KCAS, and the thrust horsepower required at these conditions is 450 hp. At what airspeed and thrust horsepower should the data be plotted at SSL conditions at the standard gross weight?

4.2 What is the minimum thrust horsepower required for the airplane in Problem 4.1 at the given altitude and OAT? At what airspeed will this occur? To do this problem, use the geometry of the Cessna 208B Caravan. You can find the information in *Jane's All the World's Aircraft, 1991–1992.*

Chapter 5 Rate-of-Climb, Time-to-Climb, and Ceilings

- Find Rate-of-Climb
- Predict Rate-of-Climb
- Determine Time-to-Climb
- Determine Ceiling

5.1 Rate-of-Climb Derived from Static Equilibrium

The rate-of-climb (R/C) is an important parameter of aircraft performance and is, of course, the rate at which an airplane can gain altitude. Knowing the R/C enables one to determine the time-to-climb and the maximum altitude, or ceiling, at which one can fly. We have already discussed in Chapter 3 (Eq. [3.21]) the fact that excess power determines the R/C. Let us now derive the same equation from the standpoint of static equilibrium.

Consider Fig. 5.1. An airplane is climbing at an angle of ϕ at a velocity of V. It has a thrust of T, a drag of D, and a weight of W. If the forces are in equilibrium, the sum of the force in the direction of travel will equal zero,

$$T - D - W \sin \phi = 0$$

and the sum of the forces normal to the flight direction will vanish.

$$L = W \cos \phi$$

Since R/C$=V \sin \phi$, it follows that

$$R/C = \frac{T - D}{W} V \tag{5.1}$$

This is identical to Eq. (3.14) for a climb at a constant speed. Now, if Eq. (5.1) is expanded, we find as before that the rate of increase in potential

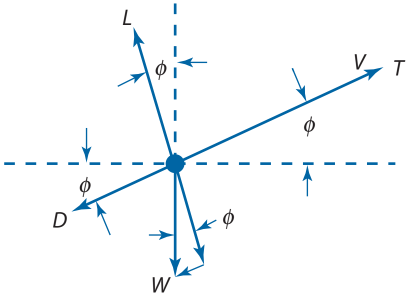

Fig. 5.1 Forces on an airplane during climb.

energy, W(R/C), is equal to the excess power, $TV - DV$, confirming the reasoning presented in Chapter 3 regarding the airborne distance during takeoff.

In Appendix B as part of the first experiment, a procedure is described for measuring the R/C at a given altitude. That procedure uses what are known as "saw-tooth" climbs, because the airplane's path during the testing looks like the teeth of a giant saw. As shown in Fig. 5.2, the airplane is trimmed, straight, and level, at some speed approximately 20 or 30 kt below the speed for best climb and at an altitude approximately 500 ft below the chosen test altitude. The throttle is then opened wide and the airspeed held constant, causing the airplane to climb. When the transition to a climb is completed and the airplane is in a steady climb, the altitude is noted and, simultaneously, the timing is started. After 30 s, the airspeed, altitude,

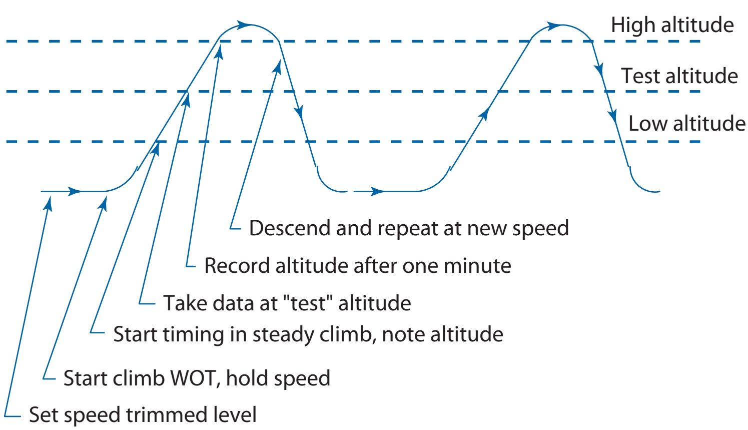

Fig. 5.2 Saw-tooth climbs.

and atmospheric state are recorded, nominally at the test altitude. After another 30 s the altitude is again recorded. The R/C, in fpm, at the test altitude, is taken to be the altitude gained during the one minute that the airplane is in a steady, WOT climb. When plotted against the airspeed, the data should look something like Fig. 5.3. In this manner it is possible to determine the maximum R/C and the speed for best climb as a function of altitude. However, it is not quite this simple.

The change in altitude is read from the altimeter, and when set to 29.92 inHg, the altimeter reads the standard pressure altitude. But the atmosphere in which the tests are being performed is not necessarily standard. So what is the actual altitude gained in a nonstandard atmosphere when Δh_p is the change in the pressure altitude? Also, what is the TAS, and how are the measurements corrected to a standard gross weight?

First, from Eq. (1.1), the change, Δ, in the pressure measured by the altimeter is given by $\Delta p = -\rho_s g \Delta h_{\text{meas}}$, where a sub "meas" refers to the measured value referenced to a standard atmosphere. But, taking the change in the pressure sensed by the altimeter as the same when we transform to the actual change in altitude and actual density, we have $\Delta p = \rho g \Delta h_{\text{meas}}$. Equation (2.2) is the equation of state for a perfect gas. Using this equation and a little algebra, the actual altitude gained in the nonstandard atmosphere can then be related to the measured change in altitude.

$$\Delta h = \Delta h_s \frac{T}{T_s}$$

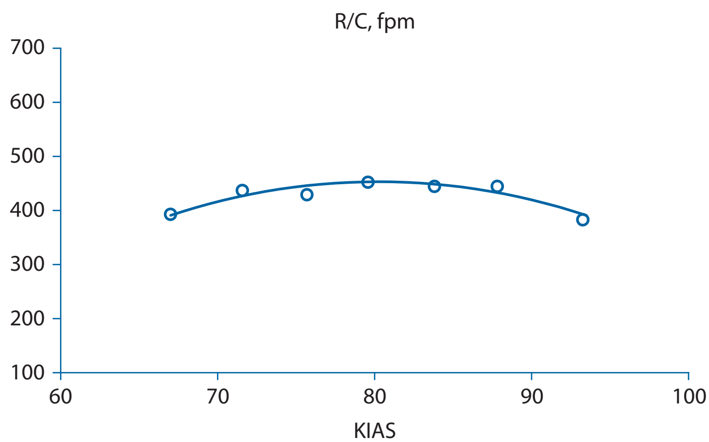

Fig. 5.3 Typical R/C data from saw-tooth climbs.

or

$$(R/C)_{\text{actual}} = (R/C)_{\text{meas}} \frac{T_{\text{actual}}}{T_s} \tag{5.2}$$

Applying these corrections to the measured R/C data and converting from KIAS to TAS, we can now make a graph similar to Fig. 5.3, except that it will be the true R/C plotted against the TAS. If you wish to compare your results with those in the POH it will be necessary to correct your data to a standard gross weight. To do this, it will be necessary to apply some basic calculus to Eq. (5.1). If you are do not understand calculus, simply skip to the final result, Eq. (5.5). Differentiating the R/C with regard to the weight by parts results in

$$\frac{\partial(R/C)}{\partial W} = -\frac{(T-D)V}{W^2} - \frac{V}{W}\frac{\partial D}{\partial W}$$

or

$$\frac{\partial(R/C)}{\partial W} = -\frac{R/C}{W} - \frac{V}{W}\frac{\partial D}{\partial W} \tag{5.3}$$

The second term on the right-hand side of Eq. (5.3) is usually smaller than the first term because the parasite drag is not a function of W. The second term can be calculated in terms of the induced drag given by Eq. (3.11). If that equation is differentiated with respect to W, the derivative of the induced drag becomes

$$\frac{\partial D_i}{\partial W} = \frac{2C_L}{\pi A e} \tag{5.4}$$

Combining Eqs. (5.3) and (5.4) results in an expression for the rate of increase of R/C with gross weight.

$$\frac{\partial(R/C)}{\partial W} = -\frac{(R/C)}{W} - \frac{V}{W}\frac{2C_L}{\pi A e} \tag{5.5}$$

Note that each term in the equations is negative, and it follows, not surprisingly, that the R/C will decrease as the weight increases.

As an example in measuring and correcting R/C, let us suppose that we were climbing in the Cessna 172R (Table 3.3) at a gross weight of 2300 lb through a desired test altitude of 5000 ft. At WOT and a KIAS of 80, we began timing when we reached an altitude of 4800 ft. After 30 s, data were read giving a pressure altitude of 5090 ft and an OAT of 65°F. After 60 s, the pressure altitude was recorded to be 5380 ft. Thus, for this test we climbed at a rate of 580 fpm according to the change in pressure altitude.

At the pressure altitude of 5090 ft, the standard OAT is 40.8°F. Adding 460 to this and to the recorded temperature and applying Eq. (5.2) corrects the R/C for nonstandard atmosphere from 580 to 608 fpm. Next, the R/C is corrected to a standard weight of 2450 lb by the use of Eq. (5.5). Because the lift coefficient must be determined and the TAS is also needed, the density will have to be calculated.

At a pressure altitude of 5090 ft, the pressure is found to equal 1754.6 psf. With an OAT of 525°R, the density is calculated from the equation of state as 0.00195 slugs/ft^3. This gives a density ratio, σ, of 0.82, resulting in a TAS of 88.3 kt (Eq. [2.3]). The dynamic pressure can be calculated directly from the KIAS because the ASI is based on sea-level density. Thus,

$$q = \frac{0.00238}{2}[(80)(1.69)]^2 = 21.8\,\text{psf}$$

The Cessna 172R has a wing planform area of 174 ft^2. The variable q in the equation then results in a wing lift coefficient of 0.944 for the test weight. With an aspect ratio of 7.48, the use of Eq. (5.4) results in the rate of change of R/C with weight.

$$\frac{\partial(R/C)}{\partial W} = -\frac{(R/C)}{W} - \frac{V}{W}\frac{2C_L}{\pi Ae} = -\frac{608/60}{(3500)} - \frac{(88.3)(1.69)}{3500}\frac{2(0.944)}{\pi(7.2)(0.6)}$$
$$= -0.0029 - 0.001 = -0.0039\ \text{fps/lb}$$

With the equation, the change in the rate of climb, Δ(R/C), can be determined from

$$\Delta(R/C) = \frac{\partial(R/C)}{\partial W}\Delta W \tag{5.6}$$

ΔW is the difference between the weight to which the test results are being corrected and the test weight. If the test weight is the higher of the two weights, ΔW is negative.

For the example, a reduction from 3500 to 2325 lb (a difference of 1175 lb) results in a correction to the R/C of 4.6 fps or 274 fpm. Therefore, our corrected data would show an R/C of 274 + 608 or 882 fpm at a TAS of 88.3 kt or 149.2 fps.

In flying saw-tooth climbs it is difficult to maintain the same test altitude. Try to be as consistent as possible. After the R/C values have been determined over a range of airspeeds, simply average the test altitudes for all the speeds and use this value for graphing R/C against airspeed for different altitudes.

5.2 Prediction of R/C

R/C is predicted using Eq. (5.1) with Eq. (4.1) for the total drag. In terms of power, the R/C can be expressed as

$$R/C = \frac{TV - DV}{W} = \frac{P_{\text{avail}} - P_{\text{reqd}}}{W} = \frac{excess\ power}{W} \tag{5.7}$$

This makes sense. You have an excess power equal to the power available from the engine *through the propeller* (P_{avail}) minus the power required (P_{reqd}) to maintain level flight. As pointed out in Chapter 3, this excess power can be used to increase the potential energy (climb) or increase the kinetic energy (accelerate) or do both. In this case, we are climbing at a constant speed, and so all of the excess power will go into R/C. The P_{reqd} is simply the product of the drag and the steady velocity at which we are climbing. The P_{avail} is the power at WOT, where the power output of the engine equals the power absorbed by the propeller. As for the takeoff calculations, this involves iterating on rpm until the power from the engine matches the power absorbed by the propeller from the C_P vs J curve. When this rpm is found, C_T for the found J is used to find the thrust and then the thrust horsepower available from TV. Remember, in using Eq. (5.7) all units must be consistent. In the English system this means that the power is in ft · lb/s (1 hp = 550 ft · lb/s), with the forces in lb and velocities in fps.

Using the procedure just described, power-available and power-required curves were constructed for the Cessna 172R as shown in Fig. 5.4.

Using these curves and Eq. (5.7), the R/C was calculated as a function of TAS and altitude for the standard weight. The results are presented in Fig. 5.5. Finally, the maximum R/C was graphed as a function of altitude and presented in Fig. 5.6 along with some experimental data from my students.

There are a few things to note about these figures. The first thing is the considerable amount of scatter that is shown for the data in Fig. 5.6. When you reserve an airplane, you take what you get, and on the particular day that the flights were performed the atmosphere was unstable and there were a lot of vertical gusts. Nevertheless, the predicted curve "plows" through the middle of the data, but the figure emphasizes that one really needs a good day to obtain good data. Another point to be made is a comparison of the predicted R/C with the POH. According to the POH, the SSL R/C at 2450 lb is 721 fpm, or about 100 fpm higher than predicted. At 10,000 ft, the POH value is 269 fpm, or almost exactly equal to the predicted

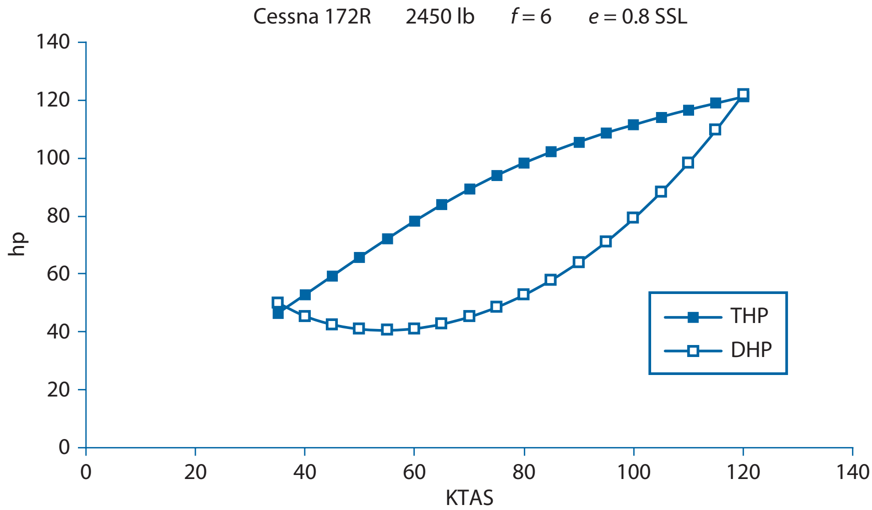

Fig. 5.4 Power required and power available for Cessna 172R.

value. The POH gives the service ceiling as 13,500 ft, whereas the predicted value is 14,234 ft. Considering the uncertainty with the drag, the propeller curve, and the engine chart, this comparison is not bad.

It was shown in Eq. (4.8) that the indicated speed for minimum required power was a constant independent of altitude and that this suggested that the CAS for maximum R/C should behave in a similar manner. However, it

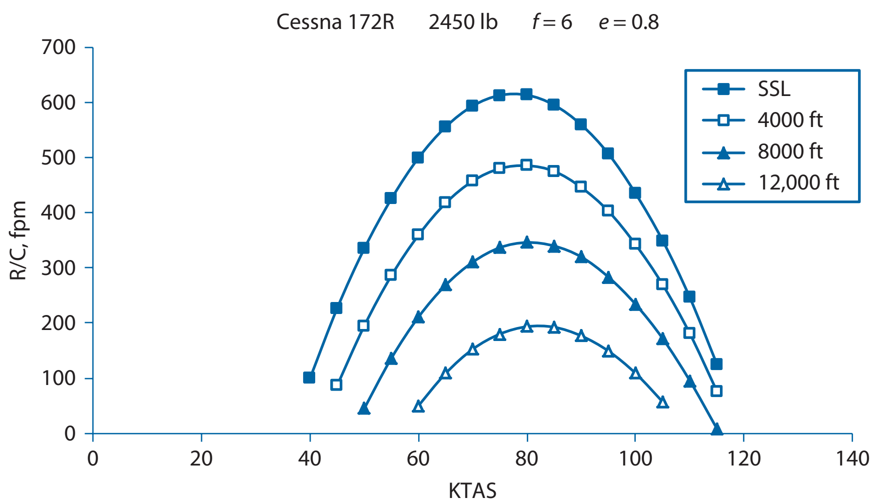

Fig. 5.5 R/C as function of altitude.

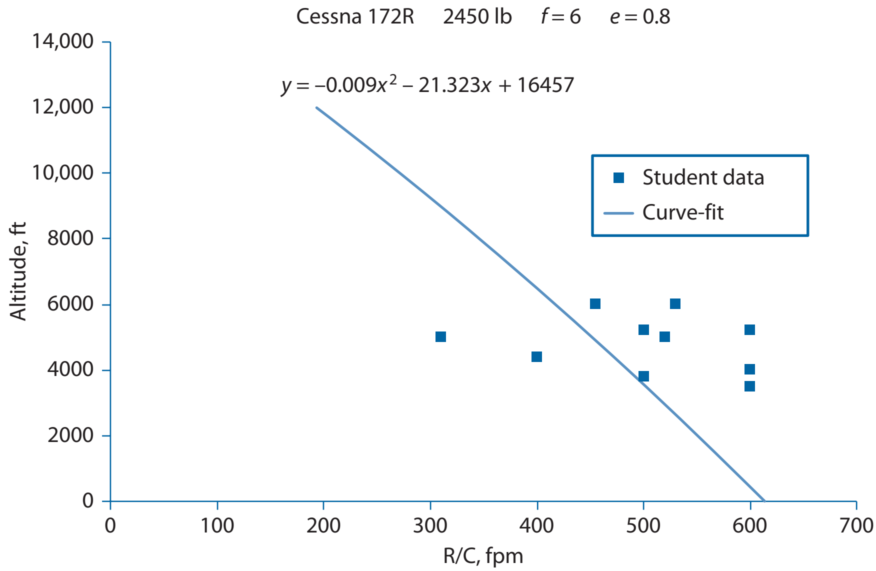

Fig. 5.6 Maximum R/C.

was pointed out that the IAS for minimum required power and the CAS for maximum excess power are not necessarily the same. Figure 5.4 addresses this point for the Cessna 172R. Here, the excess power and the required power are shown as a function of the airspeed. The required power has a minimum value at a speed of around 60 kt, whereas the excess power is a maximum at 79 kt (which is in agreement with the POH). The POH shows all climb performance at 79 KCAS because the velocity for best climb is close to being a constant with altitude. The curve of R/C vs V is nearly flat at its bottom, and so a slight variation in the climb velocity will make little difference in the R/C.

5.3 Time-to-Climb

If a linear variation of R/C with altitude is assumed, then the R/C can be written as

$$R/C = (R/C)_0 \left[1 - \frac{h}{h_{abs}}\right] \tag{5.8}$$

where h_{abs} is the absolute ceiling and $(R/C)_0$ is the sea-level R/C.

