



Electrical Engineering Applications with the TI-89

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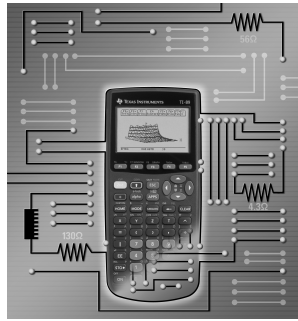
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Preface

To Students:

This book is written for electrical engineering students. It is a collection of examples that show how to solve many common electrical engineering problems using the TI-89. It is not a textbook; if you do not know how to solve the problem, look it up in your textbook first. If you do know how to solve the problem, this book will show you how to use the TI-89 to get the answer with more insight and less tedium. We show you how to use the TI-89 in class, in lab, on homework, and so forth.

Many of you may now use Maple®, Mathematica®, MATLAB®, Mathcad®, or other symbolic or numeric software. You will be pleasantly surprised to find that the TI-89 can solve many of the same problems as the big boys, but it will boot up in only a second or two, it rarely crashes, it fits your pocket book (even if you have a small one), and can fit in your pocket (if you have a big one).

You should find this book easy to use. Although we show how to use many of the features of the TI-89, we assume you already know your way around it. First read *Do This First*, then jump to the section discussing the problem you want to solve.

To Instructors:

Read the *To Students* section.

When writing this book, we resisted the temptation to show how the TI-89 can be used to solve problems in ways that differ from standard electrical engineering texts. Although it has the power and ability to approach many problems in new ways, that was not our focus.

Our focus is to help students learn the basic material better by showing them how to use the TI-89 to do the tedious things so they don't get lost in the details. Our approach was best summed up by Gottfried Wilhelm Leibniz when, in the 17th century, he said,

“It is unworthy of excellent men to lose hours like slaves in the labor of calculation.”

— *David Voltmer*

— *Mark Yoder*

About the Authors

DAVID VOLTMER (AKA Smilin' Dave) loves teaching electrical engineering at Rose-Hulman Institute of Technology. He claims to be good in the areas of electromagnetics, microwaves, antennas, communications and design. A few of his many projects designed to assist student learning include PC-based instruments, SPICE48, and Visual Electromagnetics (VEM). The writing of this book was accompanied by the sounds of clawhammer banjo music and with regular training breaks for long-distance cycling.

MARK A. YODER, Associate Professor of Electrical and Computer Engineering, received his B.S. degree in 1980 and Ph.D. in 1984, both in Electrical Engineering and both at Purdue University. While there, he did research in speech and in image processing, in addition to studying the dynamics of a disk passing through a non-viscous medium (that is, playing a lot of Frisbee®).

In 1988 he discovered that teaching was where the fun is, so he headed for Rose-Hulman in Terre Haute, Indiana. Here he pioneered the use of symbolic algebra systems in electrical engineering education and helped develop a class on computer vision. He developed an expert system for diagnosing a fiber-optic communications system for the International Centers for Communication Technology, and he hopes to work on computer assisted Bible translation. He has co-authored a book on digital signal processing for sophomores.

Dr. Yoder's biography is not complete without mention of his family. His wife Sarah has her Ph.D. in Electrical Engineering from Purdue. They have nine children aged 15, 15, 12, 10, 8, 6, 4, 2, and -0.5 – two boys, six girls, and one on the way.

Features Used

[MODE], getMode(), StoGDB, setMode(), RclGDB, NewProb, \blacklozenge 1, NewFold, getFold(), setFold(), [2nd] [VAR-LINK], \blacklozenge [Y=], [HOME], \blacklozenge [WINDOW], \blacklozenge [GRAPH], [APPS], [2nd] [MEM], [2nd] [MATH], [CATALOG], [2nd] [CHAR], \blacklozenge [EE], [STO▶]

Do This First This chapter describes the mode settings, folder structure, and operating tips needed to do many of the examples in this book. In order to reproduce the screen shots in this book, the same settings and procedures must be used.

Keystroke Instructions in This Book

Although you must press [ENTER] to execute a command on the TI-89, the keystroke instructions in this book do not include [ENTER]. You also must press [ENTER] to select certain items (from the CATALOG, for example), but we do include [ENTER] in these instructions.

Likewise, although you must press [alpha] to enter an alphabetic character on the TI-89, we have omitted the [alpha] key before each letter in the keystroke instructions to avoid loss of clarity in entering instructions.

Be sure to press [alpha] or [alpha][alpha] (for multiple letters) before entering any alphabetic characters except [X], [Y], [Z], and [T], which have their own key symbols.

When the keystroke instructions include a symbol other than an alphabetic character that requires the use of [alpha], we have included the [alpha] key symbol for clarity.

Mode Settings

READ THIS SECTION before going to other chapters. The standard settings used in this book are described in this section!

The variety of the TI-89's operational modes gives it great versatility. But, this also means that these modes must be carefully selected and set. Screens 1 - 3 show the default settings.

1. Press [MODE] to see Page 1 (screen 1).



2. Press **[F2]** to display Page 2.

3. Press **[F3]** to display Page 3.

4. If your settings are different, press **[2nd] [MEM] [F1] 3:Default** to set the defaults.

The examples in this book assume that you are using certain mode settings, unless clearly stated otherwise. The settings require only two changes from the defaults: **Display Digits** should be **FIX 2** and **Complex Format** should be **RECTANGULAR**.

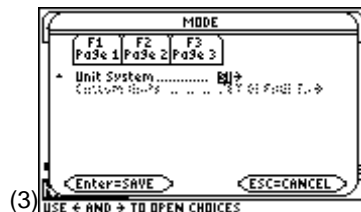
To change the settings, press **[F1]**. (If you have closed the **MODE** dialog box, press **[MODE]** first.) Press **⤵** to place the cursor on the setting you wish to change, and then press **⤵** to display the options. Select the appropriate setting and press **[ENTER]**. When your screen looks like screen 4, press **[ENTER]** to save the settings.

As you use your calculator, you may need to change your mode settings. However, saving the mode settings used in this book is easy.

1. To save the mode settings, enter **getMode([2nd] ["] all [2nd] ["] [] [STO] init** which returns a list of all the modes with the current setting and stores them in the variable **init** (middle of screen 5).
2. Enter **init** to verify that the variable contains the mode settings you want (bottom of screen 5).



(2)



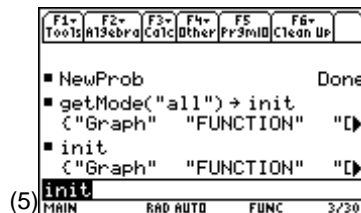
(3)

Note: In order to match the results in this book, be sure to set Display Digits and Complex Format modes.



(4)

Note: To enter the \rightarrow , press **[STO]**.



(5)

Note: The alphabetic characters are entered by pressing **[alpha]** followed by the character. The status line will display a small "a" to the left of **RAD** in screen 5. If you need to enter several alphabetic characters, press **[2nd] [a-lock]** or **[alpha] [alpha]** to stay in alpha mode. The status line will display a small inverse "a" to the left of **RAD** (screen 6). To exit alpha mode, press **[alpha]** again.


- The settings and variables used for graphing also should be saved. Enter **StoGDB ginit** to save the current graphing state in the variable **ginit** as shown in screen 6.

This variable can be recalled using the **RclGDB** command.

Defining 1

Changes in mode settings are specified in each chapter where different settings are needed. To ensure that you get the results shown in the book, it is recommended that **setMode(init)** and **RclGDB ginit** be entered before starting each chapter. These commands restore the settings saved in **init** and **ginit**.

An easy way to do this is to create a program called **kbdprgm1()**.

- Create this program by pressing **[APPS] 7:Program Editor 3:New**.
- Type **kbdprgm1** as the name of the new program variable as shown in screen 7, and press **[ENTER] [ENTER]**.
- Enter **setMode(main\init)** and **RclGDB main\ginit** (screen 8).
- Press **[HOME]** to return to the Home screen. Press ** 1** to run **kbdprgm1()**, thus setting the modes back to those stored in **init** and **ginit**; however, the screen won't change.

Also run **NewProb** before each chapter. **NewProb** clears all single-character variable names and turns off various plots.

Features Used Box

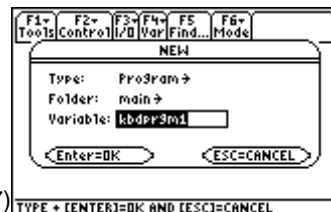
At the beginning of each chapter is a box which includes Features Used and Setup. This box describes the features you will use in the chapter. You must perform the Setup in order to execute the examples in the chapter.

Folder Structure

A new or reset TI-89 unit has a single folder named **MAIN**. The name of the current folder is displayed at the left edge of the status line at the bottom of the screen. Whenever a variable is saved, it is stored in the current folder. After an hour or two of operation, the folder may be cluttered with many variables—functions, strings, expressions, and so forth. A better strategy is to organize your work with folders for related topics. Each chapter of this book has a folder in which the work of that chapter is stored.



(6)



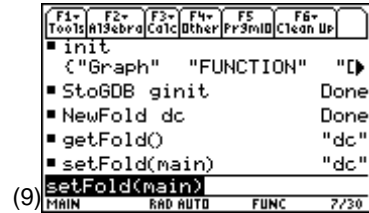
(7)



(8)

Note: To enter the \, press **[2nd] [\]**.