The time to climb a small distance, Δh, from h_1 to h_2 is given approximately by

$$\Delta t = \frac{\Delta h}{\left(\frac{(R/C)_1 + (R/C)_2}{2}\right)} \tag{5.9}$$

In words, Eq. (5.9) says the time to climb a small distance is equal to the distance divided by the average R/C over the distance. One can use a spreadsheet program and evaluate Eq. (5.9) over a range of altitudes to obtain a graph of the time to climb from one altitude to another. Using calculus, a closed-form for the time-to-climb can be obtained.

$$\begin{aligned} t &= \int_0^h \frac{dh}{(R/C)} \\ &= \frac{h_{abs}}{(R/C)_0} \ln\left[\frac{1}{1 - \frac{h}{h_{abs}}}\right] \end{aligned} \tag{5.10}$$

As an example in the use of Eq. (5.10), using the predicted value for R/C shown in Fig. 5.6, $h_{abs} = 16{,}457$ ft and $(R/C)_0 = 610$ fpm. Putting these numbers into Eq. (5.10) results in a time of 9.8 min. The time to climb to 10,000 ft is predicted to equal 25.2 min. The POH states these numbers to be 5 and 22 min respectively. Of course, the POH allows for fuel burned during warm-up, takeoff, and climb, whereas Eq. (5.10) assumes a constant weight for determining R/C. The rest of the difference between the POH and predictions is the culmination of the uncertainties in the drag, propeller, and engine characteristics.

5.4 Acceleration

To this point all of the excess power has been used to climb, but this does not have to be the case. A pilot may open throttle to go faster, or he or she may wish to both accelerate and climb. Therefore, let us go back to Fig. 5.1 and modify the equations from that figure to include acceleration. Here, we will have to resort to some calculus.

In the direction normal to V, the equation remains the same: $L = W \cos\phi$. In the direction of flight, the equation of motion becomes

$$T - D - W \sin\phi = m\frac{dV}{dt}$$

Multiplying both sides of the equation by V and recognizing that

$$V\frac{dV}{dt} = \frac{1}{2}\frac{dV^2}{dt}$$

results in

$$TV - DV = W(R/C) + \frac{d\left(\frac{1}{2}mV^2\right)}{dt} \tag{5.11}$$

In words, Eq. (5.11) states that excess power can be used to increase either the KE or the PE or both of an airplane.

5.5 Summary

The method for experimentally measuring an airplane's R/C has been described, as have methods for correcting the data to the standard atmosphere and a standard gross weight. Knowing the propeller and engine characteristics and the drag of the airplane, the relationships for calculating the R/C have been developed. Then, knowing R/C as a function of altitude, the procedure for calculating the time required to climb from one altitude to another is developed.

Problems

5.1 An airplane has a constant speed propeller that maintains an efficiency of 85 percent over its in-flight operating range. It is an aerodynamically clean, low-wing design with retractable gear. The engine develops 300 shp at SSL and the power decreases linearly with the density ratio at altitude. It weighs 3500 lb, has a wing planform area of 225 ft^2, and a total wetted area of 1000 ft^2. The wing's aspect ratio equals 7.0.

Find:

a) the service ceiling.

b) the time to climb from 2000 to 8000 ft.

5.2 An airplane weighs 2000 lb and is flying at 90 kt. It requires 85 thp to fly at this trimmed condition but, with WOT, the thp available equals 140. The pilot opens the throttle wide and begins to accelerate at a magnitude of 0.15 g. How fast is the airplane climbing in fpm at that instant?

Chapter 6 Stall, Approach, and Landing

- Determine Stalling Speed
- Predict Stalling Speed
- Determine Wing Flaps Effect
- Predict Maximum Trimmed Lift Coefficient

6.1 Stall Recovery

As the airspeed of an airplane is reduced, its angle of attack must increase in order to produce a lift equal to its weight. In so doing, at some speed, the angle of attack will attain a value at which the flow is no longer able to remain attached to the upper surface. The flow separates, a loss of lift results, and the airplane is said to have stalled. After stalling, the pilot pushes the stick forward to reduce the angle of attack and increase the airspeed. Such a stall recovery can be accomplished with only a small, insignificant loss in altitude. Practicing stall recovery is a normal part of pilot training.

Under some conditions, stalling can be dangerous. Obviously, a stall can be dangerous at a low altitude because ground impact may occur before the recovery is achieved. If the stall is done under asymmetrical conditions, as in a turn, the wing may stall first on one side causing a severe roll and possible entry into a spin. Also, at high power or full flaps, the stall may occur suddenly and, with the ailerons less effective at low speed, the airplane may roll and drop into a spin. Generally, it is good practice during a stall to stay off of the ailerons and use rudder and pitch control only.

FAR Part 23, "Airworthiness Standards: Normal, Utility, Acrobatic, and Commuter," paragraph 23.201 states that, when approaching stall, an airplane must have yaw and roll control down to the speed where the nose pitches because of stall. To demonstrate stalling performance the speed is reduced with elevator control until the speed is slightly above the stalling speed. Then the elevator is pulled back until stall is reached, keeping the reduction rate in speed at less than 1 kt/s. During the recovery from a stall, the roll should not exceed 15 deg.

Satisfactory stall performance must be demonstrated with gear and flaps up and down and with power settings off and at 75 percent of maximum continuous power. The trim speed for entering the stall must be at the minimum value or $1.5V_s$, whichever is highest. If the propeller is adjustable, the pitch should be set for high rpm.

Stalls must also be demonstrated for turning and accelerated stall.

6.2 Experimental Determination of Stalling Speed

If you are concerned with a low-wing airplane, it is interesting and informative to conduct some visual observations of stall over the wing using wool tufts. Begin by cutting tufts of black yarn approximately 3 in. long. This can be done easily by wrapping yarn around a paper tube and then cutting the tube lengthwise. Lay a 3-ft-long strip of masking tape sticky-side up on a flat surface. Place the tufts on the tape about 3 in. apart so that they do not tangle with each other. Then paste the tufted strips along one wing to show the stall pattern. Such an installation is shown in Fig. 6.1 for a Piper Warrior. Here, the airplane is stalled with flaps down. Note that the flow is separated over a large portion of the chord near the fuselage, but near the outboard end of the flap the flow is attached over the wing forward of the flap. Over the flap and most of the aileron, the flow is separated. All of the tufts are pointing upstream over the ailerons. The improvement offered to $C_{L\max}$ by the flap obviously comes from the increased pressure on the lower surface.

To determine the stalling speed at a given flap setting, trim the airplane to around 70 KIAS. Then, for a power-off stall, come back all the way on the power and back on the wheel to maintain altitude as the airplane slows down. At some point the airplane will stall. It may nose down sharply or it may begin to rock. At that point record KIAS, altitude, and OAT. If the nose drops, note the same quantities at the time it drops. As the airplane is slowing, take photos of the tufts to show the stall progression from the trailing edge to the leading edge.

6.3 Prediction of Stalling Speed and Numerical Modeling of the Wing

The maximum lift coefficient for a wing is determined by examining the spanwise distribution of the wing's section lift coefficient. As a wing's angle of attack increases, the section lift coefficients along the wing will increase. If, at some point along the wing, the section lift coefficient, C_l, is found to exceed the section maximum lift coefficient, $C_{l\max}$, then the flow will separate on the upper surface of the wing at that point and the wing will begin to lose lift. At such a point, it is said that the section has stalled

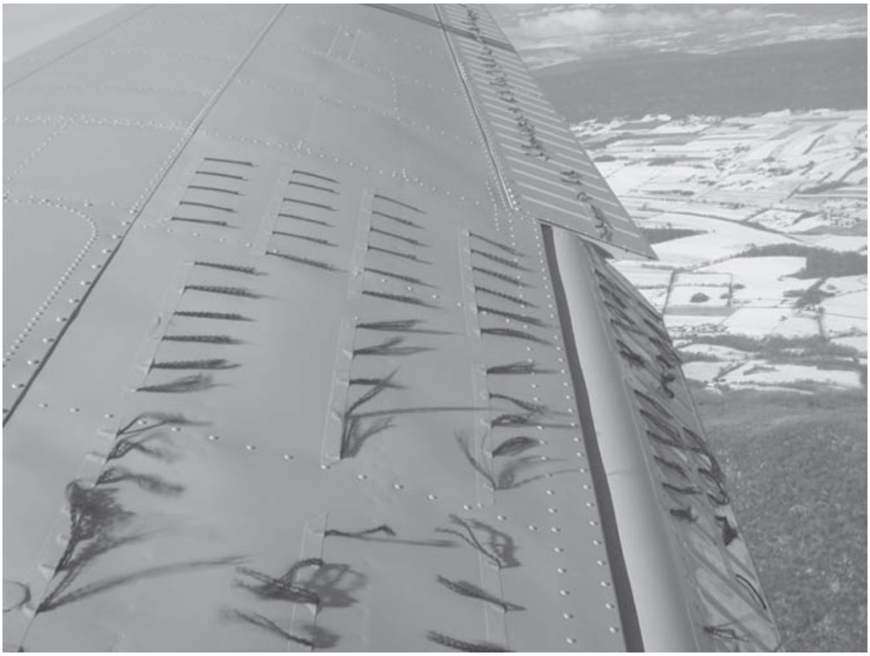

Fig. 6.1 Tufts on a Piper Warrior: two notches of flaps.

and, as it spreads over the span of the wing, the flow separation will cause the wing to stall.

Figure 1.8 depicts a vortex sheet trailing from a wing and rolling up into two trailing vortices. Equation (1.9) from the Biot–Savart law is the means for calculating the velocity induced at any point by a vortex of known strength. This is needed because the trailing vortex system induces velocities at every point along the wing and presents a flow pattern about each section of the wing that is different from that when the airfoil is in two-dimensional flow. Figure 6.2 illustrates a simplified analysis.

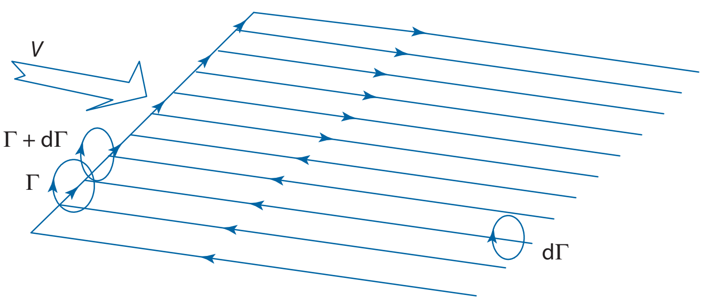

Fig. 6.2 Lifting line model of a wing.

To simplify the analysis and keep it tractable, a lifting-line model is used here to predict the spanwise C_l distribution along the wing span. The wing is replaced by a *bound vortex* from which *trailing vortices* extend and the rolling-up of the sheet is neglected. The model is pictured in Fig. 6.2.

In Fig. 6.2 a small change in the bound circulation of $d\Gamma$ can be seen feeding into the wing, causing the bound circulation to increase from Γ to $\Gamma + d\Gamma$. The directional arrows on the vortices are in accord with the right-hand rule. If the thumb of the right hand points in the direction of the arrow, the fingers curl and point in the direction of rotation. This figure illustrates vortex continuity that must be preserved in any model. But now consider Fig. 6.3. Instead of a sheet of single trailing vortex filaments, suppose the system is modeled by trailing horseshoe vortices of strength $\Gamma_1, \Gamma_2, \Gamma_3, \ldots$, etc. If the difference in strengths between neighboring trailing vortices is labeled $d\Gamma$, then we see that vortex continuity is satisfied and the model is, in effect, the same as shown in Fig. 6.1.

Figure 6.3 shows two horseshoe vortices with the centerlines of the horseshoes placed at a distance of $Y(I)$ from the left tip and trailing from the wing downstream to infinity. The width of each horseshoe vortex is ΔY. For the velocity induced by a trailing vortex filament at a point along the bound vortex line, one of the angles in Eq. (1.9) will be zero and other one will equal $\pi/2$. Thus the velocity induced vertically downward at $w(I)$ by a single vortex feeding into the wing at point J will be given by

$$w(I) = \frac{\Gamma(J)}{4\pi[Y(I) - Y(J)]} \tag{6.1}$$

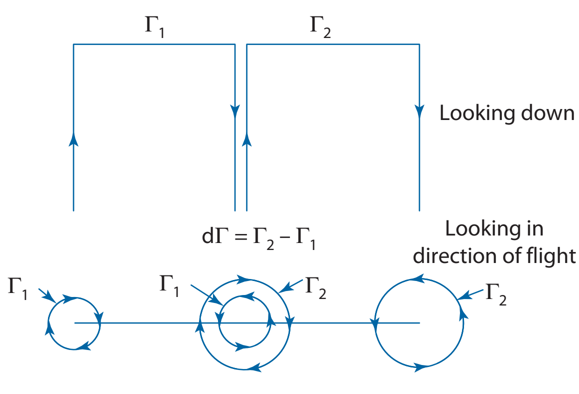

Fig. 6.3 Horseshoe vortex system.

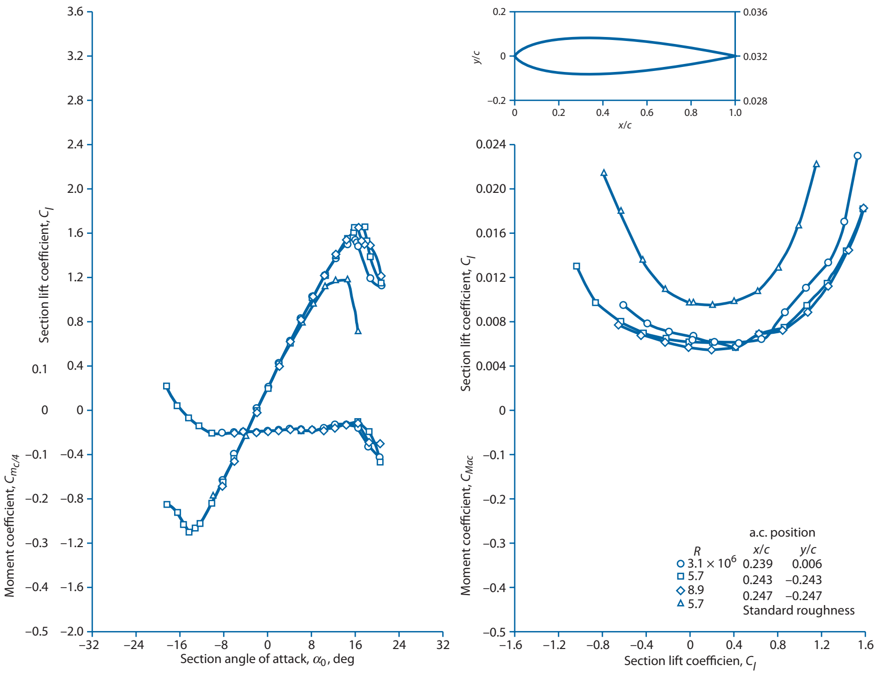

Fig. 6.4 NACA 2412 airfoil data.

The Kutta–Joukowski theorem, mentioned in Eq. (1.10), states that the lift per unit length of span is given by

$$L = \rho V \Gamma$$

But the lift per unit span can also be written in terms of the section lift coefficient as

$$L = \frac{1}{2}\rho V^2 c C_l$$

Equating the above two expressions results in a relationship between the bound circulation and the section lift coefficient.

$$\Gamma = \frac{1}{2} c C_l V \tag{6.2}$$

Now consider Fig. 6.4, which is taken directly from NACA Technical Report L5005 [4]. (This report is an excellent source of information on airfoils). You will find information in the report on the NACA four- and five-digit airfoils and on laminar flow and high Mach number airfoils (series 16).

Figure 6.4 shows data that are typical of most airfoils: there is a large region of operation where the lift coefficient varies linearly with the angle of attack. At zero angle of attack C_l equals 0.25 because the airfoil is cambered (see NACA Technical Report L5005 [4] for discussion of camber) and the angle of attack is measured relative to the chord line. The slope of the lift curve (rate of change of C_l with α) equals 0.103 C_l per degree so that α for zero-lift is -2.4 deg.

Instead of measuring the angle of attack relative to the chord line, it is convenient to use the *zero-lift line*, which, in this case, lies at an angle of 2.4 deg above the chord line. Defining the angle of attack in this manner, the section lift coefficient can be written as

$$C_l = 0.103\alpha \tag{6.3}$$

In applying Eq. (6.3) to a section of the wing, it must be remembered that when the wing is at an angle of attack of α, any section of the wing is experiencing a downward velocity, w, induced by the wing's trailing vortex system. Thus the angle of attack of the section is reduced, to a small angle approximation, by w/V, where this reduction is called the *induced angle of attack*, α_i, and is expressed in radians. Including α_i, the section lift coefficient becomes

$$C_l = a(\alpha - \alpha_i) \tag{6.4}$$

where a denotes the lift curve slope, which in this case is 0.103.

Combining Eqs. (6.1) through (6.4), an equation for the bound circulation at point J, for N points along the span, is found to be

$$\Gamma(I) = \frac{1}{2}c(I)Va\left[\alpha(I) - \frac{\sum_{J-1}^{N} W(I,J)\Gamma(J)}{V}\right] \tag{6.5}$$

$W(I, J)$ is referred to as an *influence coefficient* and is the velocity induced downward at I by a horseshoe vortex of unit strength at J. Remembering that a horseshoe vortex is composed of two single vortices of opposite direction, $W(I, J)$, from Eq. (6.1) becomes

$$W(I,\ J) = \frac{\Gamma(J)}{4\pi}\left[\frac{1}{Y(J) + \Delta Y/2 - Y(I)} - \frac{1}{Y(J) - \Delta Y/2 - Y(I)}\right] \tag{6.6}$$

Notice that α is shown as varying with the location. This is for two reasons. The wing may have some twist, and lowering the flaps or ailerons will cause a rotation of the zero-lift line.