The command **NewFold** creates a new folder and sets that folder as the current folder. This is the folder in which all variables are stored until another folder is selected. A new folder, **DC**, is created as shown in the middle of screen 9 and will be used in Chapter 1.



Note: The name of a folder is limited to 8 characters or fewer.

The name of the current folder is returned by the **getFold()** command. To transfer back to the **MAIN** folder, use the **setFold()** command (bottom of screen 9). This also returns **DC**, the current folder before the transfer. The contents of folders are displayed with **[2nd] [VAR-LINK]** (screen 10). The folders and the contents of each folder are displayed in alphabetical order.



Note: There are no variables in the **DC** folder since it was just created.

A number of useful folder operations are available from the **VAR-LINK** screen using **[F1]**, **[F2]**, **[F4]**, and **[2nd] [F6]**.

Folders and their contents can be deleted, copied, renamed, moved, or created using **[F1] File Manage**. **[F2] View** allows certain folders and type of variables to be displayed.

[F4] toggles a check by the highlighted variable to select more than one variable for later operations such as copy, delete, or move.

The contents of a folder are displayed with **[2nd] [F6]**.

To try some of the folder operations, follow these steps.

1. Press **[ESC]** or **[HOME]** to return to the Home screen.
2. Enter **[2nd] ["] 100 [2nd] ["] [STO>] str1 [ENTER]** and **[x] [^] 2 [STO>] exp1 [ENTER]** as shown in screen 11.

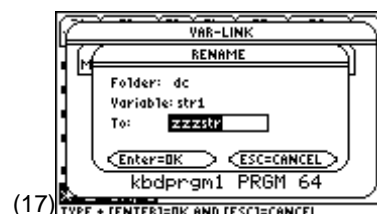
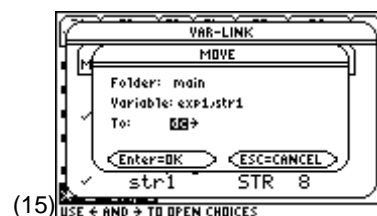
Since the current folder is **MAIN**, these variables are stored there.



3. Press **[2nd] [VAR-LINK]** (screen 12). Note that **exp1** is an expression of 7 bytes and **str1** is a string of 8 bytes.



4. To verify the contents of **exp1**, use \odot to highlight it and press **[2nd] [F6] Contents** (screen 13).
5. Press **[ENTER]** or **[ESC]** to return to the **VAR-LINK** screen. To move both of these variables to the **DC** folder, use \odot to highlight each of them and press **[F4]** to check them for later operations (screen 14).
6. Press **[F1] 4:Move**, and select **DC** as the folder to which they are to be moved (screen 15).
7. Press **[ENTER]**. The **VAR-LINK** screen shows **exp1** and **str1** in **DC** as shown in screen 16.
8. Rename variable **str1** as **zzzstr** by highlighting it, pressing **[F1] 3:Rename**, and entering **zzzstr** (screen 17).
9. Press **[ENTER] [ENTER]**. The **VAR-LINK** screen shows the renamed variable in the **DC** folder (screen 18).
10. To delete these variables, select them with **[F4]**, press **[F1] 1:Delete** (screen 19).



11. Press **[ENTER]**. The **VAR-LINK** screen shows that they have been deleted and that **DC** is empty (screen 20).



(20)

To delete a single variable at a time, highlight it and press **[←]** **[ENTER]**.

With these operations, your folders can be organized in an orderly manner.

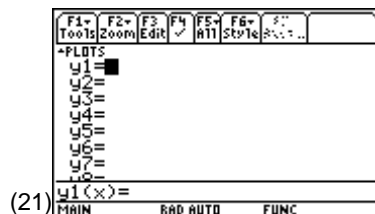
Navigating the Screens

When the TI-89 is turned on, it displays the Home screen that was in use when the calculator was turned off with **[2nd]** **[OFF]**. Most entries and operations are made from the Home screen. To change the display from any other screen to the Home screen, press **[HOME]**. Here are some other screens.

[♦] **[Y=]**

Expressions for graphing are entered using **[♦]** **[Y=]** as shown in screen 21.

This screen will change depending on the graphing mode. For now use **FUNCTION**, the default graph mode.



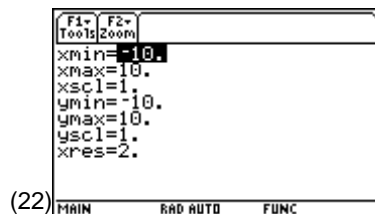
(21)

Note: The current graphing mode is displayed on the bottom right of the status line.

[♦] **[WINDOW]**

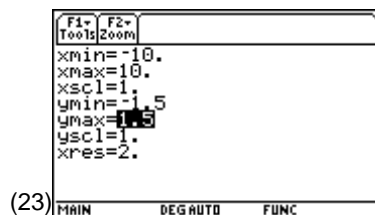
The Window Editor sets the viewing window variables—range and resolution. Press **[♦]** **[WINDOW]** to display the **FUNCTION** mode graphing parameters (screen 22).

The variables are different for each mode.



(22)

Press **[♦]** **[WINDOW]** to set the range of **x** and **y**. The **xmin** and **xmax** defaults (-10, 10) are standard, and fine. Press **[↓]** **[↓]** **[↓]** to change **ymin** to -1.5 and **ymax** to 1.5 (screen 23).



(23)

Note: The - (negative) is entered by pressing **[(-)]**.

◆ [GRAPH]

A graph is drawn using ◆ [GRAPH]. To display a graph, a function is defined by entering the equation to graph in the Y= Editor.

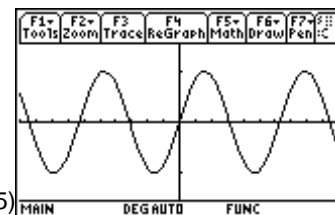
Try it by pressing ◆ [Y=] [ENTER] [2nd] [SIN] \times [] [ENTER] to plot $y=\sin(x)$ (screen 24).

(24)



To view the graph of $y=\sin(x)$, press ◆ [GRAPH]. The display should look like screen 25.

(25)



[APPS]

Press [APPS] to display the **APPLICATIONS** menu (screen 26).

(26)



The down arrow between **8** and **Text Editor** shows that there are more selections. Pressing [2nd] [DOWN] will show the other selections (screen 27).

Notice that there is now an up arrow between **2** and **Y= Editor**. This shows that there are more selections above.

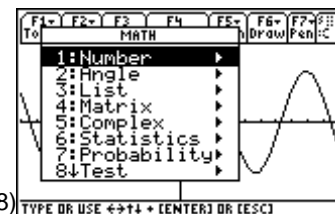
(27)



[2nd] [MATH]

The **MATH** menu provides organized menus of the TI-89's built-in math functions (screen 28).

(28)



CATALOG

The **CATALOG** provides access to all of the TI-89's built-in commands.

Move to the **crossP()** command by typing its first letter **c**, and scrolling with \odot to take single steps or $\boxed{2nd} \odot$ to take big steps. The status line shows that two vectors, **VEC1** and **VEC2**, are required as arguments (screen 29).

$\boxed{2nd}$ [CHAR]

The $\boxed{2nd}$ [CHAR] screen displays organized menus of all of the TI-89's characters (screen 30).

Press \boxed{ESC} to leave the [CHAR] screen.

\diamond \boxed{EE}

Pressing \diamond \boxed{EE} displays many of the TI-89's most common special characters (screen 31).

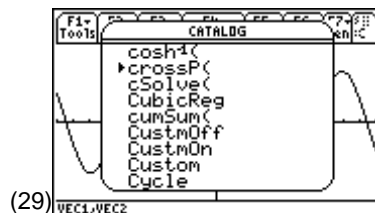
The Greek symbols are selected through $\boxed{2nd}$ [CHAR].

Saving Data

The TI-89 saves all entries from editor screens—programs, functions, matrices, and tables—when you leave the screen. Mode settings, graphing equations, and parameters are saved until a change is entered.

Often it is useful to save and later recall a particular set of data such as the mode settings, graphing data, graphics, or commands. A variety of editing tools is available with $\boxed{F1}$ on the Home screen. In particular, $\boxed{F1}$ **2:Save Copy As...** can be used in context-sensitive ways for saving textual, graphic, and tabular data. When $\boxed{F1}$ is pressed from the Home screen, all commands in the history area are saved and can be used for later execution (screen 32).

Many variables are just temporary and should be deleted from the memory regularly. From the Home screen pressing, $\boxed{2nd}$ $\boxed{F6}$ **2: NewProb** is especially useful (screen 33).



It clears all single character, unlocked variables; turns off all functions and stat plots; and clears all errors, graphs, tables, and the program I/O and Home screens (screen 34).



(34)

Note: Each chapter of this book starts with **NewProb**.

NewProb can be entered from the command line or **CATALOG** as well. To retain variables in memory independently of **NewProb**, assign them two (or more) character names or lock the variable.

Enter **1** [STO] **a** [ENTER], **2** [STO] **b** [ENTER], and **3** [STO] **c** [ENTER]. To lock a variable, press [2nd] [VAR-LINK], highlight or check (F4) the variable to be locked, and press [F1] **6:Lock Variable** (screen 35). Once locked, the variable name is marked by **Ⓛ** and can't be changed.



(35)

Variables can be unlocked with [F1] **7:UnLock Variable**.

The TI-89 has two separate areas of memory, RAM and archive. Archiving variables is a good idea when they need to be accessed but not changed. This frees up RAM for normal operations and improved operation of the TI-89. To archive a variable press [2nd] [VAR-LINK], highlight the variable to be archived, and press [F1] **8:Archive Variable** (screen 36).