To provide some insight into Eq. (6.5), let us expand it for one particular value of I.

$$\Gamma(I) = \frac{1}{2}c(I)a[V\alpha(I) - W(I,1)\Gamma(1) - W(I,2)\Gamma(2) - \cdots - W(I,I)\Gamma(I) - \cdots - W(I,N)\Gamma(N)]$$

or

$$W(I,1)\Gamma(1) + W(I,2)\Gamma(I,2) + \cdots + \left[1 + \frac{c(I)aW(I,I)}{2}\right]\Gamma(I) + \cdots + W(I,N)\Gamma(I,N) = V\alpha(I)$$

This can be written as a system of simultaneous, linear algebraic equations, where $B(I) = V\alpha(I)$:

$$A(I,J)\Gamma(J) = B(I) \tag{6.7}$$

where

$$\begin{aligned} A(I,J) &= W(I,J) && \textit{for } I \neq J \\ &= 1 + \frac{c(I)aW(I,I)}{2} && \textit{for } I = J \end{aligned} \tag{6.8}$$

Thus the task becomes one of dividing the wing into N segments and determining $Y(I)$, $W(I, J)$, $c(I)$, and $\alpha(I)$ at the center of each segment. The system of Eqs. (6.7) is then formed and solved for $\Gamma(I)$. Since lift coefficients are dimensionless, in order to simplify the numerical manipulation, V and ρ can be set equal to unity. Also, the span can be taken as 2 so that $Y(I)$ varies from -1 to $+1$. Of course, $c(I)$ and the wing area have to be adjusted

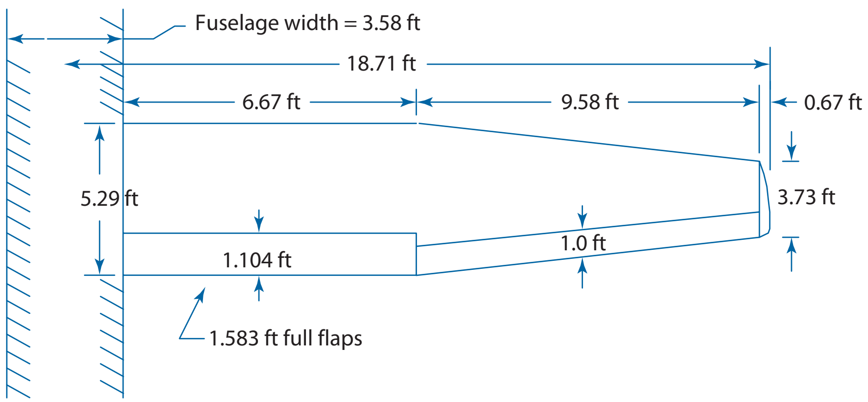

Fig. 6.5 Wing geometry for Cessna 172R.

accordingly. With this, spanwise locations will be expressed as fractions of the semispan.

As an example, consider Fig. 6.5, which presents the wing geometry measured for the Cessna 172R. These dimensions may or may not agree with those quoted by the manufacturer but will be used here as they were measured during a class lab.

If we take the semispan equal to unity, then the chord distribution as a function of the fractional span distance, x, becomes

$$
\begin{aligned}
c &= 0.282 \qquad\qquad\qquad\qquad \textit{for } 0.096 < x < 0.452 \\
c &= 0.282 - \frac{(x - 0.452)}{(1.0 - 0.452)}(0.282 - 0.198) \\
&= 0.282 - 0.153(x - 0.452) \qquad \textit{for } 0.452 < x
\end{aligned}
\tag{6.9}
$$

The flap chord ratio equals 0.209 for flaps up and 0.30 for flaps down 30 deg. This extension of the chord will be assumed to increase linearly with flap angle. The aileron chord ratio, taken at its midspan, is equal approximately to 0.22.

The wing has a 3-deg washout (i.e, the wing is twisted linearly so that the tip is nose-down 3 deg relative to the root). If the angle of attack of the wing, α_W, is measured at the centerline, the angle of attack of the Ith station will be given by $\alpha_W - 3x(I)$.

6.4 Effect of Flaps

A sketch of an airfoil lift curve with and without a flap is shown in Fig. 6.6.

The angle of attack is measured relative to the zero-lift line of the unflapped airfoil. It is seen that lowering the flap does not change the

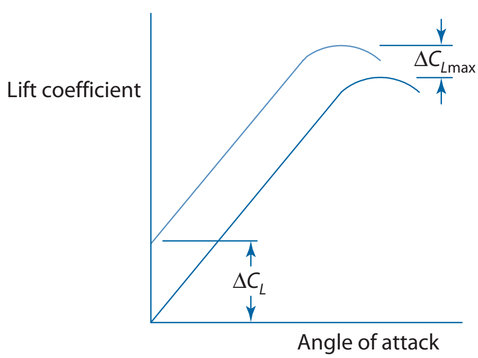

Fig. 6.6 Effect of a flap on the lift.

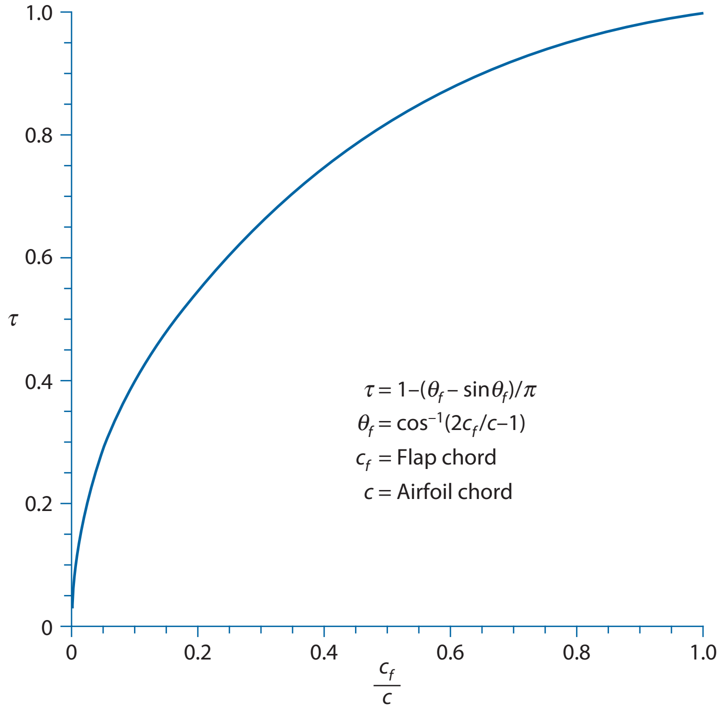

Fig. 6.7 Flap effectiveness factor, τ (from [1], reprinted with permission from John Wiley & Sons Inc.).

slope of the lift curve; it simply rotates the zero-lift line upward, thereby increasing effectively the angle of attack by $\Delta\alpha$. However, the airfoil generally stalls at a lower angle of attack with the flap so that the increase in $C_{L\max}$ is not as great as the increase in C_L in the linear part of the curve. The increase in C_L or $C_{L\max}$ can be estimated using Figs. 6.7, 6.8, and 6.9, which are taken from McCormick [1].

The factor, τ, in Fig. 6.7 is the rate of change of angle of attack with the flap angle. As one can see, the effectiveness of a flap depends on its type. Slots help to delay separation on the upper surface of a flap, resulting in higher τ values. As an example in the use of this figure, a plain flap having a 30 percent chord will have a τ of about 0.65. That means that when the flap is deflected, say, 30 deg, the zero-lift line will increase its angle by 19.5 deg.

The τ values presented in Fig. 6.7 are theoretical and can be calculated from the equations given in the figure. Theory and practice do not always agree and that is the case here. Figure 6.8, obtained after a study of flap data in many NACA reports, is a correction factor that multiplies the theoretical value of τ to obtain a more accurate measure of the change of angle of

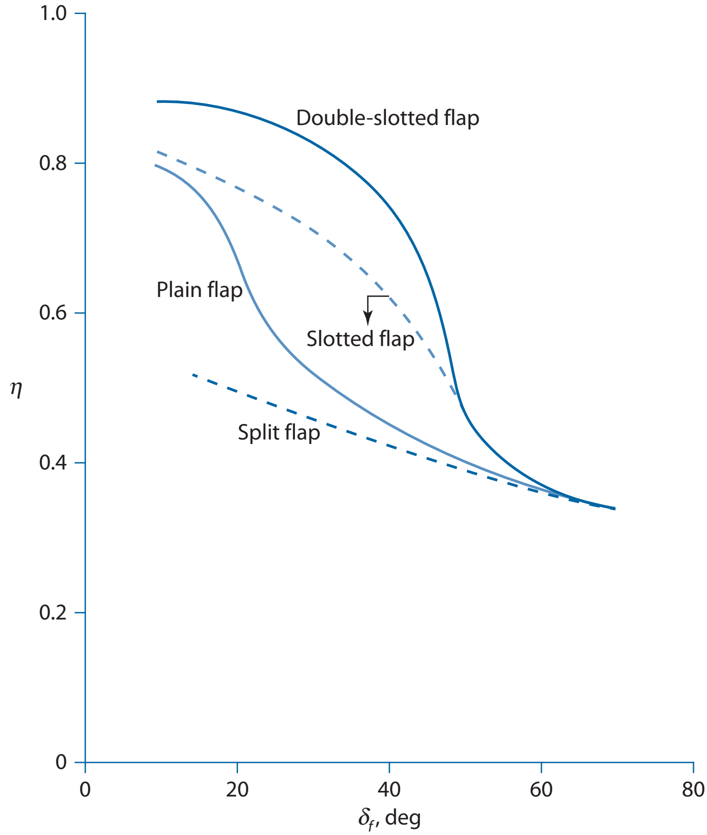

Fig. 6.8 Correction to τ (from [1], reprinted with permission from John Wiley & Sons Inc.).

attack with flap angle. Thus, for the previous example, a plain flap at an angle of 30 deg would have a correction factor of about 0.53 or a τ of only 0.34. Thus the increase in the effective angle of attack would be (30)(0.34), or 10.2 deg. For a typical lift curve slope of around 0.106, this flap deflection would produce an increase in the section lift coefficient of (10.2)(0.106), or 1.08. In order to obtain $C_{l\max}$ this increase in C_l must be multiplied by the factor from Fig. 6.9. For a 30 percent chord flap and a plain flap, this factor is about 0.65, resulting in a predicted increase in $C_{l\max}$ of (1.08)(0.65), or 0.702.

6.5 Pitching Moment

McCormick [1] shows that the change in the pitching moment due to a flap deflection is directly proportional to the change in the section lift coefficient. Further, the ratio of $\Delta C_{m1/4}$ to ΔC_l agrees well with the theoretical prediction. This ratio is given by

$$\frac{\Delta C_{m_{1/4}}}{\Delta C_l} = -\frac{2\sin\theta_f - \sin 2\theta_f}{8(\pi - \theta_f + \sin\theta_f)} \tag{6.10}$$

Therefore, the ratio found in Eq. (6.10), as well as the section lift coefficient, should be integrated across the span in order to determine the trimmed lift coefficient.

6.5.1 Trim Lift Coefficient

Figure 6.10 shows the forces and moments on an airplane. In trim the sum of the lift on the wing, L_W, and the tail, L_T, equals the weight, W. In addition, the sum of the moments about the c.g. must equal zero. Thus,

$$L_W + L_T = W$$
$$L_W l_W + M_{ac} = L_T l_T$$

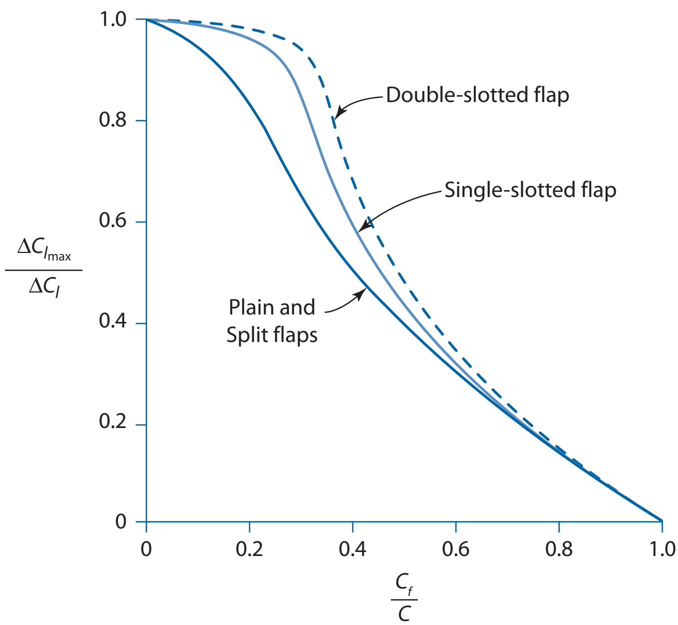

Fig. 6.9 Correction for $C_{l\,max}$ (from [1], reprinted with permission from John Wiley & Sons Inc.).

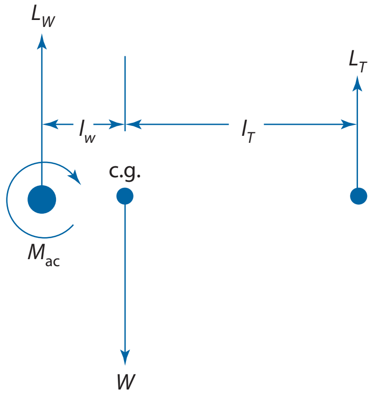

Fig. 6.10 Trim of an airplane.

Dividing by qS gives:

$$C_L = C_{Lw}\left[\frac{l_W + l_T}{l_T}\right] + C_{M_{ac}}\frac{\overline{C}}{l_T} \tag{6.11}$$

If we set the wing lift coefficient equal to its maximum value, then $C_{L\max}$ based on the weight will usually be less because $C_{M_{ac}}$ is negative and the c.g. is usually ahead of the wing's aerodynamic center (a.c.).

6.6 Formulation of Computer Program to Predict $C_{L\max}$

A computer program will now be developed predict the spanwise lift distribution for a wing. This program will utilize the analysis embodied in Eqs. (6.4)–(7.7). The simplest way to explain the program is to number the separate blocks in the program.

1. Input the wing geometry including planform shape, flap and aileron geometry, and twist. Also input the flap deflection.
2. For ease of computations, set $\rho = V = 1.0$ and the span equal to 2. Then modify the chord in proportion to the span.
3. Determine the indexed centerline locations for the N horseshoe vortices and, for each centerline, calculate the indexed chord length and unflapped indexed angle of attack.
4. Check to see if the indexed section is flapped, and, if so, increase the angle of attack for flap deflection and chord if extended by flap. Also calculate $C_{l\max}$.
5. Using Eqs. (6.8) and (6.9), calculate the determinant $A(I, J)$ and the column matrix $B(I)$.

6. Solve the equations for the values of $\Gamma(I)$ and then calculate $C_l(I)$. If $C_l(I)$ is greater than $C_{l\max}$, set $C_l(I)$ equal to $C_{l\max}$.
7. Calculate $C_{M_{ac}}(I)$ at each section that is a function of $C_l(I)$.
8. Sum over the span $(I = 1,n)$ to get the total wing lift, $\Gamma(I)\ \Delta y$.
9. Sum over the span $C(I)C_{M_{ac}}(I)/2$ to get the total wing moment.
10. Calculate the untrimmed wing lift coefficient by dividing the total wing lift by S.
11. Calculate the trimmed lift coefficient for the c.g. at the wing a.c. using Eq. (6.12).

A FORTRAN program following the guidelines was written and run for the Cessna 172R using the wing geometry shown in Fig. 6.5. Again, the numbers in the figure are not from Cessna. They were measured by students.

To illustrate the concept of spanwise loading and wing section stalling, let us refer to Fig. 6.11, which was generated by the FORTRAN program. Section C_l distributions along the span are shown for angles of attack of 10, 20, and 30 deg and for a flap angle of 30 deg. Also, $C_{l\max}$ values are

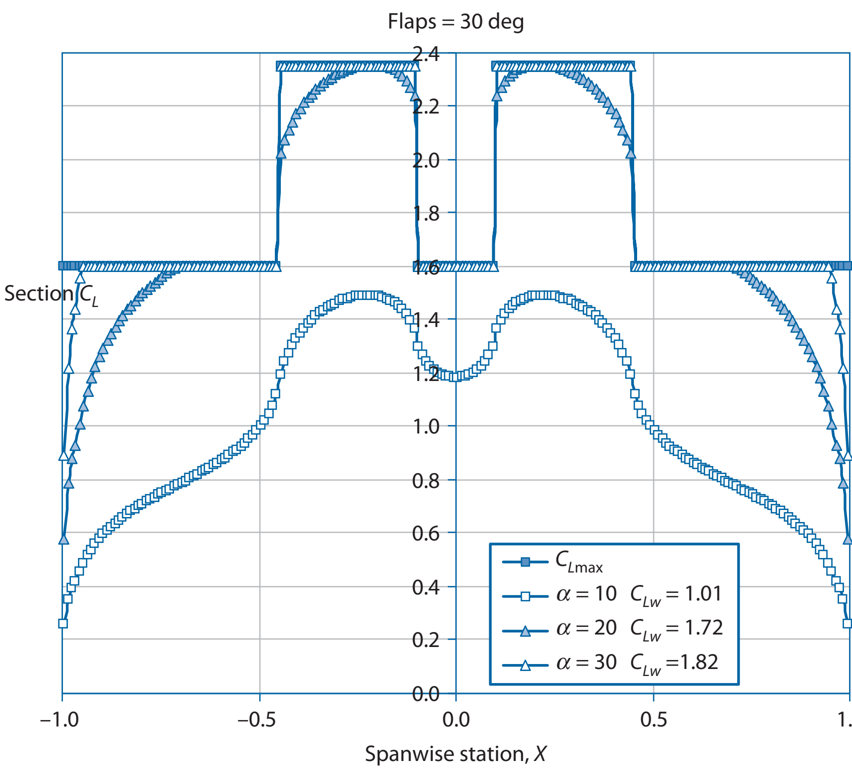

Fig. 6.11 Limiting of C_L by $C_{L\max}$.

plotted along the span. $C_{l\max}$ is seen to increase from 1.6 to 2.36 when the flaps are lowered. For an angle of attack of 10 deg, the section C_l is less than $C_{l\max}$ everywhere along the span, and thus there is no local stalling on the wing. At an angle of attack of 20 deg, the section C_l values have reached the limiting values over approximately the inboard half of the ailerons just outboard of the flaps. Because of the higher $C_{l\max}$ value over the flaps, the flaps are still unstalled but are about to stall. At 30 deg angle of attack, the entire wing, except for a very small region at the tips, has stalled.

This local stalling and spreading of the stalling gives rise to the integrated lift curves shown in Fig. 6.12. Compare the C_{Lw} vs α curve for flaps at 0 deg with Fig. 6.4. Below about 18 deg, the curve is straight, but above this angle of attack the curve begins to flatten out as the stalling develops over the inboard of the ailerons and then spreads to the rest of the wing. This method of simply limiting the section C_l values to $C_{l\max}$ results in a close estimate of $C_{L\max}$ for the wing, but the shape of the wing lift curve is not realistic. After peaking, C_L drops in the manner shown in Fig. 6.4 for an airfoil. Nevertheless, one can get a good estimate of the effect of flaps and other parameters on $C_{L\max}$ for a wing by using this numerical procedure.

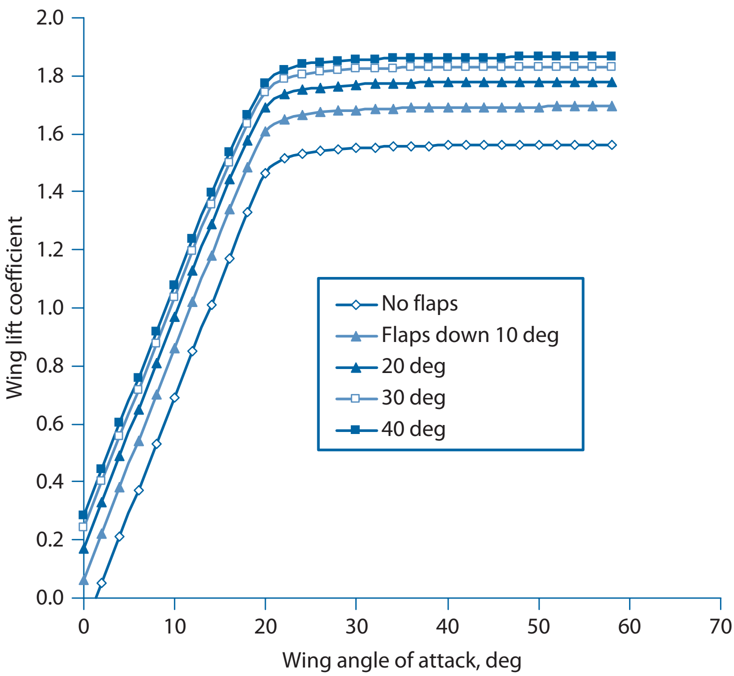

Fig. 6.12 Predicted wing lift curves at various flap angles for Cessna 172R.