(36)

Once archived, a variable is marked by **Ⓛ** and is treated as a locked variable (screen 37).

To unarchive a variable, use [F1] **9:Unarchive Variable**.

In screen 37, variables **a** and **b** are not locked and will be deleted with **NewProb**. Variable **c** is locked and will not be deleted by **NewProb**. Variables **ginit**, **init**, and **kbdprgm1** are archived and removed from RAM area, but still usable.



(37)

Tips and Generalizations

Each chapter has been written to stand alone. That is, after reading this chapter, you can jump to any other chapter. Each chapter ends with a "Tips and Generalizations" section, which includes tips on how to use the TI-89 more effectively and suggestions on how the topics of the chapter can be generalized to solve other problems.

The first chapter shows how the TI-89 solves DC circuits, which is also the first chapter of many circuit analysis books.

Features Used

NewProd, solve(),
Matrix Editor,
Simult(),
[CATALOG], [STO]

Setup

1
setFold dc

DC Circuit Analysis

This chapter shows three examples of the use of nodal analysis to solve linear circuits. The first two examples use the **solve()** command to solve a set of linear equations for a circuit. The third example shows how to write the equations in matrix form and use **simult()** to solve them.

Topic 1: Nodal Equations Using solve()

Given the circuit shown in Figure 1, find v1 and v2.

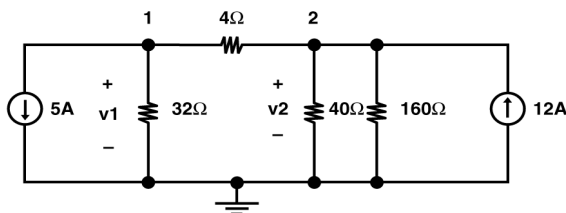


Figure 1. DC Circuit

Nodal analysis can be used to solve for the voltages of a circuit by summing the current leaving each node. Kirchhoff's current law states that the currents out of a node must sum to zero. The current through each resistor is calculated from Ohm's law by:

- Defining the voltage drop across the resistor in the direction of the current as the voltage at the node of the incoming current of the resistor minus the voltage at the node of the outgoing current of the resistor, and
- Dividing the voltage drop by the resistance of the resistor.

For a circuit with N+1 nodes (including the ground node), this process gives N equations with N unknown voltages. For the circuit above, summing the currents out of node 1 gives

$$5 + \frac{v_1}{32} + \frac{v_1 - v_2}{4} = 0$$

The sum of the currents out of node 2 is

$$\frac{v_2 - v_1}{4} + \frac{v_2}{40} + \frac{v_2}{160} - 12 = 0$$

The following series of steps leads to a solution of these two equations.

1. Clear the TI-89 by pressing $\boxed{2\text{nd}} \boxed{\text{F6}} \text{2:NewProb} \boxed{\text{ENTER}}$.

2. Enter the equation for node 1 and store it as **n1** as shown in screen 1.

$$5 + v_1 \div 32 + (v_1 - v_2) \div 4 = 0 \rightarrow n1$$

3. Enter the equation for node 2 and store it as **n2** as shown in screen 2.

$$(v_2 - v_1) \div 4 + v_2 \div 40 + v_2 \div 160 - 12 = 0 \rightarrow n2$$

4. Finally, solve for **v1** and **v2** using the **solve()** command, as shown in screen 3.

$$\boxed{\text{CATALOG}} \boxed{\text{solve}(n1, \text{CATALOG} \boxed{\text{and } n2}, \boxed{2\text{nd}} \boxed{\{ \}} \boxed{v1}, \boxed{v2} \boxed{2\text{nd}} \boxed{\{ \}} \boxed{\}} \boxed{\}$$

The two voltages are calculated as **v1 = 96 V** and **v2 = 128 V**.

Note: Press $\boxed{\alpha}$ before entering alphabetic characters.

(1)

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
NewProb Done					
$5 + \frac{v_1}{32} + \frac{v_1 - v_2}{4} = 0 \rightarrow n1$					
$\frac{9 \cdot v_1}{32} - \frac{v_2}{4} + 5 = 0$					
$5 + v_1/32 + (v_1 - v_2)/4 = 0 \rightarrow n1$					
DC		RAD AUTO		FUNC 2/30	

Note: To enter the \rightarrow , press $\boxed{\text{STO}} \rightarrow$.

(2)

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\frac{9 \cdot v_1}{32} - \frac{v_2}{4} + 5 = 0$					
$\frac{v_2 - v_1}{4} + \frac{v_2}{40} + \frac{v_2}{160} - 12 = 0$					
$\frac{-v_1}{4} + \frac{9 \cdot v_2}{32} - 12 = 0$					
$(v_2 - v_1)/4 + v_2/40 + v_2/160 - 12 = 0 \rightarrow n2$					
DC		RAD AUTO		FUNC 3/30	

(3)

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\frac{v_2 - v_1}{4} + \frac{v_2}{40} + \frac{v_2}{160} - 12 = 0$					
$\frac{-v_1}{4} + \frac{9 \cdot v_2}{32} - 12 = 0$					
$\text{solve}(n1 \text{ and } n2, \{v1, v2\})$					
$v1 = 96 \text{ and } v2 = 128$					
$\text{solve}(n1 \text{ and } n2, \{v1, v2\})$					
DC		RAD AUTO		FUNC 4/30	

Topic 2: Nodal Equations with Voltage Sources

When a voltage source is present between two nodes, Ohm's law cannot be used to calculate the current through the source (as in Topic 1). Fortunately, this difficulty can be overcome easily by giving a name to the current through the voltage source (as shown below) and treating this current as an unknown. Nodal analysis then can be used to find the solution for the voltages of the circuit shown in Figure 2.

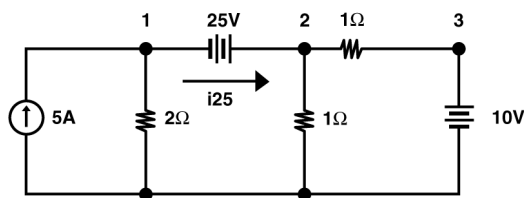


Figure 2. A DC Circuit with Three Sources

First, write the nodal equations in a form similar to that entered into the TI-89 as

$$\text{node1: } -5 + \frac{v_1}{2} + i_{25} = 0 \rightarrow n1$$

$$\text{node2: } -i_{25} + \frac{v_2}{1} + \frac{v_2 - v_3}{1} = 0 \rightarrow n2$$

$$\text{node3: } v_3 = 10 \rightarrow n3$$

Note: Some textbooks use the concept of a "supernode," which in effect combines the nodal equations for node1 and node2 into a single equation and eliminates i_{25} .

Notice that the current flowing through the 25 V source from left to right is defined as **i25**. This unknown current becomes another variable which will be found as part of the solution. The voltage drop of the 25 V battery establishes the relation between **v1** and **v2** as

$$\text{equation 1: } v_2 = v_1 - 25 \quad \text{e1} \rightarrow$$

To enter these equations into the calculator:

1. Clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.

2. Enter the equation for node1 as shown in screen 4.

$$(-) 5 \div v_1 \div 2 + i25 = 0 \quad \text{STO} \rightarrow n1$$

3. Enter the equation for node2 as shown in screen 5.

$$(-) i25 + v_2 \div 1 + (v_2 - v_3) \div 1 = 0 \quad \text{STO} \rightarrow n2$$

4. Enter the node3 equation (screen 6).

$$v_3 = 10 \quad \text{STO} \rightarrow n3$$

5. Enter the last equation for the 25 V source (screen 7).

$$v_2 = v_1 + 25 \quad \text{STO} \rightarrow e1$$

Screen 8 shows a summary of the four equations, which can be displayed by entering their names—**n1**, **n2**, **n3**, and **e1**.

6. Finally, solve for **v1**, **v2**, **v3**, and **i25** by using **solve()** as shown in screen 9.

$$\text{[CATALOG] solve(n1 [CATALOG] and n2 [CATALOG] and n3 [CATALOG] and e1 , [2nd] [t] v1 , v2 , v3 , i25 [2nd] [)] }$$

The complete result is

v1 = -14 V, **v2** = 11 V, **v3** = 10 V, and **i25** (the current through the 25 V source) = **12 A**.

(4)

(5)

*Note: Enter the first - (negative) by pressing **[(-)]** below **[3]** and the second - (subtract) by pressing **[]** to the right of **[6]**.*

(6)

(7)

(8)

(9)

Topic 3: Nodal Equations Using simt()

Another approach to solving the problem in Topic 2 is to convert the equations to matrix form. The equations as shown in screen 8 are rearranged as

$$\text{node1: } \frac{1}{2}v_1 + i_{25} = 5$$

$$\text{node2: } 2v_2 - v_3 - i_{25} = 0$$

$$\text{node3: } v_3 = 10, \text{ and}$$

$$\text{eqn1: } -v_1 + v_2 = 25$$

In matrix form they appear as:

$$\begin{bmatrix} 1/2 & 0 & 0 & 1 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_{25} \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 10 \\ 25 \end{bmatrix}$$

1. To create the square matrix on the left side, press **[APPS]** and select **6:Data/Matrix Editor** and **3:New** in sequence, as shown in screen 10.