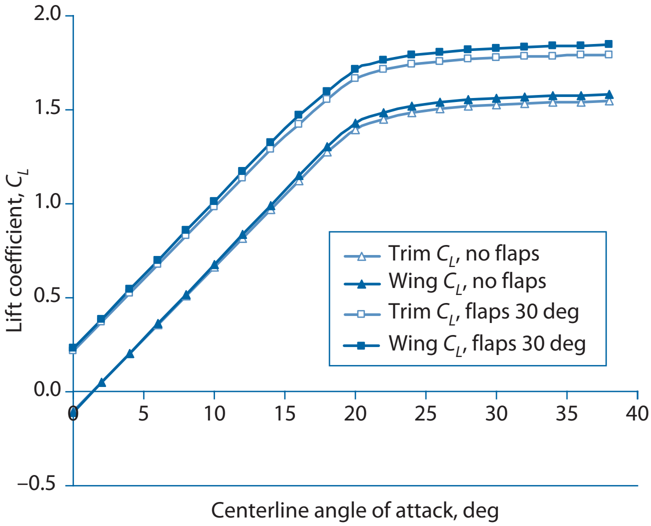

Fig. 6.13 Effect of trim on airplane lift coefficient.

The effect of trimming on $C_{L\max}$ is shown in Fig. 6.13. The figure was obtained using Eq. (6.11) and letting l_W (shown in Fig. 6.10) equal zero. Thus the difference between the wing $C_{L\max}$ and the airplane $C_{L\max}$ is due entirely to the pitching moment on the wing, $C_{M_{ac}}$. It can be seen that this difference is small. Now let us examine the effect of moving the c.g. behind the wing a.c. Because $l_W + l_T$ is a constant and because the c.g. position has only a small effect on the pitching moment from the tail, we can approximate that the decrease in the airplane $C_{L\max}$ can be written as

$$\Delta C_{L\max} = C_{L\max_0} \frac{-l_W}{l_T} \tag{6.12}$$

where $C_{L\max0}$ refers to the value with the c.g. at the wing a.c.

The distance between the wing a.c. and the tail a.c. is approximately 14 ft for the Cessna 172R. For a typical student experiment, it is estimated that the c.g. was 0.02 ft behind the wing a.c. Thus, for example, for 30 deg of flaps—from Eq. (6.12) and Fig. 6.13—it is estimated that the trimmed airplane $C_{L\max}$ is given by

$$C_{L\max} = 1.77 \frac{(-0.02)}{14} + 1.77 \cong 1.77$$

In other words, for this loading there is very little effect of c.g. position on the trim $C_{L\max}$.

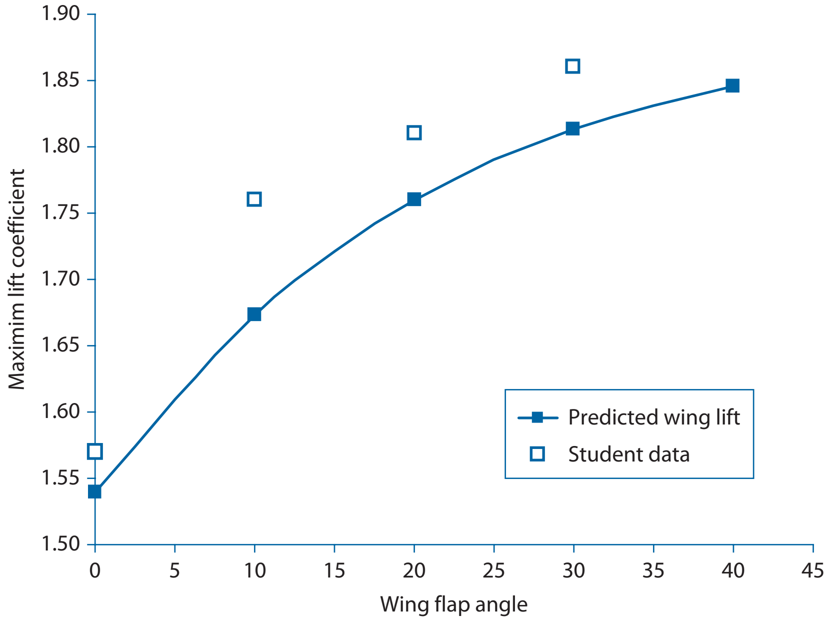

Fig. 6.14 Comparison of student data with predictions of the max wing lift coefficient for Cessna 172R.

Figure 6.14 presents some experimental measurements obtained by students for $C_{L\max}$ as a function of flap setting for a Cessna 172R. It can be seen that the data are consistently higher than the predicted values. However, the difference amounts to only a consistent 1 percent error in the stalling speed. This difference could easily be the result of airspeed calibration error at low speeds and contributions from the tail, fuselage, and propeller. (See trim lift in Chapter 4.) In summary, it appears that the numerical program closely predicts the stalling speed.

6.7 Elliptic Wing

The *elliptic wing* is one particular case of wing planform and loading for which a closed-form solution can be obtained. For ease of calculation, consider a wing with a unit semispan extending from $y = -1$ to $+1$. Suppose the wing has an elliptical spanwise distribution of circulation given by

$$\Gamma(x) = \Gamma_0 \sqrt{1 - y^2} \tag{6.13}$$

If this distribution is substituted into Eq. (6.1), the differential form of this equation will become

$$w(y_0) = \frac{1}{4\pi} \int_{-1}^{1} \frac{d\Gamma(y)}{y_0 - y} \tag{6.14}$$

Integrating Eq. (6.14), it is found, surprisingly, that the downwash is a constant along the span and given by

$$w = \frac{C_L}{\pi AR} V \tag{6.15}$$

If the wing is untwisted, the section lift coefficients along the span will be constant because the induced angle of attack is constant. It is easily shown that the wing lift coefficient, C_L, equals the section C_l if C_l is constant. Thus, for the midspan section,

$$C_L = a(\alpha - \frac{C_L}{\pi AR})$$

or

$$C_L = a\alpha \left(\frac{AR}{AR + 2} \right) \tag{6.16}$$

Thus we have an approximate correction to the slope of the lift curve for aspect ratio. Although the result applies strictly to only a wing with an elliptical distribution of Γ, it holds closely for other wings. For example, the aspect ratio of the Cessna 172R is 7.48. From Fig. 6.4, the slope of the two-dimensional C_l curve is close to 0.1 per degree and the angle of attack for zero-lift is -2 deg. From Eq. (6.16) the slope of the wing lift curve is predicted to equal 0.0789. From the numerical solution for the Cessna wing that includes the wing geometry, the slope of C_L vs α is shown in Fig. 6.12 to equal 0.080, close to the prediction based on the elliptic wing.

6.8 Quick Method for Calculating a Section C_L before Stall

Before closing this chapter, it should be noted that the section C_L can be quickly calculated if the C_L is known at that section for two values of the wing C_L. The section C_L can be expressed in terms of a basic C_{l_b} and an additional C_{l_a} as

$$C_l = C_{l_b} + C_{l_a} C_{L_w} \tag{6.17}$$

C_{l_b} is called the *basic* C_l and C_{l_a} is the *additional* C_l.

As an example, refer to Fig. 6.11 and take an angle of attack of 10 deg with $C_{L_w} = 1.01$. At a spanwise station of 0.5, the C_L equals 1.0. If we now take an angle of attack of 15 deg (not shown on the figure), the program determines a C_{L_w} of 1.47 and a C_l value of 1.41 at the 0.5 station. If Eq. (6.17) is written for these two angles, two simultaneous equations are obtained and can be solved to give

$$C_{l_b} = -0.11, C_{l_a} = 1.12$$

If we set C_l at this station equal to $C_{l\max}$ at this station and using the values in Eq. (6.17), we predict that the wing will begin to stall at this same station at a wing C_L equal to 1.53, or an α of 17 deg. Referring to Fig. 6.13, this is the value of α above which the lift curve begins to flatten.

6.9 Summary

Relationships have been derived and a computer program discussed from which one can predict the maximum trimmed lift coefficient of an airplane. From this parameter, the stalling speed can be determined. It was shown that the method predicts stalling speeds close to those measured by students.

Problems

6.1 At some point on a wing, the basic C_l equals -0.3 and the additional C_l equals 0.8. If the section maximum C_l equals 1.5 at that point, at what wing C_L will the section first stall?

6.2 Given a cambered airfoil. The slope of its lift curve is 0.106 per degree and the angle for zero-lift equals -5 deg. It is equipped with a 25 percent double-slotted chord flap. What is the predicted lift coefficient for the airfoil at an angle of attack of 3 deg with the flap lowered 35 deg?

6.3 An airplane weighs 2500 lb with a wing area of 200 ft^2. The M_{ac} equals 4 ft. The c.g. lies 1.2 ft ahead of the a.c. The distance of the tail a.c. aft of the c.g. is 15 ft. The airplane is flying at 5000 ft. The wing has a $C_{M_{ac}}$ of -0.05 and a $C_{L\max}$ of 1.4. At what calibrated airspeed in knots will the airplane stall?

6.4 Show that a flat, untwisted wing with an elliptical planform will have an elliptical lift distribution.

6.5 Prove that the wing C_L equals the section C_l if C_l is constant along the span.

Chapter 7 Cruise

- Find Rate at Which Fuel Burns
- Find Range of Airplane

7.1 Rate of Fuel Burn

We have taken off and climbed to a cruising altitude and now we are ready to go somewhere. How far can we fly before having to land to refuel, and at what speed should we fly to maximize this distance, called the range? These are questions to be addressed in this chapter. Again, those without knowledge of calculus can skip to the result, Eq. (7.4). To begin, consider the rate at which the engine is burning fuel. A quantity called the specific fuel consumption (SFC) determines this rate. The rate at which fuel is burned can be written in terms of SFC and the required engine power, P, by

$$\frac{\mathrm{d}W_f}{\mathrm{d}t} = (SFC)(P) \tag{7.1}$$

where W_f equals the fuel weight. The units of SFC, at this point, are consistent with the units of the power, P. At any instant, the power P is equal to DV divided by the propeller efficiency, η.

$$\frac{\mathrm{d}W_f}{\mathrm{d}t} = \frac{(SCF)(D)(V)}{\eta}$$

Denoting the drag-to-lift ratio with ε, the rate at which fuel is burned becomes

$$\frac{\mathrm{d}W_f}{\mathrm{d}t} = \frac{(SFC)(\varepsilon W)V}{\eta} \tag{7.2}$$

During cruise, the weight decreases as fuel is burned. Thus the right-hand side of Eq. (7.2) is a function of time. Thus,

$$W = W_i - \int \frac{dW_f}{dt} dt$$

where W_i is the initial value of W_f.

Let us go back two equations and rewrite dW_f/dt as $-dW/dt$:

$$\frac{dW}{dt} = \frac{dW}{ds}\frac{ds}{dt} = \frac{dW}{ds}V = \frac{-(SFC)(\varepsilon W)V}{\eta}$$

or

$$\frac{dW}{W} = \frac{-(SFC)\varepsilon}{\eta} ds$$

Integrating, the equation $\ell n\left[\frac{W_F}{W_I}\right] = -\frac{(SFC)\varepsilon}{\eta}R$ becomes

$$R = \frac{\eta}{(SFC)\varepsilon}\ell n\left[\frac{W_I}{W_{\text{Fin}}}\right] \tag{7.3}$$

Solving for the range, RW_I is the initial weight of the airplane and W_{Fin} is the final weight after fuel has been burned. The units of the above equation are consistent.

Let us redo this derivation using the more familiar brake specific fuel consumption (BSFC) with units in pounds per bhp-hour (lb/bhp-hr).

Equation (7.2) is divided by 550 to convert foot pounds per second to horsepower. Then it is divided by 3600 to convert the rate of fuel consumption to pounds per second. Then, if the range, R, is expressed in nautical miles, the distance, s, must be multiplied by 6080 to convert from feet to nautical miles. Thus,

$$\int_{W_E}^{W_E+W_F} \frac{dW}{W} = -\frac{(BSFC)\varepsilon\, s(6080)}{\eta(550)(3600)}$$

or

$$R = \frac{325.7\eta}{(BSFC)\varepsilon}\ell n\left[1 + \frac{W_F}{W_E}\right] \tag{7.4}$$

In Eq. (7.4), the final weight has been expressed in terms of the empty weight, W_E, plus the total fuel weight burned, W_F.

The closed-form expression for the range is referred to as the Breguet range equation. While it is instructive in illustrating the factors important to the range, it is approximate because it assumes a constant drag-to-lift ratio and a constant BSFC. However the drag-to-lift ratio (ε) is a function of C_L that changes for a constant airspeed as fuel is burned off. The speed that minimizes ε is usually slower than the speed of which an airplane is capable. Most pilots fly at a constant percentage of rated power, such as 65 or 75 percent. Thus they are willing to swap fuel for time saved. Let us pursue this tradeoff further.

An expression for the drag can be obtained by dividing the required power from Eq. (4.2) by V.

$$D = \frac{1}{2}\rho f V^2 + \frac{2(W/b)^2}{\pi\rho e}\frac{1}{V^2} \tag{7.5}$$

Differentiating this equation with respect to V and equating it to zero results in the velocity for minimum drag.

$$V^2 = \sqrt{\frac{4}{\pi f e}\frac{W/b}{\rho}}$$

Substituting this velocity into the drag results in the fact that the minimum drag is independent of altitude (does not depend on density) and, at V, the induced drag equals the parasite drag. Thus,

$$\max\,\varepsilon = \frac{D_{\min}}{W} = \frac{4}{b}\sqrt{\frac{f}{\pi e}} \tag{7.6}$$

Consider the Cessna 172R with an estimated f of 6.2 ft^2 and an e of 0.8. The maximum ε is found to be 0.174, or an L/D of 5.74. The gross weight without fuel is about 1932 lb, and the total fuel weight is about 318 lb (max). Assuming a propeller efficiency of 80 percent and a BSCF of 0.5 lb/bhp-hr results in a calculated range from the Breguet equation of 456.4 n miles. The optimum speed for flying this range would be about 120 kt, thus requiring a time of 3.8 h. It might be better to assume an average weight for the mission of 2091 lb. The optimum speed then becomes 110.8 kt.

Of course the calculated ranges are not practical because one would never fly until the fuel is exhausted. Instead, range is quoted for an airplane plus some reserve, such as 45 min, for example. Thus, the fuel must be calculated first to fly for 45 min at the nearly empty fuel weight. This amount is then subtracted from the fuel supply and the reduced quantity used to calculate the range. Another factor affecting the range is the allowable gross weight. It is possible with most airplanes to exceed the maximum allowable gross weight if the airplane is loaded with full fuel, passengers, and cargo. For example, the empty weight of the Cessna 172R is approximately 1639 lb. Suppose there are four passengers at an average weight of 180 lb and each has a bag weighing 50 lb. Full fuel tanks will bring the gross weight to approximately 2877 lb, exceeding the allowable gross weight by 427 lb. Let's get rid of the bags, and the excess weight goes down to 227 lb. To compensate for this we will reduce the fuel by 37.8 gal. But this means we only have 12.2 gal to use, and so we are not going far. Let's kick one of the passengers off. That gives us another 30 gal, and so we are getting close to the maximum fuel volume of 53 gal. In general, if passengers plus baggage is called payload, we see that the range is affected by payload. Generally, the range decreases slightly as the payload is increased due to the increase in induced power. However, a payload is reached where any further increase will require a reduction in fuel. Beyond this payload, the range decreases rapidly.

7.2 Range–Payload Curve

Let us construct a range–payload curve for the Cessna 172R. To check against the numbers given in the POH, we will cruise at 75 percent of rated power at a standard altitude of 6000 ft. To do this, start with Eq. (4.2), which shows that

$$P = \frac{1}{2}\rho f V^3 + \frac{2(W/b)^2}{\pi \rho e}\frac{1}{V} \tag{7.7}$$

In using this equation, one must remember that the units of V are in feet per second and the BSFC in pounds per bhp-hour. Thus BSFC must be divided by 3600 to calculate the expended fuel rate in pounds per second. Also of course, P must be divided by 550 and by the propeller efficiency to express it as brake horsepower. Further, if V is taken as knots, then V must be multiplied by 1.69 to convert it into feet per second for use in Eq. (7.7).

At a standard pressure altitude of 6000 ft, $\rho = 0.001987$. The payload will vary between zero and the difference between the allowable gross weight and the empty weight, or 811 lb. The maximum fuel weight equals 318 lb, leaving 493 lb for passengers and baggage before the fuel has to be reduced. The BSFC is estimated from Fig. 7.1 to equal 0.5 lb/bhp-hr. A propeller efficiency of 80 percent is assumed for cruising.

The weight of fuel for cruising will be dictated by the payload, PL.

$$\begin{array}{ll} PL < 493\,\text{lb} & W_F = 318\,\text{lb}(53\,\text{gal}) \\ PL > 493 & W_F = 318 - (PL - 493)\text{lb} \end{array}$$

The rate at which fuel is burned will equal the product of brake horsepower and BSFC. And the cruising speed will be determined by Eq. (7.7) for

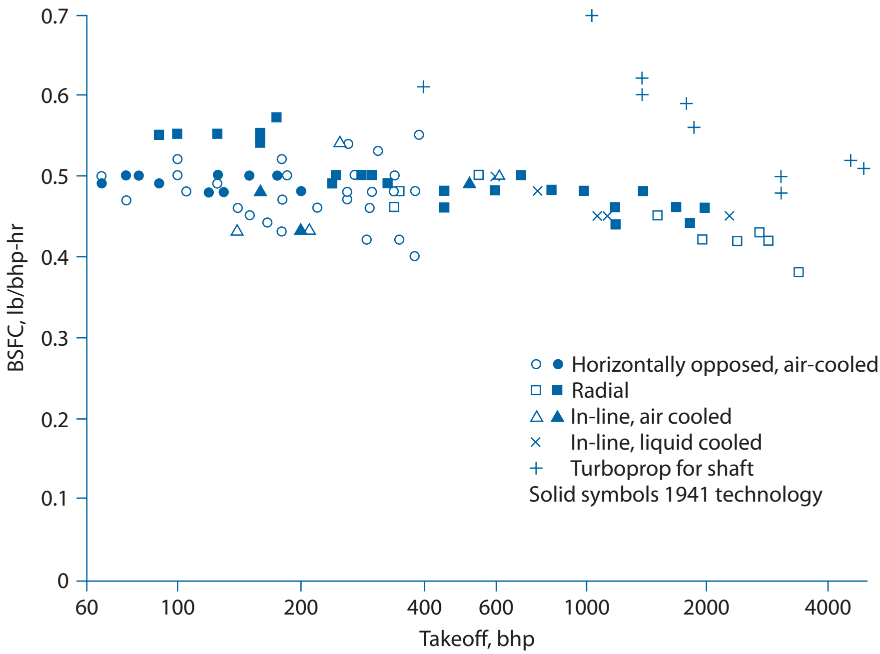

Fig. 7.1 BSFC for piston and turbo shaft engines (from (1), reprinted with permission from John Wiley & Sons Inc.).

the set power of 75 percent of the rated value, or 120 bhp. As the fuel is burned, the weight will decrease and, for a fixed power, the TAS will increase. Thus, the determination of the range will require calculations at small time increments and allowing for the change in weight, fuel burned, and distance gained over each time increment. Such an exercise is best done on a computer. It can be done using spreadsheet software or the equations can be programmed, preferably with a simple language such as FORTRAN. The latter course of action was chosen here resulting in the range–payload curve presented in Fig. 7.2.

As can be seen, the range is decreasing slightly as the payload is increased until a value of 493 lb is reached. Beyond that payload, it is necessary to offload fuel so that the range begins to decrease rapidly. Figure 7.2 does not include any reduction in the range for reserve. Again, it is emphasized that this calculation or any of the others in this book are not necessarily approved by the manufacturer.

If one has the money and time, it is fun for students to do an experiment to check the range. This means flying a significant distance and carefully measuring the fuel expended and the flight conditions during the trip. Obviously a whole class cannot do this. I always held a lottery, and the winners were then flown for a couple hundred miles to someplace

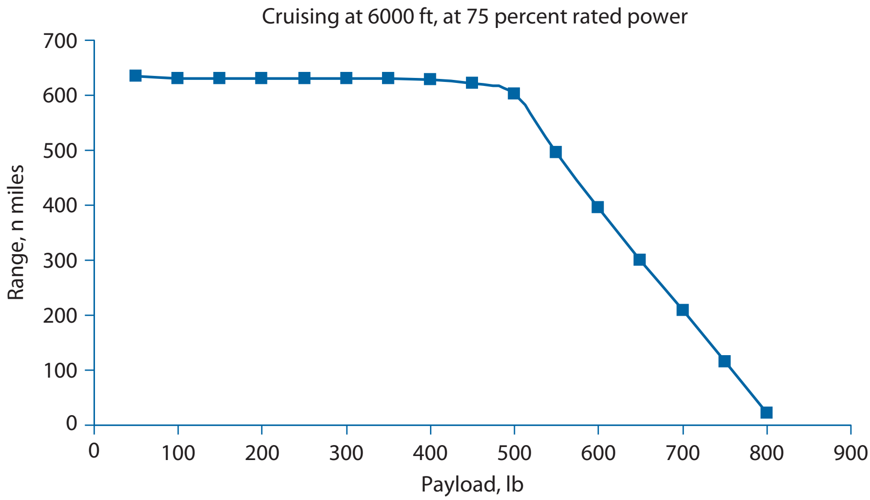

Fig. 7.2 Predicted range–payload curve for Cessna 172R.

interesting and educational such as NASA's Langley Research Center or the National Museum of the United States Air Force at the Wright-Patterson Air Force Base. As payment for the trip, they were obliged to supply their data to the rest of the class with a small visual presentation. Then everyone had to make predictions to compare with the measured values of expended fuel.

7.3 Summary

The classic Breguet equation has been developed for calculating the maximum range of an airplane. This usually results in a low cruising speed, and so, instead, the range was determined for operating at a constant fraction of the engine's rated power. The range–payload curve for an airplane was developed to show that when a certain takeoff weight is exceeded, the range will decrease rapidly as the weight is increased further. This is because fuel must be offloaded so that the maximum allowable gross weight is not exceeded.

Problems

7.1 Correct Fig. 7.2 to allow for a 45-min reserve to be flown at a speed for maximum endurance.

7.2 Redo Problem 7.1 using a cruise at 65 percent of the rated power used in Fig. 7.2.

Chapter 8 Static and Dynamic Stability and Control

- Understand Principles of Static Stability and Control
- Understand Principles of Dynamic Stability and Control

8.1 Introduction

Simply put, an airplane is stable, statically or dynamically, if it tends to return to its trimmed position after being disturbed. *Statically* means that, in essentially steady flight, if something such as a gust changes the angle of attack or angle of yaw, the resulting moments will tend to rotate the airplane back to its original static orientation. *Dynamically* means that if the motion of the airplane is disturbed, the ensuing unsteady aerodynamic forces and moments will tend to restore the airplane to its original flight condition.

8.2 Static Stability

There are elegant equations that can be found in many texts, including McCormick [1], to define and calculate static stability. However, a lot can be gained by looking at basic concepts. The whole airplane will be modeled by simply a wing and a tail, and we will consider only longitudinal stability (i.e., motion only in the vertical plane of symmetry). This neglects the lift and moments produced by the fuselage, nacelles, and propellers. These can be fairly important, but their effects are normally small. Consider Fig. 8.1, which shows a wing and a tail, the c.g., and their relationship to each other. Both lifting surfaces are represented by average chord lengths called *mean aerodynamic chords*.

If the airplane is in equilibrium, or trim, the total lift must equal the weight and the moments about the c.g. must balance. Thus,

$$L_w + L_t = W \tag{8.1}$$

$$L_w l_w + M_{\mathrm{ac}} - L_t l_t = 0 \tag{8.2}$$

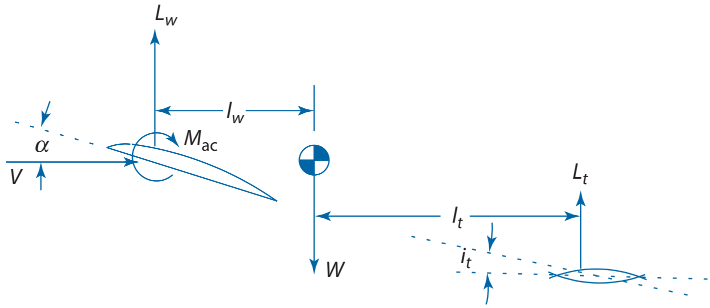

Fig. 8.1 Wing–tail model.

M_{ac} is the moment about the aerodynamic center for the wing. The aerodynamic center is located to the quarter-chord point. By definition the aerodynamic moment is constant about the aerodynamic center, independent of angle of attack. L_w and L_t are the lifts on the wing and tail respectively. W is the gross weight of the airplane acting downward at the c.g. as shown.

Suppose we now increase the angle of attack of the airplane so that the airplane is no longer in trim. Remembering that the pitching moment is defined positive nose-up, the untrimmed moment becomes

$$\Delta M = (L_w + \Delta L_w)l_w + (M_{ac} + \Delta M_{ac}) - (L_t + \Delta L_t)l_t$$

Here, a Δ denotes a change in each quantity. But ΔM_{ac} is zero (since M_{ac} by definition is constant), and the remaining unchanged terms balance out. Thus,

$$\Delta M = \Delta L_w l_w - \Delta L_t l_t \tag{8.3}$$

A lot can be learned about the static stability by simple reasoning. Look at Fig. 8.2, which, schematically, is a graph of the pitching moment, M, about the c.g. vs the angle of attack, α. We can reason that, when the airplane is flying in trim, M must equal zero and the angle of attack must be some positive number. Thus the point A must be along the M axis as shown. Two possible variations from the point A are shown. For the dashed line, M increases as α increases. For the solid line, M increases as α decreases. Which line do we want for static stability? The answer is obviously the solid line. As α increases above the trimmed value, we want a nose-down, or negative, moment to rotate the nose back down. Thus for static stability, the change in pitching moment for a change in α should be negative. Now consider the solid line for a zero α. At this point, the moment must be positive. But the wing lift is zero, which means that the tail lift at this point must be downward to give a nose-up pitching moment. Thus, we see the reason for the incidence angle, i_t, shown in Fig. 8.2. It is needed to trim an airplane that is statically stable. Usually, therefore, L_t is negative and is referred to as tail download.

If the c.g. is located at a location referred to as the neutral point, the aerodynamic moment about the c.g. will not change as the angle of attack increases, thus resulting in neutral static stability. The neutral point can be thought of as the aerodynamic center for the entire airplane. This point can be determined simply by setting Eq. (8.3) to zero and expressing the wing lift and tail lift in terms of lift coefficients.

$$\frac{1}{2}\rho V^2 S l_w \Delta C_{Lw} = \frac{1}{2}\rho V^2 S_t l_t \Delta C_{Lt}$$

The tail lies in the wake of the wing and, thus, experiences a downwash, w. To a small angle approximation, the downwash angle, ε, is given by w/V and varies linearly with the wing angle of attack. This rate of change is denoted by ε_α so that the angle of attack of the tail is $\alpha(1 - \varepsilon_\alpha)$. The change in C_L, ΔC_L, due to a change in α will equal the product of the slope of the lift curve and the change in α. Thus,

$$S l_w a_w = S_t l_t a_t (1 - \varepsilon_\alpha)$$

It follows that the neutral point will be the c.g. location for which the following holds true:

$$\frac{l_t}{l_w} = \frac{S a_w}{S_t a_t (1 - \varepsilon_\alpha)} \tag{8.4}$$

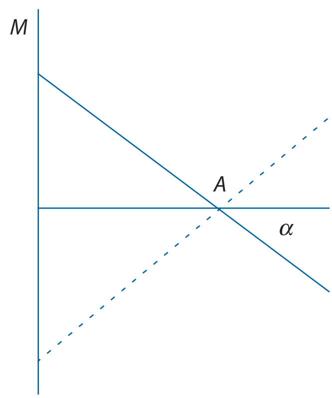

Fig. 8.2 Static pitching moment vs angle of attack.

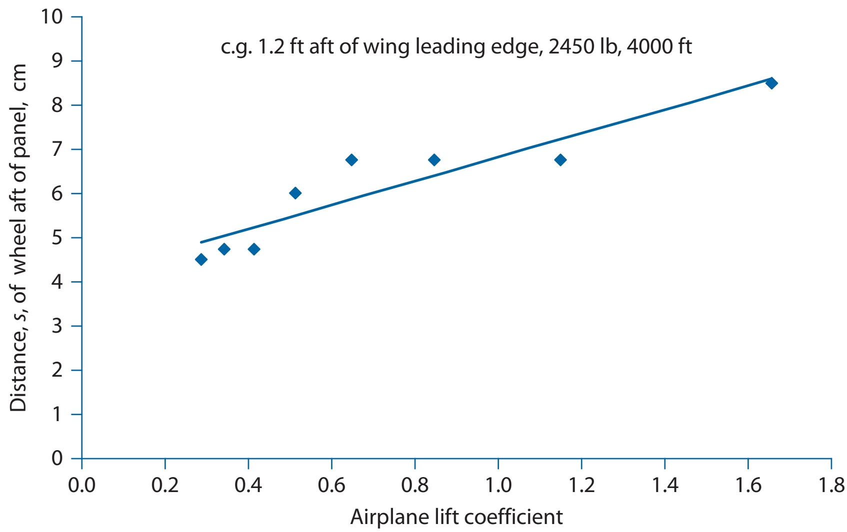

Fig. 8.3 Stick gradient for Cessna 172R.

Consider the Cessna 172R. Assuming that the two-dimensional lift curve is approximately the same for the wing and the tail, the ratio of a_w to a_t is determined from the correction to the two-dimensional slope for aspect ratio. The horizontal tail planform area is 36.1 ft^2 with a span of 11.3 ft, giving an aspect ratio for the tail of $A_T = 3.54$. From the geometry used previously, $A_w = 7.5$. In this chapter it will be shown that the slope of the lift curve for an airfoil is reduced when it is incorporated into a wing. If a is the two-dimensional slope then the three-dimensional slope is given by

$$\frac{a_w}{a} = \frac{A}{A + 2\left(\frac{A+4}{A+2}\right)} \tag{8.5}$$

The ε_α is typically around 0.4 for a light aircraft. This quantity is difficult to calculate and is best obtained from experiment. Analysis of this parameter can be found in McCormick [1]. Using the Cessna 172R values in Eq. (8.4), it is found that $a_w/a = 0.756$ and, for the neutral point, $l_t/l_w = 6.07$. But $l_w + l_t$ equals approximately 14 ft. Thus the distance of the neutral point behind the wing a.c. is predicted to be 1.98 ft.

The neutral point can be determined experimentally. Moving the stick fore and aft will change the trim angle of attack. Thus, plotting stick position vs trim C_L gives a measure of how the pitching moment varies with C_L. Since C_M does not vary when the c.g. is at the neutral point, it follows that the rate of change of trim C_L with stick position will be zero. Thus, to determine the neutral point, measure the stick position, s, for different airspeeds and plot s vs C_L. Repeat this process for different c.g. locations and graphically find the c.g. that results in no change in s as V varies.

Figure 8.3 presents student-measured test results where the stick position was measured over a range of trim airspeeds. The results are shown here with the distance of the wheel aft of the instrument panel plotted against the airplane lift coefficient. The slope of the curve is positive (i.e., the wheel is moved back to fly slower, and therefore the airplane is statically stable for this c.g. location.

8.3 Dynamic Stability

For most dynamic systems, one usually applies Newton's second law, $F = ma$, to the system. For an airplane, it is a little different because all of the aerodynamic forces are determined with respect to the airplane axes that are moving. Such axes are called *noninertial axes*. To illustrate this further, consider Fig. 8.4.

An airplane is pictured moving along a circular path. At time equal to zero, the airplane has velocity components of U and W along its x and z axes. These axes are aligned with fixed axes, x and z initially. Now picture the airplane at a small time, Δt, later. If Q is the pitch rate, the airplane has rotated nose-upward through a small angle $Q\Delta t$, and now the velocity along the airplane's x axis is $U + \Delta U$ and $W + \Delta W$ along the z axis. Relative to the fixed axes, the velocities are now $(U + \Delta U)\cos(Q\Delta t) + (W + \Delta W)\sin(Q\Delta t)$ along the fixed x axis and $(W + \Delta W)\cos(Q\Delta t) - (U + Q\Delta t)\sin(Q\Delta t)$ along the fixed z axis. The axes moving with the airplane will be chosen so that initially W is zero. Remembering that acceleration is equal to the change

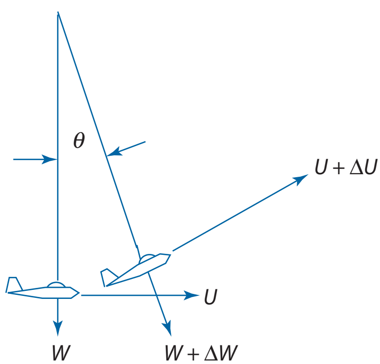

Fig. 8.4 Pitching airplane.

in velocity divided by the time increment, the acceleration on the fixed x axis becomes

$$a_x = \frac{(U + \Delta U)\cos(Q\Delta t) + (W + \Delta W)\sin(Q\Delta t) - U}{\Delta t}$$

As Δt becomes very small, $\cos(Q\Delta t)$ approaches 1.0 and $\sin(Q\Delta t)$ becomes the angle itself, $(Q\Delta t)$. Thus, the acceleration found in the equation relative to the fixed x axis becomes

$$a_x = \frac{\mathrm{d}u}{\mathrm{d}t} + QW \tag{8.6}$$

The term $\mathrm{d}u/\mathrm{d}t$ is the time rate of change of the velocity component along the airplane's x axis. Similarly, the acceleration along the fixed z axis is written

$$a_z = \frac{\mathrm{d}w}{\mathrm{d}t} - QU \tag{8.7}$$

Notice that the angular velocity, Q, causes the acceleration about a fixed axis to include an additional term equal to the product of Q and the velocity along the other airplane axis.

There is a third acceleration to be considered for longitudinal motion, namely the angular acceleration,

$$a_\theta = \mathrm{d}Q/\mathrm{d}t \tag{8.8}$$

The equations defining the longitudinal motion of an airplane are obtained by equating the forces divided by the mass to the linear accelerations and the moments divided by the mass moment of inertia to the angular acceleration. The formulation and solution of these equations is beyond the scope of this book. Those interested in delving deeper into this subject are directed to McCormick [1]. Let us, however, consider some interesting airplane behavior that is observed and is predicted by these equations.

To understand the concepts to be presented, consider first the simple mechanical system pictured in Fig. 8.5. Two pendulums are coupled together by a spring. If both pendulums are pulled to the same side and released they will swing as if they are not connected. If one is pulled in one direction and one in the other direction, the spring is stretched and, when released, the pendulums will swing back and forth, or oscillate, much faster than before. The two motions ensuing from these different initial displacements are called the *normal modes* of the system. The motion resulting from any arbitrary displacement of the pendulums will be a combination of the two normal modes.

The longitudinal motion of an airplane also displays two normal modes. The first is a short period, heavily damped mode and the other one is lightly damped with a long period. The long-period mode is called the phugoid mode. When an airplane is trimmed and then perturbed (by a gust, for instance), its airspeed, altitude, and pitch angle will all begin to oscillate. If the airplane is dynamically stable, the oscillation will damp out. If it is unstable, the oscillation will diverge. Normally, the short period is heavily damped with a very short period and the pilot will not feel it. The ups and downs of the phugoid are long enough, however, to be recognized by the pilot. The airplane's airspeed, altitude, and pitch angle will all oscillate but usually at a frequency and amplitude that can be easily controlled by the pilot.

An actual phugoid for a Cessna 172R is shown in Fig. 8.6. This trace was obtained using a small accelerometer linked to a PC, and the actual time is arbitrary. The airplane was trimmed at 4000 ft pressure altitude at 100 KCAS. Without changing the trim, the wheel was pulled back and held until it was climbing steadily at 80 KCAS. Then the wheel was returned rapidly to its trimmed position and held. The airplane begins to lose altitude and gain speed as it accelerates downward. After about 10 s it levels out and begins to climb and accelerate upward. The cycle is then repeated with the same timing but with smaller values of the acceleration. The period of the oscillation is the time from one peak to another. In this case it is about 25 s. A curve drawn through the peaks defines the amplitude of the vibration. In addition to the period, another important parameter defining the oscillation is the time for the amplitude to damp to half of its value. This time constant will be the same regardless of the reference point along the curve.

Denoting the vertical acceleration by x, the motion shown in Fig. 8.6 is of the form

$$x = Ae^{-at}\cos(\omega t)$$

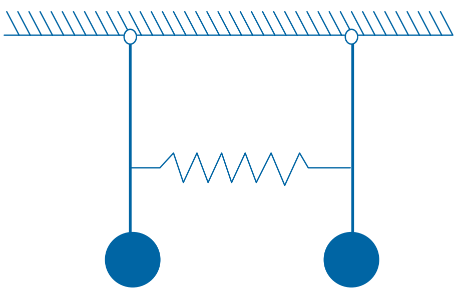

Fig. 8.5 Dynamic system with two degrees of freedom.