2. Press **[ENTER]** to display screen 11.

3. To enter a matrix, press **⬇** and **⬅** to highlight **2:Matrix** (screen 12) and press **[ENTER]**.

4. Press **⬅** twice and enter the Variable name. (For convenience, call it **matA**.) Using **⬅**, fill in **Row dimension: 4** and **Col dimension: 4** as shown in screen 13.



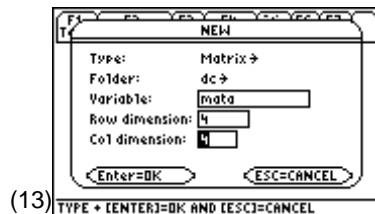
(10) DC RAD AUTO FUNC 10/30



(11) DC RAD AUTO FUNC



(12) DC RAD AUTO FUNC



(13) TYPE + [ENTER]=OK AND [ESC]=CANCEL

5. Press **[ENTER]**. You will see screen 14.

(14)

F1 Tools	F2 Plot Setup	F3 Cell	F4 Equar	F5 Func	F6 Unit	F7 Stat
MAT 4x4						
	c1	c2	c3			
1	0	0	0			
2	0	0	0			
3	0	0	0			
4	0	0	0			
r1c1=0						
DC RAD AUTO FUNC						

6. To see all four columns, press **[◀]** **[I]** and set the cell width to **5** (screen 15).

(15)

F1 Tools	F2 Plot Setup	F3 Cell	F4 Equar	F5 Func	F6 Unit	F7 Stat
MAT 4x4						
FORMAT						
Cell Width: 5						
Enter=SAVE NCEL						
r1c1=0						
DC RAD AUTO FUNC						

7. Press **[ENTER]** twice to see screen 16.

(16)

F1 Tools	F2 Plot Setup	F3 Cell	F4 Equar	F5 Func	F6 Unit	F7 Stat
MAT 4x4						
	c1	c2	c3	c4		
1	0	0	0	0		
2	0	0	0	0		
3	0	0	0	0		
4	0	0	0	0		
r1c1=0						
DC RAD AUTO FUNC						

8. Fill in the rows and columns with the numbers from the circuit matrix as shown in screen 17.

(17)

F1 Tools	F2 Plot Setup	F3 Cell	F4 Equar	F5 Func	F6 Unit	F7 Stat
MAT 4x4						
	c1	c2	c3	c4		
1	1/2	0	0	1		
2	0	2	-1	-1		
3	0	0	1	0		
4	-1	1	0	0		
r4c4=0						
DC RAD AUTO FUNC						

9. To create the column matrix on the right side of the matrix equation, press **[F1]** and select **3:New**. Define it as: **Type: Matrix, Variable: colb, Row dimension: 4, and Col dimension: 1** (screen 18).

(18)

F1 Tools	F2 Plot Setup	F3 Cell	F4 Equar	F5 Func	F6 Unit	F7 Stat
NEW						
Type: Matrix						
Folder: dc						
Variable: colb						
Row dimension: 4						
Col dimension: 1						
Enter=OK ESC=CANCEL						
TYPE * (ENTER)=OK AND (ESC)=CANCEL						

10. Press **[ENTER]** and fill in the values (screen 19).

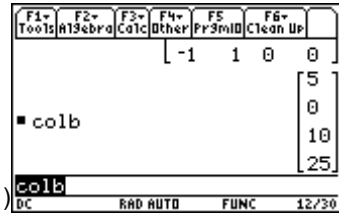
(19)

F1 Tools	F2 Plot Setup	F3 Cell	F4 Equar	F5 Func	F6 Unit	F7 Stat
MAT 4x1						
	c1	c2	c3	c4		
1	5					
2	0					
3	10					
4	25					
r4c1=25						
DC RAD AUTO FUNC						

11. Press **[HOME]** to return to the Home screen and check the contents of **mata** and **colb**, shown in screens 20 and 21.

(20)

F1 Tools	F2 Plot Setup	F3 Cell	F4 Equar	F5 Func	F6 Unit	F7 Stat
14 and v2 = 11 and v3 = 10						
$\begin{bmatrix} 1/2 & 0 & 0 & 1 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix}$						
mata						
mata						
DC RAD AUTO FUNC 11/20						



(21)

12. Enter `CATALOG` `simult(mata, colb)` (screen 22).

Note: The `simult()` command returns a column vector that contains the solutions to a system of linear equations.



(22)

Referring to the matrix equation for the circuit as shown below, the values returned by `simult()` correspond to the variables in the first column vector. The solution is $v_1 = -14$ V, $v_2 = 11$ V, $v_3 = 10$ V, and $i_{25} = 12$ A, the same answer as in Topic 2.

$$\begin{bmatrix} 1/2 & 0 & 0 & 1 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_{25} \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 10 \\ 25 \end{bmatrix}$$

Tips and Generalizations

There are many ways a command can be entered on the Home screen. For example, to enter `solve()`:

- Type it: `alpha` `alpha` `solve` `alpha` `(` `)`. Here `alpha` `alpha` locked the `alpha` key and the single `alpha` unlocked it.
- Use the function key menus: `F2` `1:solve(`.
- Use the catalog: `CATALOG` `s`. Pressing `s` scrolls to the first command that begins with `s`. If needed, press `⏮` to get to the desired command.
- Use `2nd` `[MATH]` `9:Algebra, 1:solve(`.
- If it has been used before, press `⏮` on the Home screen until the desired command is highlighted and then press `ENTER`.

Summary

In this chapter, nodal analysis was used to generate equations to solve a circuit. Loop analysis (or any method that produces N equations and N unknowns) also can be used to produce equations for the TI-89 to solve. The equations can include complex values (Chapter 4) and do not have to be linear. In fact, they also can include derivatives as shown in Chapter 2.

Features Used

deSolve(), **expand()**, **fMax()**, **zeros()**, **[Y=]**, **[WINDOW]**, **[GRAPH]**, **NewProb**, **tCollect()**, **limit()**, **[']**, **ZoomFit**, **Trace**, **[ANS]**, **[e]**

Setup

1, NewFold transym

Transient Circuit Analysis: Symbolic

This chapter describes how to use the differential equation solver, **deSolve()**, to solve first- and second-order circuits containing resistors, capacitors, inductors, DC sources, and exponential sources. It also shows how to graph the solutions and find the zero crossing and peak values.

Topic 4: RC First-Order Circuit

In the circuit in Figure 1, the switch has been open for a long time. At time $t = 0$, the switch closes. Find $v(t)$ after the switch is closed, that is, for $t > 0$.

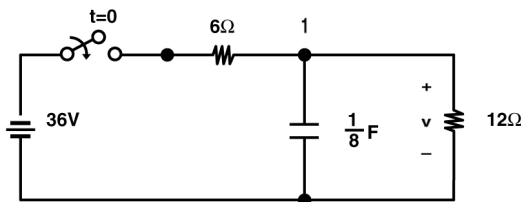


Figure 1. A simple RC circuit

Kirchhoff's current law is applied to the circuit; therefore, the sum of the currents out of node 1 is zero. Current flow in the direction of the voltage drop across a capacitor is

$$i = C \frac{dv}{dt} \text{ or } i = Cv'$$

Therefore, the nodal equation at node 1 is

$$\frac{v-36}{6} + \frac{1}{8} v' + \frac{v}{12} = 0$$

1. Clear the TI-89 by pressing $\boxed{2\text{nd}} \boxed{\text{F6}} \text{2:NewProb} \boxed{\text{ENTER}}$.
2. Enter this equation as shown in screen 1.

$$\boxed{\boxed{v} \boxed{-} \boxed{36} \boxed{)} \boxed{\div} \boxed{6} \boxed{+} \boxed{v} \boxed{2\text{nd}} \boxed{['] \div} \boxed{8} \boxed{+} \boxed{v} \boxed{\div} \boxed{12} \boxed{=} \boxed{0} \boxed{\text{STO}} \boxed{\text{►}} \boxed{\text{n1}}$$
3. Since the switch has been open for a long time, any charge originally on the capacitor has been discharged through the 12Ω resistor. This initial condition is expressed as $v(0) = 0$.

Enter the nodal equation, **n1**, and the initial condition, $v(0) = 0$, into the **deSolve()** command as shown in screen 2.

$\boxed{\text{CATALOG}} \text{deSolve}(\text{n1} \boxed{\text{CATALOG}} \text{ and } v \boxed{(} \boxed{0} \boxed{)} \boxed{=} \boxed{0} \boxed{,} \boxed{t} \boxed{,} \boxed{v} \boxed{)} \boxed{\text{STO}} \boxed{\text{►}} \boxed{\text{eqn}}$

The solution is $v(t) = 24 - 24e^{-2t}$ V, where t is in seconds.

Screen 1 shows the TI-89 calculator interface. The equation $\frac{v-36}{6} + \frac{v'}{8} + \frac{v}{12} = 0$ is entered and stored to **n1**. The screen also shows the simplified equation $\frac{v'}{8} + \frac{v}{4} - 6 = 0$.

Note: To enter $[']$, press $\boxed{2\text{nd}} \boxed{[']}$; it represents d/dt .

Screen 2 shows the TI-89 calculator interface. The **deSolve** command is entered with **n1** and $v(0)=0$, and the solution $v = 24 - 24e^{-2t}$ is displayed.

Topic 5: Graphing First-Order Solutions

A graph of this solution may help you understand it — graph $v(t)$ for $0 \leq t \leq 3$ seconds.