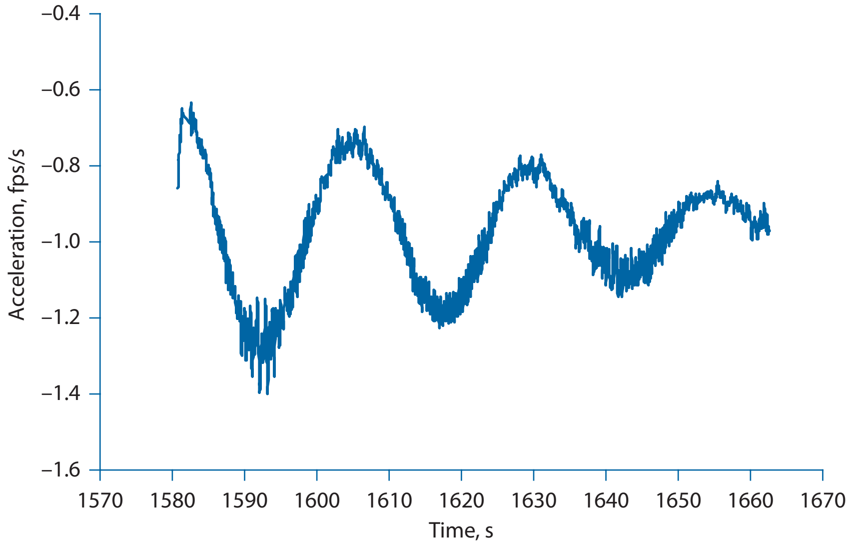

Fig. 8.6 Cessna 172R phugoid.

The circular frequency, ω, and the period, T, are related by $\omega T = 2\pi$ while the parameter, a, and the time-to-damp to one-half are related by

$$t_{1/2} = \ell n(2)/a \tag{8.9}$$

The value for Eq. (8.9) is easily obtained by letting the cosine term equal to one and solving for the time difference to go from one peak to another. As an example in the use of these relationships, consider Fig. 8.5 again. At a time of 1604 s, the amplitude, relative to -1.0, equals 0.24. At the next peak, the time equals 1629 s with an amplitude of 0.18. The period is the time between peaks, or 25 s. The damping is obtained by writing

$$0.25 = Ae^{-1604a}$$

$$0.18 = Ae^{-1629a}$$

Dividing the first equation by the second one gives $1.389 = e^{25a}$, or $a = 0.0131$. This gives a time-to-damp to one-half equal to 53 s. The results are based on only two points, and a more accurate approach is to curve-fit the entire curve as best as possible using a spreadsheet program. Figure 8.7 illustrates this approach for another test where the trim speed was about 80 KCAS with a 20 kt decrement. After some trial and error, a period of 32 s and a time-to-damp to one-half of 64 s were chosen as giving the best fit to the data.

One does not need an accelerometer to perform this experiment. A simple handheld recorder will do. When the wheel is pushed forward to its trimmed position, start calling out the airspeed and altitude continuously into the recorder. Then play the recordings back and time the callouts to obtain plots of altitude and airspeeds against time. These parameters have the same frequency and damping as the acceleration. The graphs may be a little rough, but they will disclose the basic behavior. One will find that the airspeed and altitude are 180 deg out of phase. When the altitude is at a high peak, the airspeed is at a low valley.

As a result of the aerodynamics and dynamics of the airplane, the phugoid is a motion at, essentially, a constant angle of attack. If α is assumed constant, then the equations of motion can be reduced to give a simple approximation for the phugoid. Assuming α to be constant results in a second-order differential equation, the roots of which are given by

$$\sigma^2 - X_U\sigma - \left(\frac{gZ_u}{U_o + Z_q}\right) = 0 \tag{8.10}$$

X_U is the change in the X force with respect to the x velocity divided by the airplane mass. Z_U is the same in the z direction. Z_q is the change in the Z force with respect to the pitch rate divided by the airplane's mass moment of inertia.

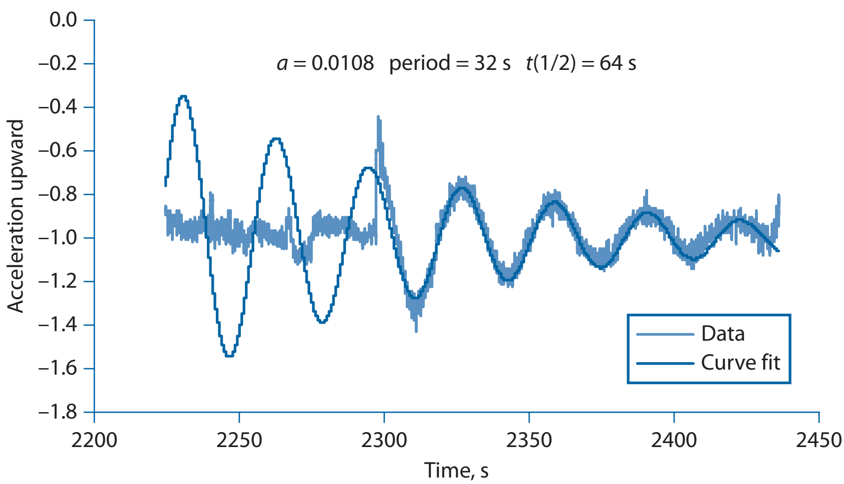

Fig. 8.7 Curve fit for Cessna 172R phugoid.

Solving Eq. (8.10) using the classic quadratic solution gives

$$\sigma = \frac{X_U}{2} \pm \frac{1}{2}\sqrt{X_u^2 + 4\left(\frac{gZ_u}{U_0 + Z_q}\right)} \tag{8.11}$$

McCormick [1] gives expressions for the stability derivatives as

$$X_U = -\frac{3}{2m}\rho U_0 C_{D0} S$$

$$Z_u = -\frac{1}{m}\rho U_0 S C_{L0}$$

$$Z_q = -\frac{1}{2m}\rho U_0 a_t S_t l_t$$

The dimensions and areas for the Cessna 172R are found in Table 3.3. As an example, the distance from the tail to the c.g., l_t, will be taken as 14 ft. This is the same as that for Fig. 8.3. Thus, for a standard altitude of 4000 ft, a weight of 2450 lb, and 90 KCAS, the stability derivatives become

$$X_U = -0.0600 \qquad Z_U = -0.403 \qquad Z_q = -3.63$$

Substituting these into Eq. (8.11) gives

$$\sigma = -0.0300 \pm 0.294\,i$$

Thus, the approximate predicted period equals $2\pi/0.294$, or 21.3 s, and the time-to-damp to one-half is $\ell n(2)/0.0300 = 23.1$ s. These numbers are significantly different from the experimental values shown in Fig. 8.7. This suggests that the approximation of a constant α for the phugoid is not accurate or that the simple model shown in Fig. 8.1 needs to be improved. Also, there are many places where the stability derivatives can be in error. All in all, it is rather difficult to predict the dynamics of an airplane. This has led to the development of a technique named *parameter identification* to extract the stability derivatives from experimental data.

8.4 Stick-Free Neutral Point

Before closing, one other facet of static stability should be mentioned. The neutral point as used thus far is, more specifically, the stick-fixed neutral point. The controls are not free to move as they wish in response to aerodynamic and inertial moments on them. There is another neutral point, called the stick-free neutral point. This point is aft of the stick-fixed neutral point. The situation is considerably more dangerous if the c.g. is located at the stick-free neutral point. At that location, it does not require any stick *force* to change the angle of attack. The airplane is nearly uncontrollable. To experimentally determine the stick-free neutral, one measures the stick force as a function of C_L for different c.g. locations. For a c.g. ahead of the stick-free neutral point, it will require a pull on the stick to fly slower. The slope of stick force vs C_L is plotted against c.g. and extrapolated to the value of c.g. that gives a zero slope.

8.5 Summary

Relationships treating both static and dynamic stability of an airplane have been presented. Techniques have been covered for studying both of these phenomena. It was shown that a c.g. location, known as the neutral point, exists such that the airplane will exhibit control reversal and static instability if the c.g. is placed behind this point.

Problems

8.1 An airplane has a wing loading of 40 psf. Measurements of stick position vs airspeed were taken for two c.g. locations with the results given in the table. From these results, calculate the location of the neutral point.

Trim airspeed	c.g. location	Stick position, *s*
100 KCAS	45 in.	5.0 in.
90 KCAS	45 in.	6.5 in.
100 KCAS	48 in.	4.5 in.
90 KCAS	48 in.	5.0 in.

8.2 An airplane has a period of 35 s and a 45-s time-to-damp to one-half. It is trimmed at 100 KTAS, but it was disturbed and begins to undergo a phugoid motion. At 10 s it has a peak velocity of 115 KTAS. What will its velocity be at a time of 38 s?

Appendix A General Data

A.1 Conversion Factors

Multiply	By	To get
pounds (lb)	4.448	newtons (N)
feet (ft)	0.3048	meters (m)
slugs	14.59	kilograms (kg)
horsepower (hp)	0.7457	kilowatts (kw)
pounds per square foot (psf)	47.88	pascals (Pa)
nautical mile (n mile)	6080	feet
knots (kt)	1.69	feet per second (fps)
miles per hour (mph)	1.467	feet per second (fps)

A.2 Standard Atmosphere

Altitude (h) ft	Temperature degrees R	Pressure psf	Density slugs/ft^3	Kinematic viscosity (ν) ft^2/s
0	518.7	2116	0.002377	0.0001572
5000	500.9	1761	0.002048	0.0001776
10,000	483.0	1456	0.001755	0.0002013
15,000	465.2	1195	0.001496	0.0002293
20,000	447.4	973.3	0.001267	0.0002623
25,000	429.6	786.3	0.001066	0.0003017
30,000	411.9	629.7	0.000891	0.0003488
35,000	394.1	499.3	0.000738	0.0004058
40,000	390.0	393.1	0.000587	0.0005056
Altitude m	**Temperature degrees K**	**Pressure N/m^3**	**Density kg/m^3**	**Kinematic viscosity m^2/s**
0	288.2	101.3	1.225	1.461E-05
1800	276.5	81.5	1.027	1.687E-05
3600	264.8	64.9	0.854	1.960E-05
5400	253.1	51.2	0.705	2.290E-05
7200	241.4	40.0	0.577	2.695E-05
9000	229.7	30.8	0.467	3.196E-05
10,800	218.1	23.4	0.374	3.820E-05
12,600	216.7	17.7	0.284	5.008E-05

A.3 Temperature and Perfect Gases

Degrees Rankine (R)	F + 460
Degrees Kelvin (K)	C + 273
Degrees centigrade (C)	C is (F − 32)(5/9)
Universal gas constant (p is ρRT)	R is 1716 for English system R is 287.0 for SI system

A.4 Definitions for the SI System of Units

newton (N)	Force that gives a mass of 1 kg and acceleration of 1 m/s/s
joule (J)	Work done when a force of 1 N moves 1 m
watt (W)	Power that produces energy at rate of 1 J/s
pascal (Pa)	1 N/m^2

Appendix B Instructions for Experiments

B.1 Experiment Standards

For every experiment, record the following:

1. Date
2. Airplane type and N number
3. Altimeter
4. Fuel gauges at beginning and end of recording data set
5. Weight of occupant of each seat

Each group is responsible for preparing data sheets and ensuring that the experiment is being flown properly. If students feel that the procedure being flown is not correct they should advise the pilot.

Important: Set altimeter to 29.92 inHg to read pressure altitude.

B.1.1 Experiment 1: Ground Roll, ASI Calibration, Steady Climb, Power-Required, and Lift Curve Slope

For this experiment, the ceiling should be at least 5000 ft or higher.

Ground Roll

1. Set brake at first runway light on south side of runway 24 or 6. Record headwind. Advance to full throttle. Select folder A and start voice recorder. (Look at directions for recorder.) Count “4, 3, 2, 1” and “release brake.” At the first sign of motion or tire rotation, say “mark” into recorder. Note initial engine rpm into voice recorder.
2. Call out “mark” as the second runway light is passed. Then “mark” again as each light is passed. Hold on ground until approximately 75 KIAS. Also, record the rpm and KIAS when the “mark” is called. Read the ASI and tachometer as accurately as possible.
3. Abort takeoff run, go back and repeat steps 1 and 2. Put record in folder B of voice recorder. However, this time, take off and climb to 800 ft above ground level.

ASI Calibration

4. Turn and fly downwind holding altitude, airspeed, and heading parallel to runway. Record time passing nearest end of runway. Then record time passing far end of runway.
5. Holding airspeed and altitude, turn 180 deg and fly upwind holding heading parallel to runway. Again record time interval to traverse runway length.

Steady Climb

6. After completing upwind pass of runway, climb out at WOT at a steady speed for best climb according to the POH. Climb to smooth air, preferably to an altitude of at least 2000 ft above ground level (AGL) or higher. Record pressure altitude, OAT, KIAS, and rpm every minute up to maximum altitude. Record fuel at end of climb.

Power-Required

7. Search for the airspeed where stall begins. Then trim the airplane in level flight at a speed just above the stalling speed. Make it a number divisible by five. When satisfied that the airplane is trimmed in level flight, go to next step.
8. Holding the trim and not touching the throttle, start timing and record the altitude. After 30 s, record rpm, KIAS, pressure altitude, and OAT. After 60 s, record the altitude.
9. Increase the trim airspeed by 5 kt and repeat the procedure in step 7. At each speed, measure and record the pitch angle by placing a protractor with a bubble level on a flat surface such as a windowsill.
10. Repeat step 9 up to the maximum speed at WOT. Record fuel at end of last test.

B.1.2 Experiment 2: Rate-of-Climb and Stall

Rate-of-Climb

1. From experience, someone should have an idea of the speed for best climb. Or, with WOT fly the airplane quickly over a range of airspeeds and note the R/C meter to find approximately a speed for best climb. At approximately 500 ft below test altitude, reduce speed to 10 or 20 kt below speed for best climb—make it a round number such as 60 or 70 kt. The speed should be high enough so you can hold a trim speed without difficulty. The airplane should be trimmed at a constant airspeed, then go to WOT and hold the airspeed. The airplane will climb.
2. Begin timing when you reach a trimmed, steady state somewhere below the test altitude, preferably 200 or 300 ft below. Once timing begins don't

touch the throttle; hold the airspeed constant. At start of timing ($t = 0$) record altitude. After 30 s, record the altitude, OAT, IAS, and rpm. Exactly after 1 min, record the altitude. During the run, note any gusts encountered. Read ASI and tachometer as accurately as possible.

3. Drop back down to the initial altitude and repeat steps 7–9 from Experiment 1 over a range of airspeeds in approximately 5-kt increments up to a speed where a noticeable drop in R/C occurs. Each group should fly at different altitudes for R/C tests. If time permits, repeat data points.

Stall

4. Cut tufts of black yarn approximately 3 in. long. This can be done quickly by wrapping yarn around a paper tube and then cutting the tube lengthwise. Place a strip of masking tape, sticky-side up and about 3 ft long, on a table and then place the tufts on the tape about 3 in. apart. Then place the tufted strips on one wing to show the stall pattern. (See Fig. 6.1.)
5. At a safe altitude, with flaps up, trim the airplane to around 70 KIAS. Then come back all the way on the power and then back on the wheel to maintain altitude as the airplane slows down. At some point the airplane will stall. It may nose down sharply or it may begin to rock. If it rocks, hold the wheel back and record KIAS, altitude, and OAT. If the nose drops, note the same quantities at the time it dropped. If the airplane has a low wing, take photos of the tufts to show the stall progression from the trailing edge to the leading edge.
6. Repeat step 5 at each flap setting.

B.1.3 Experiment 3: Static and Dynamic Stability

Dynamic Stability: Phugoid

1. Make sure you have a notebook PC, an accelerometer, and a flash drive.
2. Record the usual numbers to determine gross weight and c.g. position.
3. Set altimeter to 29.92 inHg.
4. Climb to a comfortable altitude, at least 2000 ft AGL, where the air is smooth.
5. Trim airplane at around 80 KIAS. Then, the pilot should grasp the wheel column against the instrument panel and slowly pull it out until plane slows to a *steady* 20 KIAS below the trim speed, making sure to keep the position on the column for trim speed. Don't change the trim setting or power. At this point the airplane will probably be climbing. Now, thrust the column rapidly forward holding it steady at the trimmed position. At the same time the recording of the accelerometer output should begin and continue until the oscillations damp out. Again, do not change the

trim during this procedure, and keep the wings level with the rudder. Stay off the wheel. When the wheel is pushed forward the airplane will nose down and build up speed. Without any control input, it will bottom out, nose up, and begin to slow down. This oscillation will be measurable for several cycles but eventually it will damp out. As a backup to the accelerometer, call out the airspeed and altitude continuously into handheld recorders and time the callouts to obtain a graph of these state parameters against time.

6. Repeat step 5 at another trim speed of approximately 100 KIAS. Copy your files to a storage device.

Static Stability: Stick Position

7. Again, at a good altitude, slow the airplane to the lowest speed at which it can be positively controlled.
8. Using a gauge, measure the distance from the instrument panel to the back, flat face of the control wheel.
9. Increase the speed about 10 kt and repeat the measurement. For this test, the longitudinal trim can be used to trim the airplane. Do this procedure up to approximately *V*-maximum and plot stick position vs KIAS.

Appendix C Solutions to Problems

1.1 An airplane has a wing loading of 100 psf and is flying at a TAS of 200 kt at 10,000 ft standard. What is the airplane's lift coefficient?

Solution:

$$C_L = \frac{W/S}{q} = \frac{100}{\frac{1}{2}(0.001756)[(200)(1.69)]^2} = 0.997$$

1.2 The airplane in Problem 1.1 is flying at a KIAS of 200 with an OAT of 10°F. What is the airplane's C_L?

Solution: Assuming no ASI error, $q = \rho_0(200 \times 1.69)^2/2 = 136.0$ psf. $C_L = W/S/q = 100/136.0 = 0.735$

1.3 An airplane weighs 40,000 N and is flying at 2000 m standard. If the airplane C_L equals 0.3 and the TAS equals 150 kt, what is the wing planform area?

Solution: $V = 150(0.5151) = 77.3$ m/s $\rho = 1.007$ kg/m^3

$$C_L = \frac{2W}{\rho V^2 S} \quad \text{or}$$

$$S = \frac{2W}{\rho V^2 C_L} = \frac{2(40{,}000)}{(1.007)(77.3)^2(0.3)} = 44.3\ \text{m}^2$$

1.4 A Cessna 172 with a wing span of 36 ft is making an approach at 90 KTAS when it becomes aligned with the center of one vortex trailing from a B-767 that has a span of 156 ft a gross weight of 387,000 lb and is flying at 135 KTAS. How much would the angle of attack of the Cessna's wing be increased at the left tip by the B-767's vortex trailing from its left wing? The pressure altitude is 2000 ft and the OAT equals 60° F.