1. Since **y** is always graphed versus **x** on the TI-89, **v** must be converted to **y** and **t** to **x**. This is accomplished in the Y= Editor ($\boxed{\blacklozenge} \boxed{\text{Y=}}$) using the “with” operator, $\boxed{[]}$. To convert **t** to **x** and **v(t)** to **y1(x)**, enter the expression as shown in screen 3.

$v \boxed{[]} \text{eqn} \boxed{\text{CATALOG}} \text{ and } t \boxed{=} \boxed{x}$

This expression causes **v** to be graphed using its value given in variable **eqn** and to use **x** instead of **t**.

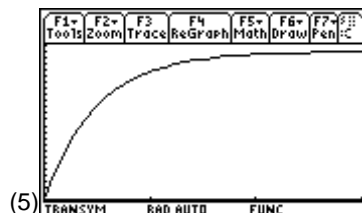
2. Press $\boxed{\blacklozenge} \boxed{\text{WINDOW}}$ to set the graph range of **t** from 0 to 3 seconds and **v** from 0 to 25 V, as shown in screen 4.

The y-axis tick marks are set with **yscl**. The resolution or “closeness” of the pixels representing adjacent calculated values is set by **xres**. Picking **xres=5** will complete the graphs more quickly, but also gives more jagged results.

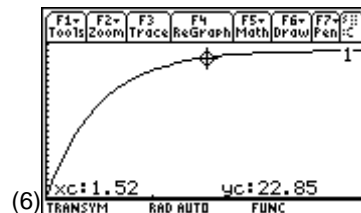
3. Press $\boxed{\blacklozenge} \boxed{\text{GRAPH}}$ to see a graph of the solution (screen 5).

Screen 3 shows the TI-89 calculator interface. The Y= Editor is displayed with the expression $y1(x) = v[eqn] \text{ and } t = x$.

Screen 4 shows the TI-89 calculator interface. The **WINDOW** settings are displayed, showing the range for **x** (0 to 3) and **y** (0 to 25), and the resolution settings (**yscl=1**, **xres=5**).



4. Press **F3** to trace the graph and to display the cursor location, **xc** and **yc**. The cursor can be moved with the cursor keys, **↑** and **↓** (screen 6).



Note: **xc** and **yc** are variables that can be accessed from the Home screen.

5. Tracing helps to verify that the voltage is approaching 24 V for large t as the equation shows, too. This can be verified with the **limit()** command as well (screen 5).

HOME **CATALOG** **limit**(**v** | **eqn** , **t** , **∞**)



Topic 6: First-Order Circuit with an Initial Condition

Consider the circuit of Topic 5 (Figure 1) with an initial voltage on the capacitor of -10 V at time $t = 0$. Figure 2 shows this circuit just after the switch has closed.

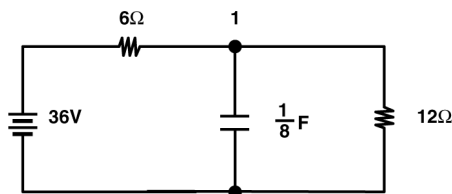
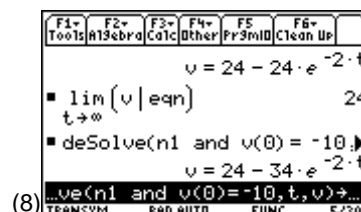


Figure 2. Circuit of Figure 1 just after the switch is closed

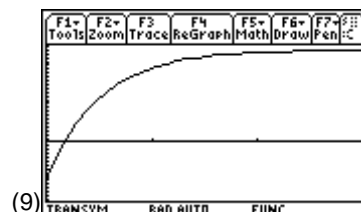
The circuit equation is the same as before. The initial condition is changed to $v(0) = -10$ V.

1. Enter the equation as shown in screen 8.

CATALOG **deSolve**(**n1** **CATALOG** **and** **v** (**0**) = (**-**) **10** , **t** , **v**) **STO** **eqn**



2. Since the capacitor voltage starts with $v(0) = -10$ V, use **♦** **[WINDOW]** and set **ymin** to -15. There is no need to reset the other window variables.
3. The results are graphed with **♦** **[GRAPH]** as shown in screen 9.



4. The graphical form of the solution makes it easy to find the instant of time when the capacitor voltage is 0.

Move the cursor to the vicinity of the zero crossing by pressing **[F3]** (the Trace tool) and using the cursor keys **⬅** and **➡** (screen 10).

At time $t = 0.19$ seconds, the voltage is small, 0.74 V, but not close enough.

More digits can be displayed by pressing **[MODE]** and changing **Display Digits** to **5:Fix 4**, but **yc** will not be any closer to 0.

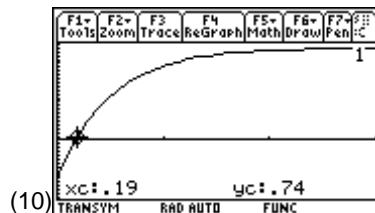
5. To get closer, press **⬇** **[WINDOW]** and set **xres** to a smaller value (try **xres=2**), but it will take longer to complete the graph (screen 11). This result may be closer to 0, but maybe not close enough due to graphical resolution.
6. A “cleaner” approach to finding the zero crossing is through the **Zero** command. Press **[F5]** **2:Zero** (screen 12.)

7. Position the cursor to the left of the zero crossing to set a Lower Bound and press **[ENTER]** (screen 13). Note that the location is marked on the screen.

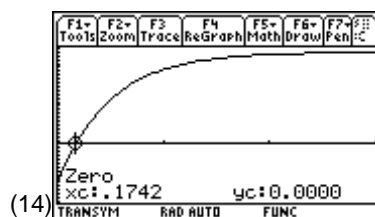
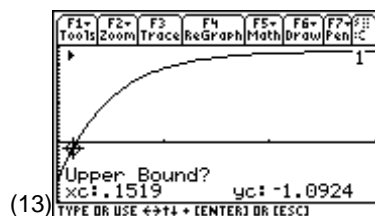
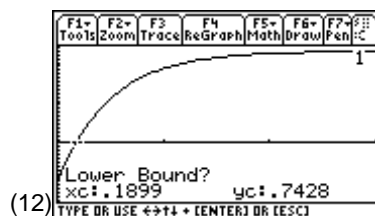
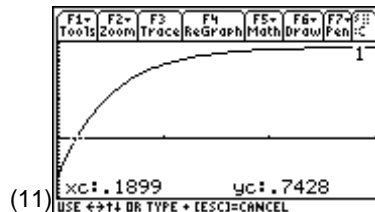
8. Move the cursor to the right of the zero crossing and press **[ENTER]**.

A much more accurate value for the time at which the voltage is zero is now displayed in screen 14.

This more accurate time is $t = .1742$ seconds.



Note: Press **[2nd]** **⬅** or **[2nd]** **➡** to take big steps, or hold down **⬅** or **➡**.



9. Return to the Home screen and find this value using the **zeros()** command (top of screen 15).

HOME **CATALOG** **zeros**(**v** **|** **eqn** , **t**)

10. Press \diamond \approx to get a floating point approximation of the exact solution (bottom of screen 15).

Note that the more accurate graphical answer agrees with the floating point answer through the four significant figures.

11. Reset **Display Digits** (**MODE**) to **Fix 2** and **xres** to **5**

(\diamond **WINDOW**).

(15)

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$v = 24 - 34 \cdot e^{-2 \cdot t}$					
$\text{zeros}(v \text{eqn}, t)$					
$\left\{ \frac{\ln(17/12)}{2} \right\}$					
$\text{zeros}(v \text{eqn}, t) \quad \{.1742\}$					
$\text{zeros}(v \text{eqn}, t)$					
TRANSYM RAD AUTO FUNC 7/30					

Topic 7: First-Order Circuit with a Time Varying Source

Modify the circuit of Topic 6 to include $v(0) = -10$ V with the battery and switch replaced by a source with $v_s(t) = 36e^{-3t}$ V for $t \geq 0$ s.

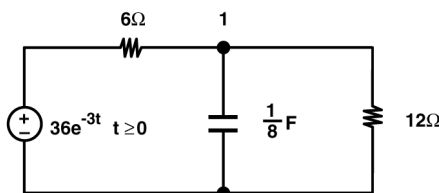


Figure 3. The circuit of Figure 2 with a time varying source

1. Edit **n1** to include this different source by substituting $36e^{-3t}$ for 36. Copy the original equation to the command line and edit it by using \ominus repeatedly to move up the history area to the original equation and pressing **ENTER**. Or just enter the equation as shown in screen 16.

v **=** **36** \diamond $[e^x]$ \diamond **3t** **/** **6** **+** **v** **2nd** **[']** **/** **8** **+** **v** **/** **12** **=** **0** **STO** **n1**

(16)

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\text{zeros}(v \text{eqn}, t) \quad \{.1742\}$					
$\frac{v - 36 \cdot e^{-3 \cdot t}}{6} + \frac{v'}{8} + \frac{v}{12} = 0$					
$e^{-3 \cdot t} \cdot \left\{ e^{3 \cdot t} \cdot (v' + 2 \cdot v) - 4 \right\}$					
$\dots (-3 \cdot t) \cdot \left\{ e^{3 \cdot t} \cdot (v' + 2 \cdot v) - 4 \right\} / 6 + v' / 8 + v / 12 = 0 \rightarrow n1$					
TRANSYM RAD AUTO FUNC 8/30					

Note: $[e^x]$ is entered by pressing \diamond $[e^x]$ and - is entered by pressing \ominus .

2. Use **deSolve()** to get the solution as shown in screen 17.

CATALOG **deSolve**(**n1** **CATALOG** and **v** **(** **0** **)** **=** \ominus **10** **,** **t** **,** **v** **)** **STO** **eqn**

(17)

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\frac{e^{-3 \cdot t} \cdot \left\{ e^{3 \cdot t} \cdot (v' + 2 \cdot v) - 4 \right\}}{8}$					
$\text{deSolve}(n1 \text{ and } v(0) = -10)$					
$v = 2 \cdot (19 \cdot e^{-t} - 24) \cdot e^{-3 \cdot t}$					
$\dots n1 \text{ and } v(0) = -10, t, v) \rightarrow \text{eqn}$					
TRANSYM RAD AUTO FUNC 9/30					

3. Use **expand()** to put **eqn** in a more familiar form (screen 18).

CATALOG **expand**(**eqn**)

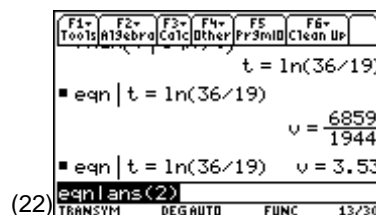
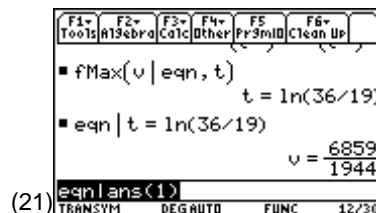
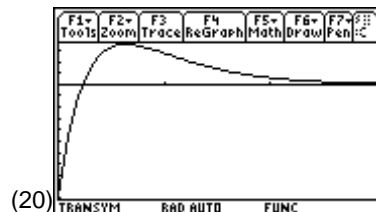
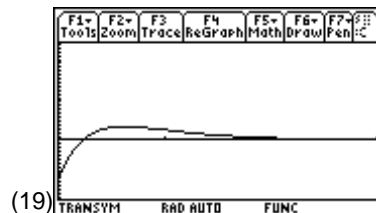
The solution is $v(t) = 38e^{-2t} - 48e^{-3t}$ V

(18)

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\text{deSolve}(n1 \text{ and } v(0) = -10)$					
$v = 2 \cdot (19 \cdot e^{-t} - 24) \cdot e^{-3 \cdot t}$					
$\text{expand}(\text{eqn})$					
$v = \frac{38}{(e^t)^2} - \frac{48}{(e^t)^3}$					
$\text{expand}(\text{eqn})$					
TRANSYM RAD AUTO FUNC 10/30					

4. Press \diamond [GRAPH] to graph the solution since **eqn** is still defined as the equation to graph in the Y= Editor (screen 19). Note that the peak value is much different than previously.
5. The range of the graph can be altered to see more of the behavior by pressing [F2] **A:ZoomFit** to make the curve fit the window (screen 20).
6. Find the zero crossing using [F5] **2:Zero** as before.
7. Find a good first estimate of the peak value and its time with Trace, [F3], where $v = 3.52$ V at time $t = 0.66$ seconds.
8. Find a more accurate value [F5] **4:Maximum**. Use it the same way as [F5] **2:Zero** to get $v = 3.53$ V at $t = 0.64$ seconds.
9. Return to the Home screen and use the **fMax()** command to find an exact answer (top of screen 21).
10. Substitute the result of the **fMax()** command into the original equation (**eqn**) using the “with” operator, [] (bottom of screen 21).
11. Find the floating-point solution using \diamond [\approx] (screen 22). Note that the **ans(2)** command was used to get the answer from second line of the history area.

The accurate graphical answer agrees with the floating point answer.



Topic 8: RLC Second-Order Circuit

Given the circuit in Figure 4 in which the current at time $t=0$ s is 10 A, ($i(0)=10$), and the time derivative of the current at $t=0$ s is 0, ($i'(0)=0$), find $i(t)$ for $t>0$ s.

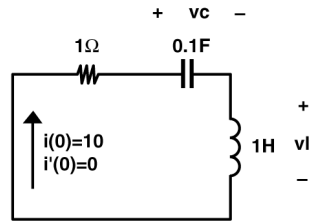


Figure 4: A simple RLC circuit

Kirchhoff's voltage law states that the sum of the voltages around every closed loop is zero. The voltage drop across a capacitor in the direction of current flow is

$$v_c = \frac{1}{C} \int i dt$$

and the voltage drop across an inductor in the direction of current flow is

$$v_l = L \frac{di}{dt}$$

Therefore, Kirchhoff's voltage law for the single loop of the circuit above is given as

$$iR + L \frac{di}{dt} + \frac{1}{C} \int i dt = 0$$

A derivative with respect to time of this equation and a rearrangement of terms gives

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0$$

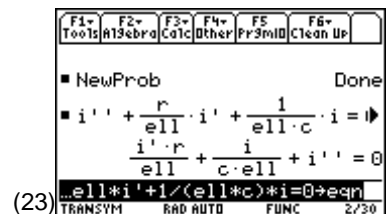
The differential equation can be rewritten as

$$i'' + \frac{R}{L} i' + \frac{1}{LC} i = 0$$

with the notation of $i'(t)$ and $i''(t)$ as the first and second time derivatives, respectively.

1. Clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
2. Enter the equation as shown in screen 23.

i **[2nd] [']** **[2nd] [']** **+** **r** **÷** **ell** **×** **i** **[2nd] [']** **+** **1** **÷** **(** **ell** **×** **c** **)** **×** **i** **=** **0** **[STO] eqn**



Note: "ell" is entered instead of "l" because "l" and "1" look very similar.

3. Enter the numeric values of **r**, **c**, and **ell** as shown in screen 24.

1 [STO] r

.1 [STO] c

1 [STO] ell

4. Set the radian mode before running **deSolve()**; otherwise, the solution may appear differently than expected. \blacktriangledown 1 should have set the radian mode. To be sure, press [MODE] and set **Angle** to **RADIAN**.

5. Use **deSolve()** to find **i** as shown in screen 25.

[CATALOG] **deSolve**(eqn [CATALOG] and i [2nd] ['] [0] [0] [=] 0

[CATALOG] and i [0] [=] 10, t [0] i [0] [STO] eqn2

The result of **deSolve()** has been stored in a variable called **eqn2**, although only a small portion is visible in the entry line. To see the rest of the result, press \odot then \odot until the rest of the line is visible, as shown in screen 26. The complete result is

$$i(t) = 10.00e^{-\frac{t}{2}} \cos(3.12t) + 1.60e^{-\frac{t}{2}} \sin(3.12t) \text{ A}$$

If **c** is entered as an exact value of 1/10 rather than 0.1, the answer will be in exact form. As a result, it may contain square roots and fractions that aren't always as easily interpreted. For decimal results, be sure that at least one of the element values is entered as a decimal number.

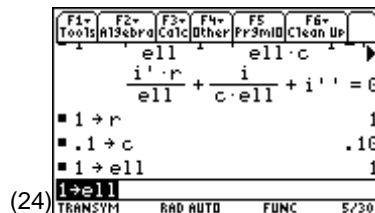
Engineers generally express values in terms of a single sinusoid, cosine or sine, and an angle rather than the sum of a cosine and a sine.

6. Convert the solution from **deSolve()** to the more common form with **tCollect()** (screen 27).

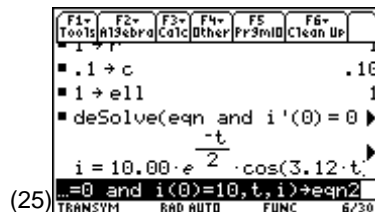
[CATALOG] **tCollect**(eqn2 [0] [STO] eqn3

The complete result is

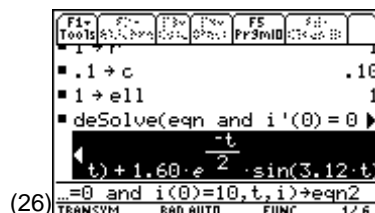
$$i(t) = 10.13e^{-\frac{t}{2}} \sin(3.12t + 1.41) \text{ A}$$



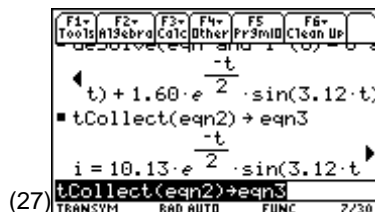
(24)



(25)



(26)



(27)

To convert the angle from radians to degrees, multiply the angle by $180/\pi$, that is, $1.41 \cdot 180/\pi = 80.79^\circ$. Therefore, an alternate form of the current answer is

$$i(t) = 10.13e^{-\frac{t}{2}} \sin(3.12t + 80.79^\circ) \text{ A}$$

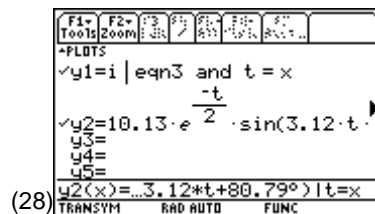
Let's graph it.

1. Press \diamond [Y=] and enter the equation as **y2** (screen 28).

10.13 \times \diamond [e^x] \diamond [-] \diamond t \div 2 \diamond \times [2nd] [SIN] 3.12 t \div 80.79 [2nd] [°]
 \diamond t = x

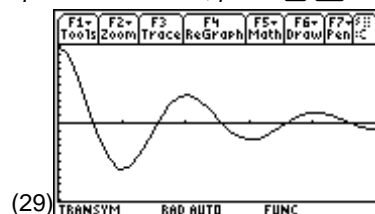
From the differential equation solution, it is clear that the time constant is 2, so the graph should extend for several time constants.