Solution:

B-767:

$$\text{distance between trailing vortices} = \pi b/4 = 122.5\,\text{ft}$$
$$\text{TAS} = V = 1.69(135) = 228\,\text{fps}$$
$$\text{at 2000 ft, } p = 1974\,\text{psf (by interpolating Table A.2)}$$
$$T = 60 + 460 = 520^\circ\text{R}$$
$$\text{from equation of state, } \rho = p/\text{R}/\text{T} = (1977)/(1716)/(520)$$
$$= 0.00221\,\text{slugs/cu. ft}$$
$$\Gamma_0 = \frac{W/b}{\pi\rho V} = \frac{4(387{,}000/156)}{\pi(.00221)(228)} = 6269\,\text{ft}^2/\text{s}$$

Cessna 172:

$$\text{TAS} = v = 1.69(90) = 152.1\,\text{fps}$$

The left wing tip of the Cessna 172 is 43.3 ft to the right of the B-767 left wing tip. Therefore, the downwash, w, at the Cessna's tip from the B-767's vortex can be calculated as:

$$w = \frac{\Gamma_0}{2\pi r} = \frac{6269}{2(3.142)(43.3)} = 23.0\,\text{fps}$$

This downward velocity is added vectorially to the forward velocity of the Cessna 172 to give an upward flow of the resultant velocity at an angle of $\tan^{-1}(23.3/152.1) = 8.7$ deg. Thus the angle of attack at the tip is increased by this amount due the B-767's trailing vortex. Actually, the angle here will be larger because the vortex from the B-767's other tip will also induce a downwash.

1.5 Each vortex of a pair of trailing vortices induce a downward velocity on the other given by Eq. (1.8). How rapidly would you predict the pair of vortices from the B-767 in Problem 1.4 to descend?

Solution: From Problem 1.4, the strength of each trailing vortex equals 6269 ft^2/s. The distance from one vortex to the other is 122.5 ft. Thus the velocity induced downward at one vortex by the other is

$$w = \frac{\Gamma_0}{2\pi r} = \frac{6269}{2\pi\,(122.5)} = 8.14\,\text{fps}$$

This is the velocity at which the pair will initially descend. Ultimately, viscosity and turbulent diffusion will slow it down. Empirically, vortices from large airplanes do not descend more than approximately 1000 ft.

2.1 Prepare a data sheet for the ASI calibration flight test. The sheet should allow for input on airplane ID, payload, weights, distances, headings, times, and temperatures.

DATA SHEET

DATE_____________ TIME _________ GROUP NO.___ NAMES________________________
AIRPLANE MODEL________ N__________ ________________________
WEIGHTS: FRONT SEATS_______ ________________________
REAR SEATS _______
ALTIMETER _________ (**AFTER READING, SET ALTIMETER TO 29.92)**
REMARKS____

MARKER DIST. FT	RUN #1 TIME TO MARKER	RUN #2 TIME TO MARKER	AIRSPEED KIAS	RPM

____
____
____
____
____
____

GROUND ROLL, AIRSPEED AND R/C EXPERIMENTS
RUNWAY LENGTH
__________ FT

TIME	ALTITUDE

WIND _______KT FROM
________DEGS

$h_{p,}$ ALTITUDE __________ FT

KIAS ______ OAT_____^{0}F

DOWNWIND TIME________

UPWIND TIME ________

2.2 An ASI flight test is performed and the following data are obtained:

measured distance	2 n miles
pressure altitude	10,000 ft
OAT	50°F
headings	060° and 240°
time to fly first heading	61.2 s
time to fly reciprocal heading	53.2 s

What is the IAS in knots if the ratio of CAS to IAS is 1.05?

Solution:

$$TAS = \frac{s}{t} = \frac{2}{\frac{(61.2+53.2)}{2}} 3600 = 125.8\,\text{kt}$$

At 10,000 ft, standard $p = 1455.5$ psf. Therefore,

$$\rho = \frac{p}{RT} = \frac{1455.5}{1716(50+460)} = 0.00166\,\text{slugs/cu. ft.}$$

Now $\text{TAS} = \text{CAS}/(\sigma)^{1/2}$ and $\sigma = 0.00166/0.00238 = 0.697$. Thus $\text{CAS} = (125.8)(0.697)^{1/2} = 105.0$ kt, and $\text{IAS} = 105.0/1.05 = 100$ kt.

2.3 In Problem 2.2, assume that the difference between CAS and IAS is due only to position error. If the pilot reads a pressure of 10,000 ft, what is his or her actual pressure altitude? (Remember: The altimeter and ASI are connected to the same static pressure source.)

Solution: The density for Problem 2.2 is 0.00166 slugs/cu. ft and the TAS equals 125.8 kts or 212.6 fps. The IAS is 100 kt or 169 fps. Writing Bernoulli's equation from the freestream to the point where the static pressure is sensed gives

$$p + \frac{\rho_0 V_i^2}{2} = p_\infty + \frac{\rho V_T^2}{2}$$

or

$$p - p_\infty = \frac{(0.00166)(212.6)^2}{2} - \frac{(0.00238)(169)^2}{2} = 3.52\,\text{psf}$$

Because the pressure at the static pressure is higher than the freestream static pressure, the altimeter is reading lower by

$$\Delta h = \Delta p/g/\rho = 3.52/32.2/0.00166 = 66\,\text{ft}$$

3.1 Estimate the total takeoff distance for the Cessna 172R over a 50-ft obstacle in Denver, Colorado, where the standard altitude is 5300 ft.

Solution: Using a standard atmosphere, the mass density, ρ, at Denver is found from Eqs. 1.2 and 1.3 and a lapse rate of 3.57° R per 1000 ft to equal 0.00203 slugs/cu. ft. This number can also be obtained approximately from Fig. 1.1.

The approximate method for calculating ground roll distance presented in Sect. 3.10 will be used in lieu of a computer program. To begin, we must obtain a stall speed so a reasonable value of $C_{L_{\max}}$ will be assumed. For the Cessna 172R, the following numbers are given:

$$W = 2450\,\text{lb} \quad \text{propeller} = 6.25\,\text{ft}$$

$$S = 174\,\text{ft}^2$$

$$A = 7.5 \qquad e = 0.8$$

The wing loading is W/S or 14.08 psf. Because $C_{L_{\max}} = (W/S)/q$, the dynamic pressure q equals 8.28. But

$$q = \frac{\rho V^2}{2}$$

Therefore

$$V_S = \sqrt{\frac{2q}{\rho}} \text{ or } V_S = \sqrt{\frac{2(8.28)}{0.00203}} = 90.3\,\text{fps}$$

The airplane will lift off at 30% above this speed or 117.4 fps.

To use the method of Sect. 3.10, an average acceleration is calculated at the liftoff speed divided by $(2)^{1/2}$ or a speed of 83 fps. As stated in Chapter 3, it is reasonable to assume a C_L of 0.4 for the 172R when rolling along the runway. The C_{D_i} when rolling is thus $C_L^2/(\pi Ae)$. With e = 0.8, $C_{D_i} = 0.0594$. For this average condition, $q = (.00203)(83)^2/2.0 = 6.99$ so the induced drag, D_i equals qSC_{D_i}, or 72.2 lb. Because of ground effect this drag is approximately halved to 36.1 lb. The parasite drag equals qf or $(6.99)(6) = 41.94$ lb. The lift, for the rolling C_L equals qSC_L or 486.5 lb. The net force on the wheels is equal to the weight minus the lift, or 1963 lb. The rolling friction equals the product of the coefficient of rolling friction and the force on the wheels, or, $(0.02)(1963) = 39.3$ lb. Thus the total retarding force, F_R, is

$$F_R = 39.3 + 41.94 + 36.1 = 117.3\,\text{lb}$$

The accelerating thrust, T, is estimated from Fig. 3.13 by observing that the RPM during ground roll is nearly constant (Fig. 3.15) at around 2250 RPM. At the average speed, this gives a propeller advance ratio of $J = 83/(2250/60)/(6.25) = 0.35$. From Fig. 3.13, $C_T = 0.092$ and $C_P = 0.076$. This gives a propeller efficiency of $\eta = (C_T)(J)/C_P = 0.42$. From Fig. 3.3, the engine BHP is read as 130 for this RPM and altitude. Multiplying this power by the efficiency results in $\text{THP} = \eta\text{BHP} = 54.6$. But $\text{THP} = \text{TV}/550$ so that $T = (54.6)(550)/83 = 361.8$ lb. The net accelerating force is $T - T_R$ or 244.5 lb. Thus, the average acceleration during the ground roll equals this force divided by the mass, or $a = 244.5/(2450/32.2) = 3.21$ fps/s. It takes 117.4/3.21 or 36.6 s to attain the liftoff speed of 117.4 fps. The ground roll distance corresponding to this time and acceleration will be $s = at^2/2 = (3.21)(36.3)^2/2 = 2150$ ft.

At the instant of liftoff, $v = 117.4$ fps and RPM = 2250. The propeller J now becomes 0.50 and $C_T = 0.085$ and $C_P = 0.074$. The thrust for climbing now becomes $T = C_T \rho n^2 D^4 = (0.085)(0.00203)(2250/60)^2(6.25)^4 = 370.3$ lb. During climb there is no rolling friction and the $C_L = W/(qS)$. For the takeoff speed, $q = 13.98$ psf and $C_L = 1.0$. Thus, $C_{D_i} = 0.0424$ resulting in an induced drag of 103.2 lb. The parasite drag, qf, is 83.88 lb.

Thus the excess thrust to climb is $T - D$ or 183.3 lb. The rate of climb, R/C, equals this excess thrust multiplied by the velocity and divided by the weight.

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{(T - D)V}{W} = \frac{(183.3)(117.4)}{2450} = 8.78\,\text{fps}$$

At this R/C, it will take 5.69 seconds to climb 50 ft. During this time the airplane will travel forward (117.4)(5.69) or 668 ft. Thus the total distance to roll and climb is estimated to be $s = 2150 + 668 = 2818$ ft.

3.2 Formulate a computer program and validate Figs 3.14, 3.15, and 3.16.

Hints:

1. Using Fig. 3.3, engine chart, plot WOT power at SSL vs rpm and get equation for straight trend line using Excel.

2. Plot rate of decrease of WOT power with altitude and get equation of straight trend line using Excel.
3. From above. Get equation for WOT engine brake horsepower as function of rpm and altitude.
4. Plot propeller using Excel and get equations for trendlines for C_T and C_P vs J.
5. For a given V, iterate using equations for engine power and propeller power until powers match. This gives prop operating J.
6. Calculate propeller thrust, drag, lift, and frictional forces leading to acceleration. Use integration scheme such as that shown in Problem 3.4 to integrate ground roll.

3.3 A propeller has a 5.5-ft diam propeller and is powered by a 300-bhp engine. What is the maximum static thrust that the propeller can produce at SSL conditions?

Solution: The momentum theory of propellers predicts the maximum performance of a propeller.

Propeller disc area: $A = \pi D^2/4 = 23.75$

Induced velocity, w: $T = 2\rho A w^2$

Power: $P = Tw = (550)(300) = 165{,}000$ ft-lb/s

Therefore: $P = T^{3/2}/(2\rho A)^{1/2}$ or $T = P^{2/3}(2\rho A)^{1/3}$

Or $T = (165000)^{2/3}(2 \times 0.00238 \times 23.75)^{1/3} = 1454$ lb

3.4 Here is an exercise in numerically integrating equations of motion. The drag coefficient of a smooth golf ball is 0.4. When dimpled, C_D reduces to 0.17. A golfer drives a ball and it leaves the ground at an initial angle of 30 deg above the level and at some initial velocity. Under these conditions, the golfer can drive a dimpled ball 200 yd. How much less would the golfer drive it if the ball were smooth? Use SSL conditions. A golf ball has a diameter of 1.68 in. and weighs 1.62 oz.

Hints:

1. Use $F = ma$ in range and height direction and resolve drag and weight in those directions.
2. For numerical integration,

$$x(t + \Delta t) = x(t) + \frac{dx}{dt}\Delta t + \frac{dv}{dt}\frac{(\Delta t)^2}{2}$$

$$v(v + \Delta t) = v(t) + \frac{dv}{dt}\Delta t$$

3. Use $\Delta t = 0.01$ s but only print out results every 0.1 s.
4. Check to see if $h < 0$. If so, check range and increase initial resultant velocity if range less than 200 yd. (An initial velocity of 200 fps will give 200 yd.)

3.5 Given: Cessna 172R propeller

hp	5000 ft
IAS	60 kt
rpm	2400

Find: The amount of thrust and power required by the propeller.

Solution: At 5000 ft, $\rho = 0.002048$, $\sigma = 0.861$,
$V = 60 \times 1.69/(0.861)^{0.5} = 94$ fps
$n = 2400/60 = 40$ and $J = V/n/D = 0.38$, therefore,
$C_T = 0.077$, $C_P = 0.050$, and $\eta = 0.585$
$T = \rho n^2 D^4 C_T = 365.7$ lb
$= HP = \rho n^3 D^5 C_P/550. = 106.5$ hp

3.6 Show that the work done on a system equals the change in the total energy of the system.

Solution: This requires the use of calculus. Referring to Fig. C1, a force is inclined upward through the angle ϕ and is acting on a

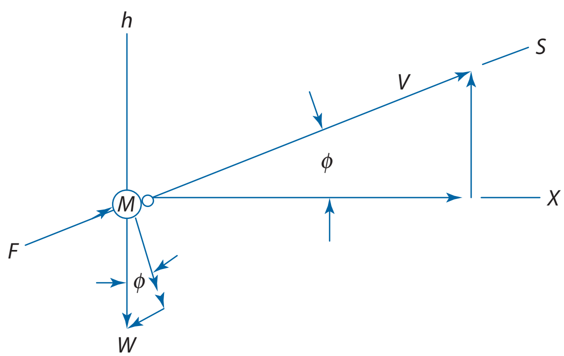

Fig. C1 Inclined force acting on a mass.

mass. In the s-direction,

$$F - W \sin \phi = m\frac{dV}{dt}$$

But,

$$\frac{dV}{dt} = \frac{dV}{ds}\frac{ds}{dt} = V\frac{dV}{ds} = \frac{1}{2}\frac{dV^2}{ds}$$

Thus,

$$F - W \sin \phi = \frac{d\left(\frac{1}{2}mV^2\right)}{ds}$$

Multiplying the above by ds and using the facts that

$$\int F ds = \text{work}$$

$$ds \sin \phi = dh$$

$$\int W dh = \text{change potentialenergy}$$

$$\int d\left(\frac{1}{2}\rho V^2\right) = \text{change in kineticenergy}$$

leads to the desired proof.

4.1 An airplane is flying at a pressure altitude of 6000 ft, where the OAT is 10°F. It has an operating weight of 5350 lb, but its standard weight is 6000 lb. The airplane is traveling at an airspeed of 180 KCAS, and the thrust horsepower required at these conditions is 450 hp. At what airspeed and thrust horsepower should the data be plotted at SSL conditions at the standard gross weight?

Solution: The standard temperature for 6000 ft is found from the lapse rate of 3.57°R per 1000 ft. Thus, $T = 519 - (6 \times 3.57)$, or 498°R. This gives a standard temperature ratio, θ, of 0.960. From Eq. (1.3) this gives a pressure ratio corresponding to the pressure altitude of $\delta = 0.807$, or a pressure of 1707 psf.

We now use the equation of state, $p = \rho RT$ (Eq. [2.2]), to get the density. Using the nonstandard temperature,

$$\rho = \frac{p}{RT} = \frac{1707}{(1716)(10 + 460)} = 0.00212 \text{ slugs/ft}^2$$

This gives a density ratio, $\sigma = 0.891$. The true airspeed is arrived at by dividing the KCAS by the square root of the density ratio. The ratio of the standard weight to the operating weight equals 6000/5350, or 1.121. Thus, from Eq. (4.3), remembering that the calibrated airspeed equals the true airspeed multiplied by the square root of σ, the equivalent airspeed becomes

$$V_{\text{ew}} = V\sqrt{\sigma}\sqrt{\frac{W_S}{W}} = (180)(1.121) = 202\,\text{kt}$$

The equivalent power is obtained from Eq.(4.4):

$$\frac{P_{\text{ew}}}{P} = \sqrt{\sigma}\left(\frac{W_S}{W}\right)^{3/2} = \sqrt{0.891}(1.121)^{3/2} = 1.120$$

or

$$HP_{\text{ew}} = 504$$

4.2 What is the minimum thrust horsepower required for the airplane in Problem 4.1 at the given altitude and OAT? At what airspeed will this occur? To do this problem, use the geometry of the Cessna 208B Caravan. You can find the information in *Jane's All the World's Aircraft, 1991–1992.*

Solution: From *Jane's*, $b = 52.1$ ft and $A = 9.6$. Using the density found above, the true airspeed is $(180)/(0.891)^{0.5} =$ 190.7 kt, or 322 fps. The dynamic pressure, q, is $q = 0.5 \times 0.00212 \times 322^2 = 109.9$ fps. Now the thrust horsepower $=$ $DV/550$. Thus, $D = 450 \times 550/322 = 769$ lb.

Equation (4.7) can be written as

$$D = qf + \frac{(W/b)^2}{q\pi e}$$

We will assume that Oswald's efficiency is equal to 0.8, a reasonable value for high wing airplanes. Thus the equation becomes

$$769 = 109.9f + \frac{(5350/52.1)^2}{(109.9)(\pi)(0.8)}$$

Solving the above for f gives $f = 6.65$ ft. We can now solve from Eq. (4.9) for the velocity for minimum power:

$$V = \left[\frac{4(W/b)^2}{3\pi\rho^2 ef}\right]^{1/4} = \left[\frac{4(5350/52.1)^2}{3\pi(0.00212)^2(0.8)(6.65)}\right]^{1/4}$$
$$= 117.0 \text{ fps}$$

Thus, the true airspeed for minimum thrust horsepower is predicted to be 69.2 kt.

5.1 An airplane has a constant speed propeller that maintains an efficiency of 85 percent over its in-flight operating range. It is an aerodynamically clean, low-wing design with retractable gear. The engine develops 300 shp at SSL and the power decreases linearly with the density ratio at altitude. It weighs 3500 lb, has a wing planform area of 225 ft^2, and a total wetted area of 1000 ft^2. The wing's aspect ratio equals 7.0.

Find:

a) the service ceiling.
b) the time to climb from 2000 ft to 8000 ft.