2. Press \diamond [WINDOW] and set **xmin** = 0, **xmax** = 5, **ymin** = -10, and **ymax** = 10.
3. Press \diamond [GRAPH].



Note: The ° symbol is essential for correct graphing as it “overrides” the radian mode setting.

Note: To see how to enter other special characters, press \diamond [EE].



In screen 29, **y1** graphs the results from the **tCollect()** command while **y2** graphs the simplification. They both should be the same graph if the simplification was done correctly; however, **y2** graphs more quickly. Refer to the Tips section to see how to speed up the graphing of **y1**.

The result looks like a nice, under-damped, second-order response.

Tips and Generalizations

Faster Graphs

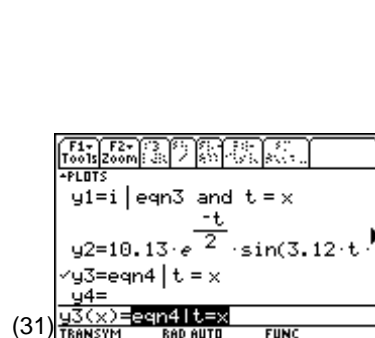
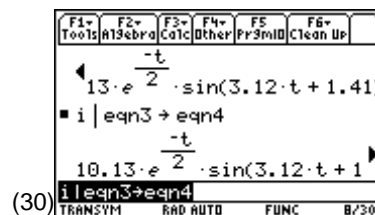
Graphing **y1** as shown in the previous section is a handy way to graph the output of **deSolve()** (or **solve()**), but it graphs more slowly than just retyping the equation into the Y= Editor. One way to graph **i** more quickly without reentering it is to use the “with” operator, [I], to extract the equation for **i** prior to graphing and to store it as another variable. This appears to occur when the “with” operator, [I], is used for the **y** function within the Y= Editor. Screen 30 shows how to extract **i** from **eqn3** and rename it as **eqn4**.

i [I] eqn3 [STO] eqn4

Now press \diamond [Y=] and enter **y3** as shown in screen 31.

In the Graph screen, the graph of **y3** should be the same as the last two graphs, but it should appear about twice as fast.

Since **y3** is the only function checked, it is the only one that is graphed.



Note: [F4] is used in the Y= Editor to select and deselect the functions to be graphed.

Exact vs. Approx

If the exact/approx mode ($\boxed{\text{MODE}}$ $\boxed{\text{F2}}$) hasn't been changed, the TI-89 will produce exact (rational rather than decimal) results when it can. If a decimal value is used on the input, the output will be approximate (contains decimal values). If the input has no decimal values, the result will be exact (in terms of rationals). If a decimal result is desired, press $\boxed{\blacktriangleleft}$ $\boxed{[\approx]}$ rather than just $\boxed{\text{ENTER}}$.

Summary

Although an RC circuit was shown here, any first-order equation can be solved and graphed using these techniques. **zeros()** and **fMax()** also were used. **fMin()** works that same way to find the minimum of a function. Check the other commands under the $\boxed{\text{F5}}$ menu in the Graph screen, which includes Minimums, Derivatives, Tangents, and so forth.

The **solve()** command also has a numeric version called **nSolve()** which finds a numeric solution rather than symbolic. This shouldn't be needed for linear equations, but **nSolve()** might be the only way of getting a solution of non-linear circuits.

In addition, solving for the current in a series RLC circuit has been shown, but these techniques can be used for finding voltage as well. A parallel RLC circuit could be solved for voltages or currents, too. In fact, any second-order circuit can be solved by these methods.

Although the TI-89 can find symbolic solutions for up to second-order differential equations, it can find numeric solution for systems of any number of first-order equations, as shown in Chapter 3.

Features Used

Differential Equations
Graphing Mode
NewProb

Setup

1

NewFold trannum

Transient Circuit Analysis: Numeric

This chapter describes how the differential equation plotter is used to solve the second-order circuit presented in Chapter 2 (Topic 8) by expressing it as a system of first-order differential equations.

Topic 9: RLC Circuit: Direction Field

Find $i(t)$ for $t > 0$ s for the circuit in Figure 1 in which the current at time $t = 0$ s is 10 A, that is, $i(0) = 10$ A, and the voltage across the capacitor at $t = 0$ s is 0, that is, $v_c(0) = 0$ V.

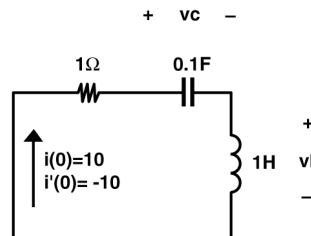


Figure 1. An RLC circuit

Kirchhoff's voltage law states that the sum of the voltages around every closed loop is zero. The voltage drop across a capacitor in the direction of current flow is

$$v_c = \frac{1}{C} \int i dt \quad (1)$$

and the voltage drop across an inductor in the direction of current flow is

$$v_l = L \frac{di}{dt}$$

The **DIFF EQUATIONS** plot requires that the equations be expressed in terms of $v_c'(t)$ and $i'(t)$. To do this, write equation (1) as

$$i(t) = C \frac{dv_c(t)}{dt} \text{ or } i(t) = C * v_c'(t)$$

Solve for $vc'(t)$ as

$$vc'(t) = \frac{i(t)}{C}$$

and for $i'(t)$ as

$$i'(t) = \frac{vl(t)}{L}$$

All equations must be entered as functions **y1(x)**, **y2(x)**, and so on. Therefore, let

$$vc = y1 \text{ and } i = y2$$

vl must be expressed in terms of **y1** and **y2**. To do this, sum the voltages around the loop to get

$$i(t)R + vc(t) + vl(t) = 0$$

which is solved for $vl(t)$

$$vl(t) = -i(t)R - vc(t)$$

Substitution of **y1** and **y2** gives

$$vl(t) = -y2 * R - y1$$

or

$$i'(t) = y2' = \frac{vl(t)}{L} = \frac{-y2 * R - y1}{L}$$

As a result, these differential equations are

$$y1' = \frac{y2}{C} \text{ and } y2' = \frac{-y2 * R - y1}{ell}$$

The corresponding initial conditions are

$$y1(0) = 0 \text{ and } y2(0) = 10$$

To enter these equations, follow these steps.

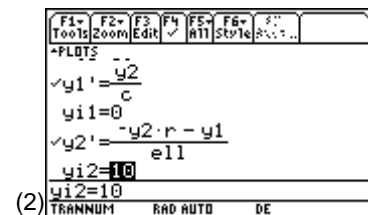
1. Clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
2. Press **[MODE] [6] 6:DIFF EQUATIONS [ENTER]** to select the differential equation graphing mode as shown in screen 1.
3. Press **[♦] [Y=]** and enter the equations as shown in screen 2.

The **y1=0** entry sets $vc(0)=0$ (remember **y1** is $vc(t)$). The **y2=10** entry sets $i(0)=10$.

Note: "ell" is used to avoid confusion between "l" and "1."



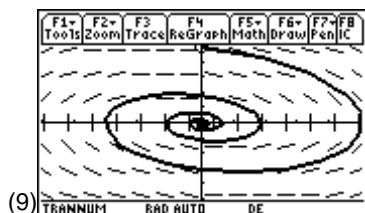
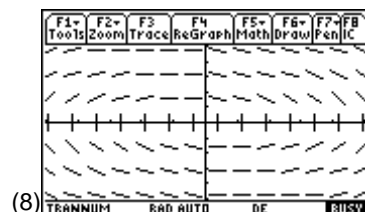
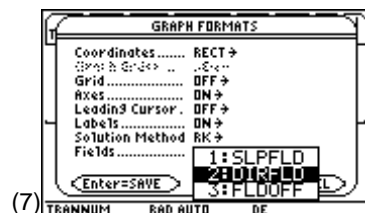
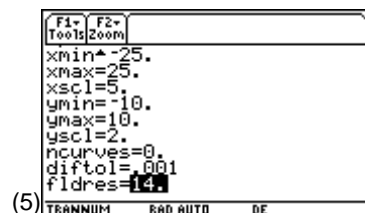
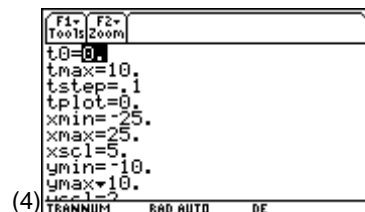
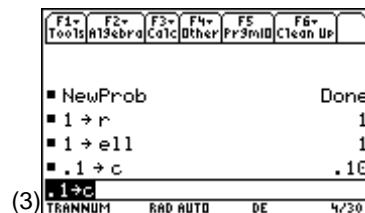
(1)



(2)

4. Press **HOME** and enter values for **r**, **ell**, and **c** as shown in screen 3.
5. Press **WINDOW** and set the window variables to the values as shown in screens 4 and 5.
6. Press **GRAPH** to graph the results. An error message appears first (screen 6). Slope fields are used for 1st-order equations only.
7. Press **ESC** **GRAPH** to set the proper graph format. Move the cursor down to **Fields** at the bottom and select **2:DIRFLD** as shown in screen 7.
8. Press **ENTER**. The graph first shows a direction field (screen 8). The short lines indicate the direction in which **i** and **vc** change with respect to each other at each point in the solution space. This plot has **vc** as **y1** on the x-axis and **i** as **y2** on the y-axis. Initial values were entered in the Y= Editor.