Solution: From Table 3.2, $C_f = 0.0067$. Therefore, $f = C_f S = 6.7$ ft^2. Assume $e = 0.8$. At SSL, $\rho = 0.00238$ slugs/ft^3. Therefore, with V in knots,

$$q = \frac{0.002378(V\text{kt} \times 1.69)^2}{2} = 0.00340\, V\text{kt}^2$$

The lift coefficient becomes $C_L = (W/S)/q = 4575/V\text{kt}^2$, and the induced drag coefficient $C_{Di} = C_L^2/\pi/AR = (4575)^2/\pi/7.0/V\text{kt}^4 = 0.00951/(V\text{kt}/100)^4$. Thus the drag at SSL in terms of Vkt becomes

$$D = qf + qSC_{Di} = 228(Vkt/100)^2 + 72.8/(Vkt/100)^2$$

Multiplying the equation by the speed (in fps) and dividing by 550 gives the thrust horsepower to maintain level flight:

$$\text{thrust horsepower} = D(1.69\, V\text{kt})$$
$$= 70.1(Vkt/100)^3 + 22.4/(Vkt/100)$$

This power has a minimum value for $V\text{kt} = 57.0$. This number can be found graphically or by setting the derivative of P with respect to V equal to zero and solving for V kt. Substituting this value into thrust horsepower gives a minimum power to maintain level flight at SSL of 61.8 hp. The thrust horsepower available at SSL will be the shaft horsepower multiplied by the propeller efficiency, or 255 hp. The excess power for climb is the difference between the available power and the required power, or 106,260 ft · lb/s. Equating this to the work done in raising the weight of the aircraft at the R/C (Eq. [5.1]) gives, at SSL, R/C = 1822 fpm.

Now repeat the preceding at, say, 10,000 ft standard. For 10,000 ft., $\rho = 0.00176$ slugs/ft^3. Here, the minimum thp = 61 at 71.8 kt. The engine brake horsepower, assuming it is proportional to the density ratio, becomes 222 bhp. Including the propeller efficiency gives an excess thrust horsepower to climb of 116.9 hp. Again, using Eq. (5.1), the R/C at 10,000 ft is predicted to be 1102 fpm.

From these two altitudes, the absolute ceiling is found to be 25,306 ft. Thus the R/C becomes

$$R/C = 1822\left(1 - \frac{h}{25306}\right)$$

a) Setting R/C equal to 100 fpm gives a service ceiling of 23917 ft.
b) The time to climb from 200 to 8000 ft is found by using Eq. (5.9). The time to climb to 2000 ft (from Eq. [5.9]) equals 1.14 min. To climb to 8000 ft requires 5.28 min. Therefore, the time to climb from 2000 to 8000 ft is the difference of 4.14 min.

5.2 An airplane weighs 2000 lb and is flying at 90 kt. It requires 85 thp to fly at this trimmed condition but, with WOT, the thp available equals 140. The pilot opens the throttle wide and begins to accelerate at a magnitude of 0.15 g. How fast is the airplane climbing in fpm at that instant?

Solution: Equation (5.10) reads

$$TV - DV = W(\mathrm{R/C}) + \frac{\mathrm{d}\left(\frac{1}{2}mV^2\right)}{\mathrm{d}t}$$

But $TV = 550(140)$ and $DV = 85(550)$ and

$$\frac{\mathrm{d}\left(\frac{1}{2}mV^2\right)}{\mathrm{d}t} = mV\frac{\mathrm{d}V}{\mathrm{d}t}$$

dV/dt is given as 0.15×32.2 and $m = W/32.2$. Thus, substituting these numbers into the equation and solving for the R/C gives R/C $= -461$ fpm. Surprise! The pilot chose to accelerate when he or she opened the throttle, and instead of climbing, the airplane descended.

6.1 At some point on a wing, the basic C_l equals -0.3 and the additional C_l equals 0.8. If the section maximum C_l equals 1.5 at that point, at what wing C_L will the section first stall?

Solution: The section lift coefficient is given by Eq. (6.17). Thus for this section,

$$C_l = -0.3 + 0.8C_L$$

Equating the section lift coefficient to 1.5 and solving for the wing lift coefficient gives $C_L = 2.25$, at which that station will first stall.

6.2 Given a cambered airfoil. The slope of its lift curve is 0.106 per degree and the angle for zero-lift equals -5 deg. It is equipped with a 25 percent double-slotted chord flap. What is the predicted lift coefficient for the airfoil at an angle of attack of 3 deg with the flap lowered 35 deg?

Solution: From Fig. 6.7, theoretically, $\tau = 0.6$. The correction factor to τ is $\eta = 0.8$, giving an actual τ of 0.48. Thus the effective increase in the angle of attack due to lowering the flap is $\tau\eta\delta_f$, or 16.8 deg. At 3 deg angle of attack, the angle of attack of the zero-lift line is 8 deg. Added to the change in α due to the flap gives an angle of attack of 24.8 deg. Multiplying this by the slope of the lift curve results in a final airfoil lift coefficient of 2.63.

6.3 An airplane weighs 2500 lb with a wing area of 200 ft^2. The M_{ac} equals 4 ft. The c.g. lies 1.2 ft ahead of the a.c. The distance of the tail a.c. aft of the c.g. is 15 ft. The airplane is flying at 5000 ft. The wing has a $C_{M_{ac}}$ of -0.05 and a $C_{L_{max}}$ of 1.4. At what calibrated airspeed in knots will the airplane stall?

Solution: Using Eq. (6.11),

$$C_{L_{max}} = 1.4\left[\frac{1.2 + 15}{15}\right] + (-0.05)\frac{4}{15} = 1.5$$

6.4 Show that a flat, untwisted wing with an elliptical planform will have an elliptical lift distribution.

Solution:

$$\frac{dL}{dy} = qcC_l = qc_0\sqrt{1-\left(\frac{2y}{b}\right)^2}C_l$$

Since C_l is constant,

$$\frac{dL}{dy} = (qc_0C_{l_0})\sqrt{1-\left(\frac{2y}{b}\right)^2}$$

Thus, dL/dy is elliptical since $(qc_0C_{l_0})$ is constant.

6.5 Prove that the wing C_L equals the section C_l if C_l is constant along the span.

Proof:

$$L = \int qcC_l dy = qSC_L = qC_l\int c dy = qC_lS \quad or \quad C_L = C_l$$

7.1 Correct Fig. 7.2 to allow for a 45-min reserve to be flown at a speed for maximum endurance.

Solution: Referring to Fig. 7.2, the range–payload curve breaks due to the off-loading of fuel at for a payload of approximately

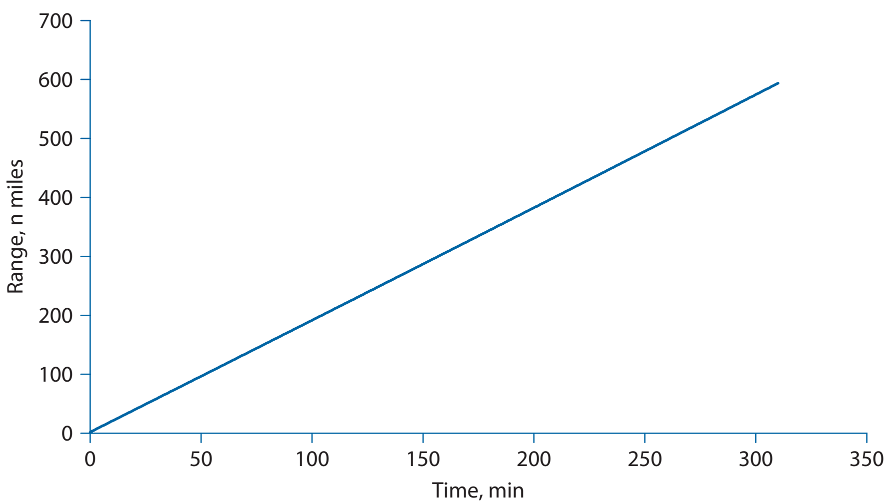

Fig. C2 Range vs time for 500-lb payload, 75-percent rated power.

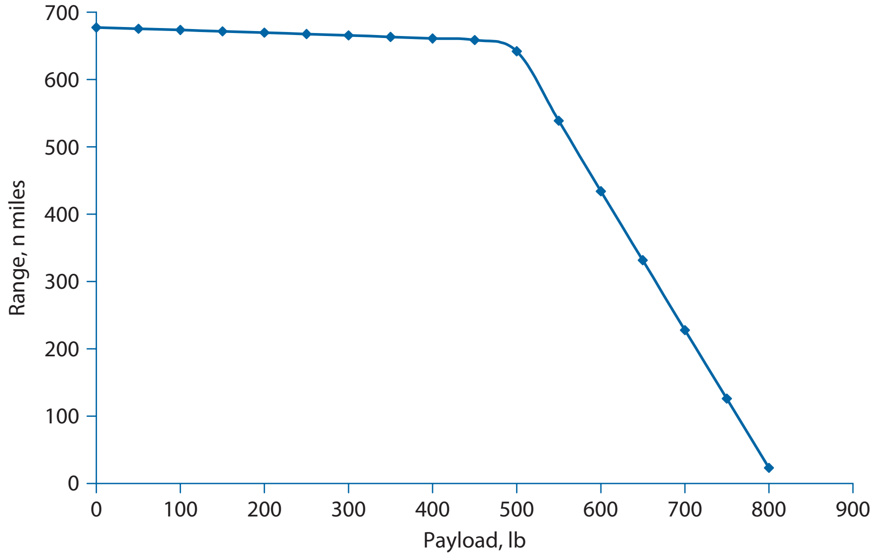

Fig. C3 Estimated range–payload for Cessna 172R cruising at 6000 ft and 65 percent rated power.

500 lb. Using a FORTRAN program as described in Chapter 7, Fig. C2 was prepared and presents the calculated distance flown as a function of time until the fuel is exhausted at 310 min and 600 n miles. This is the range shown for this payload. To find the range for this payload for a 45-min reserve of fuel at cruising speed, we simply back off from the 310 min to a time of 310 − 45 min, or 265 min. This results in a range from Fig. C3 of approximately 510 n miles. This is therefore the range for this payload with a 45-min reserve. Because the rate of fuel consumption varies little with gross weight at the cruising speed, this decrement of 90 n miles in the range for a 45-min reserve is nearly constant for all payloads. Therefore, a close approximation to more exact calculations is obtained for the range–payload curve for a 45-min reserve by simply subtracting 90 n miles from the curve of Fig. 7.2.

7.2 Redo Problem 7.1 using a cruise at 65 percent of the rated power used in Fig. 7.2.

Solution: Figure C3 was prepared using the same program as before for range, only the cruising power was reduced. The results are seen to be very close to the ones for 75% power.

8.1 An airplane has a wing loading of 40 psf. Measurements of stick position vs airspeed were taken for two c.g. locations with the results given in the table. From these results, calculate the location of the neutral point.

Trim airspeed	c.g. location	Stick position, *s*
100 KCAS	45 in.	5.0 in.
90 KCAS	45 in.	6.5 in.
100 KCAS	48 in.	4.5 in.
90 KCAS	48 in.	5.0 in.

Solution: We are looking for the c.g. location that will produce no change in stick position for a change in the lift coefficient. For a constant weight, the lift coefficient is inversely proportional to the square of the velocity. Therefore, simply determine that

$$\frac{\Delta s}{\Delta V^2} = \frac{5 - 6.5}{100^2 - 90^2} = -0.0007895 \text{ at CG} = 45 \text{ in.}$$

$$= \frac{4.5 - 5.0}{100^2 - 90^2} = -0.0002632 \text{ at CG} = 48 \text{ in.}$$

Thus, it follows that the slope of the stick position with respect to V^2 is equal to zero when the c.g. is at 49.5 in. This is the neutral point.

8.2 An airplane has a period of 35 s and a 45-s time-to-damp to one-half. It is trimmed at 100 KTAS, but it was disturbed and begins to undergo a phugoid motion. At 10 s it has a peak velocity of 115 KTAS. What will its velocity be at a time of 38 s?

Solution: The oscillation in V will be of the form $\Delta V = Ae^{-at}\cos(\omega t)$, where $A = 5$ KTAS if we measure t starting at 10 s. As stated in Chapter 7, the exponent, a, is obtained from the time-to-double or -halve while ω and the period, T, are related by $\omega T = 2\pi$. Thus,

$$t_{1/2} = \ell n(2)/a$$

or $a = 0.0154$. Thus,

$$\omega = 2\pi/35 = 0.1795$$

Relative to 10 s, the time is 28 s. Therefore, at this time,

$$\Delta V = 5\,e^{-0.0154(28)} \cos((0.1795)(28)) = 4.807$$

Adding this to the trimmed V gives a V of 14.807 KTAS at 38 s.

Appendix D Nomenclature, Abbreviations, and Acronyms

Nomenclature

A	amplitude of oscillatory motion
A	area
A	aspect ratio
$A(I, J)$	matrix in Eq. (6.7) for $\Gamma(I)$
A_T	aspect ratio of horizontal tail
A_W	aspect ratio of wing
a	acceleration
a	damping coefficient
a	slope of airfoil lift curve ($\mathrm{d}C_l/\mathrm{d}\alpha$)
a_t	slope of tail lift curve ($\mathrm{d}C_L/\mathrm{d}\alpha$)
a_w	slope of wing lift curve ($\mathrm{d}C_L/\mathrm{d}\alpha$)
$B(I)$	matrix in Eq. (6.7) for $\Gamma(I)$
b	wing span
b'	span of trailing vortices
C_D	three-dimensional drag coefficient
C_{D_i}	induced drag coefficient
C_d	two-dimensional drag coefficient
C_f	flap chord
C_f	skin-friction drag coefficient based on "wetted" area
C_L	lift coefficient of airplane or wing
C_l	airfoil section lift coefficient
C_{l_a}	additional sectional lift coefficient multiplier due to planform
C_{l_b}	basic section lift coefficient due to twist
$C_{L\max}$	maximum wing lift coefficient
$C_{l\max}$	airfoil section maximum lift coefficient
C_m	pitching moment coefficient
C_M	airplane pitching moment coefficient
$C_{m1/4}$	C_m about $C/4$ point
C_P	power coefficient
C_P	pressure coefficient; power coefficient
C_T	thrust coefficient (Eq. [3.4])
C_{pp}	profile power (due to profile drag)
c	wing section chord

D	diameter
D	drag
D_i	induced drag
D_{min}	minimum drag
D_{par}	parasite drag
e	Oswald's efficiency factor for induced drag
F	force
f	equivalent flat "drag" area (Eq. [3.10])
g	acceleration of gravity
h	altitude
h_{abs}	absolute ceiling
h_p	pressure altitude
i_t	tail incidence angle, positive nose-down
J	propeller advance ratio
L	lift
L_T	lift of horizontal tail
L_t	horizontal tail lift
l_t	distance of horizontal tail a.c. aft of c.g.
L_w	wing lift
l_w	distance of wing a.c. ahead of c.g.
M_{ac}	moment about aerodynamic center
m	mass
P_{avail}	power available to propeller from engine
P_{ew}	equivalent power for standard weight at SSL
P_{reqd}	power required by airplane to maintain level flight
p	pitch
p	pressure
p_0	free-stream pressure; SSL pressure
Q	airplane's pitching velocity about y axis
q	dynamic pressure
R	range
R	universal gas constant
r	radius
S	area (frequently planform area)
S	planform area of wing
S_t	planform area of horizontal tail
s	distance
s	distance of control wheel (stick) aft of reference
T	period of oscillatory motion
T	temperature
T	thrust
T_S	standard temperature

T_0	temperature for zero altitude
t	time
$t_{1/2}$	time to damp motion to half amplitude
U	x-component of airplane velocity relative to airplane's axes
u	perturbation of U-velocity
V	velocity
V_c	calibrated airspeed
V_{ew}	equivalent velocity for standard weight at SSL
V_i	indicated airspeed
V_{S1}	single engine stall speed
V_S	stalling speed
V_T	tangential velocity at propeller section
V_{to}	takeoff velocity
V_w	wind speed
V_0	free-stream velocity
V_θ	induced tangential velocity
W	weight
W	z-component of airplane velocity relative to airplane's axes
$W(I, J)$	influence coefficient equal velocity induced at I by vortex at J
W_E	empty weight
W_F	fuel weight burned
W_{Fin}	final weight
W_f	fuel weight
W_I	initial weight
W_i	initial value of the fuel weight
W_S	standard weight
w	downwash velocity
w	perturbation of W-velocity
w	propeller induced velocity
X_U	change of X-force with U-velocity
x	dimensionless radius or distance; unknown quantity
Y	spanwise location
Z_q	change of Z-force with pitching velocity, Q
Z_U	change of Z-force with W-velocity
Γ	circulation
Γ_0	midspan value of circulation
Δ	denotes increment
α	angle of attack
α_i	induced angle of attack
β	blade section pitch angle
δ	pressure ratio
δ_f	flap angle

ε	drag-to-lift ratio
ε_α	change of downwash angle at tail with respect to wing α
η	efficiency
η	empirical correction to τ
θ	heading angle
θ	pitch angle
θ	temperature ratio
θ_f	parameter, function of C_f/C (Fig. 8.7)
μ	coefficient of (rolling) friction
ρ	mass density
σ	complex root for oscillatory motion
σ	density ratio
σ	propeller solidity $(A/\pi/R^2)$
τ	change in section angle of attack with flap angle
ϕ	angle of climb; resultant flow angle for prop section
ω	angular velocity, radians per second

Abbreviations and Acronyms

a.c.	aerodynamic center
AGL	above ground level
ASI	airspeed indicator
AR	aspect ratio
BSFC	brake specific fuel consumption, lb/bhp-hr
CAS	calibrated airspeed
c.g.	center of gravity
°F	degrees Fahrenheit
FAA	Federal Aviation Administration
FAR	Federal Aviation Regulations
GPS	global positioning system
IAS	indicated airspeed
KE	kinetic energy
KCAS	knots calibrated airspeed
KIAS	knots indicated airspeed
KTAS	knots true airspeed
MAP	manifold pressure gauge
NACA	National Advisory Committee for Aeronautics (forerunner of NASA)
OAT	outside air temperature
POH	pilot's operating handbook
PE	potential energy
°R	degrees Rankine

R/C	rate-of-climb
SFC	specific fuel consumption
SSL	standard sea level
SI	Système International
TAS	true airspeed
THP	thrust horsepower
WOT	wide-open throttle

REFERENCES

[1] McCormick, B. W., *Aerodynamics, Aeronautics and Flight Mechanics*, 2nd ed., John Wiley & Sons Inc., New York, 1995.
[2] Stickle, G. W., *Measurement of the Differential and Total Thrust and Torque of Six Full-Scale Adjustable-Pitch Propellers*, NACA TR 421, 1933; http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930091495_1993091495.pdf [retrieved 25 May 2011].
[3] Hoerner, S. F., *Fluid-Dynamic Drag*, published by author, Midland Park, NJ, 1965.
[4] Abbott, I. H., von Doenhoff, A. E., and Stivers, Jr., L. S., *Summary of Airfoil Data*, NACA-ACR-L5005, 1945; http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19930092747_1993092747.pdf [retrieved 25 May 2011].

Bibliography

Anderson, J. D., *Fundamentals of Aerodynamics*, 4th ed., McGraw-Hill, New York, 2006.
Askue, V., *Flight Testing Homebuilt Aircraft*, Iowa State Univ. Press, Ames, IA, 1992.
Jane's All the World's Aircraft, 1992–1993, Jane's Information Group, Inc., Alexandria, VA.
Kimberlin, R. D., *Flight Testing of Fixed-Wing Aircraft*, AIAA, Reston, VA, 2003.
Lowry, J. T., *Performance of Light Aircraft*, AIAA, Reston, VA, 1999.
Smith, H. C., *Introduction to Aircraft Flight Test Engineering*, Jeppesen Sanderson, Inc., 1981.

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