After a short time, the trajectory of the solution for the given initial conditions appears (screen 9).



It takes a few seconds to complete this plot. The time can be reduced by pressing \blacksquare [WINDOW] and setting **tmax** to a smaller value or setting **tstep** to a larger one. The smaller **tmax** causes the plotting to stop sooner. The larger **tstep** results in a less smooth plot.

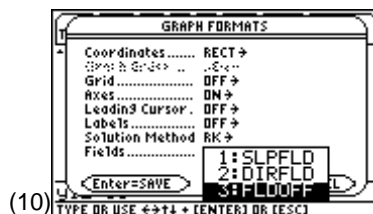
The thicker solution line starts at $v_c = 0$ and $i = 10$ (screen 9). It then circles down and to the right, spiraling clockwise to 0,0. This shows how the energy moves back and forth between the

current in the inductor $E_L = \frac{Li^2}{2}$ and the voltage on the capacitor $E_C = \frac{Cvc^2}{2}$. The resistor is converting some of the energy to heat; so after a long time all the energy is lost as heat and $i(t)$ and $vc(t)$ are 0.

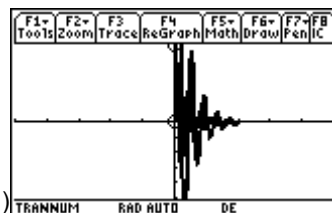
Topic 10: RLC Circuit: Time Domain

A more traditional way of viewing the voltage across the capacitor (**y1**) and current through the inductor (**y2**) is to plot them versus time.

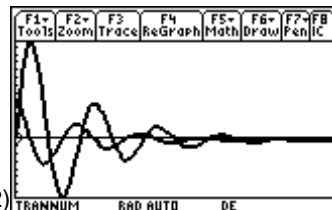
1. You can do this easily from the Graph screen by pressing \blacksquare [1] and selecting **Fields 3:FLDOFF** (screen 10).
2. Press [ENTER] to save the change, and screen 11 appears.
3. Press \blacksquare [WINDOW] and set **xmin** to 0, **xmax** to 10, **ymin** to -15, and **ymax** to 25.
4. Press \blacksquare [GRAPH] to display a clearer plot, as shown in screen 12.



(10)



(11)



(12)

Note: Pressing [F2] **A:ZoomFit** will produce a graph with **ymin** and **ymax** automatically set to display the whole function.

Topic 11: RLC Circuit: Multiple Initial Conditions

The Y= Editor can be used to set multiple initial conditions. The first example (Topic 9) initially set the current, i (**y2**), to 10 A and the voltage vc (**y1**) to 0 V. The plot for the current $i = 0$ A and the voltage $vc = 20$ V is added to the original plot by entering lists of initial conditions in the Y= Editor.

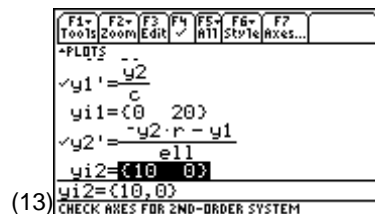
1. Press \blacklozenge [WINDOW] to set **xmin** back to -25, **xmax** to 25, **ymin** to -10, and **ymax** to 10, and press \blacklozenge [1] to set **Fields** back to **DIRFLD**. This will enable redrawing the direction field and **y1** versus **y2** solution.

2. The lists of initial conditions are added (in the Y= Editor) as shown in screen 13.

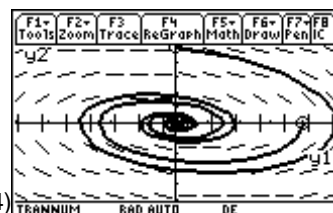
y1: [ENTER] [2nd] [t] [0] [,] [20] [2nd] [1] [ENTER]

y2: [ENTER] [2nd] [t] [10] [,] [0] [2nd] [1] [ENTER],

3. Pressing \blacklozenge [GRAPH] shows both solutions (screen 14). Notice the second set of initial conditions produces a second curve which begins on the positive x-axis.



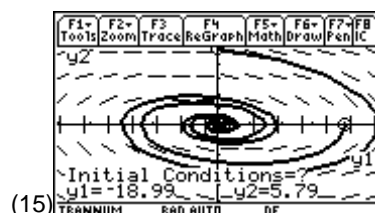
(13)



(14)

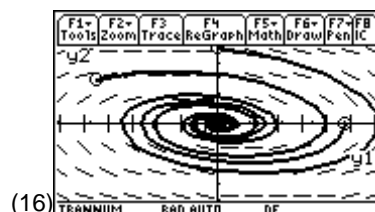
Note: Pressing \blacklozenge [1] and selecting **Labels 2:ON** activates the **y1** and **y2** axes labels.

4. Initial conditions can also be added graphically. Press [2nd] [F8] and use the cursor keys to move to the desired initial conditions in the **y1-y2** plane (screen 15).



(15)

5. Press [ENTER] to add the graph of this new set of initial conditions to the display (screen 16).



(16)

Topic 12: RLC Circuit: Adjusting the Circuit Parameters

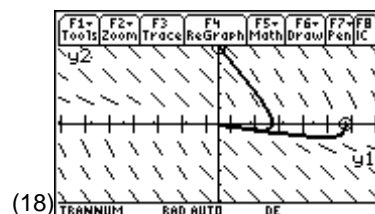
If the resistor, R, is increased, the energy in the circuit should be dissipated in the resistor more rapidly. This change can be investigated by simply changing **r**.

1. Press [HOME] and increase **r** to 10 by entering **10** [STO] **r** as shown in screen 17.
2. Press \blacklozenge [GRAPH] to display the solutions to the previous sets of initial conditions as shown in screen 18.

Notice that the direction field has changed dramatically, and now the energy quickly goes to zero from both sets of initial conditions.



(17)

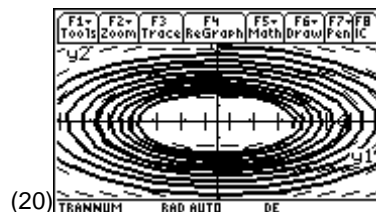


(18)

3. Making r smaller will cause the current and voltage to die out more slowly. Return to the Home screen, and try $r = 0.1$. Also, reset the initial conditions to the original single set, that is, $y1=0$ and $y2=10$ (screen 19) to keep the graph from becoming too cluttered.

4. Press \blacklozenge [GRAPH] to see the result (screen 20).

What would happen if r were set to 0 or to a negative value? Try it!



Tips and Generalizations

The examples presented here show how to plot the numeric solution to a second-order differential equation by expressing it as a system of two first-order differential equations. The numeric differential equation solver is not limited to two equations, so higher-order differential equations also can be solved.

The response of most circuits is composed of two parts, transient and steady-state. Chapters 2 and 3 have considered transient responses.

Often the steady-state response is of more interest. Chapter 4 shows how the TI-89 manipulates complex numbers (phasors) to find sinusoidal steady-state solutions.

Features Used

cSolve(), [\angle], [$^\circ$], [\square],
abs(), angle(), Numeric
Solver, when(), log(),
DelVar, DrawFunc, DrawInv,
NewProb, [\square], [\diamond][Y=],
[\diamond][GRAPH], [\diamond][WINDOW]

Setup

[\diamond] 1, NewFold steady,
setMode("Angle", "Degree")

Steady-State Circuit Analysis And Filter Design

This chapter shows how the TI-89 implements *phasors* to perform sinusoidal steady-state analysis. The focus is on how to enter and display complex numbers. This chapter also shows a typical steady-state application—how to use the Numeric Solver to find the required order of lowpass Butterworth and Chebyshev filters in making a standard “handbook” filter design.

Topic 13: Phasor Analysis

Given the circuit shown in Figure 1, find v , the voltage across the current source.

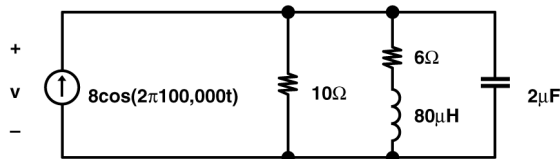


Figure 1. A circuit in steady-state

The first step is to convert the actual circuit to its phasor equivalent. The circuit shown in Figure 2 includes these conversions.

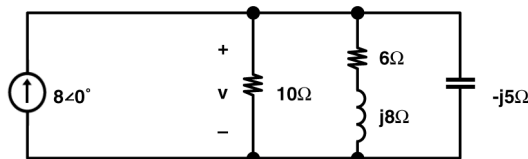


Figure 2. The phasor equivalent of the Figure 1 circuit

Only one nodal equation is needed to solve for v

$$-8\angle 0^\circ + \frac{v}{10} + \frac{v}{6 + j8} + \frac{v}{-j5} = 0$$

1. Clear the TI-89 by pressing $\boxed{2\text{nd}} \boxed{\text{F6}} \text{2:NewProb} \boxed{\text{ENTER}}$.
2. Be sure the **Complex Format** mode is set to **Rectangular**.
Be sure the **Graph** mode is set to **Function**.
3. Enter the equation as shown in screen 1.

$\boxed{(-)} \boxed{8} \boxed{+} \boxed{v} \boxed{\div} \boxed{10} \boxed{+} \boxed{v} \boxed{\div} \boxed{6 + i \cdot 8} \boxed{+} \boxed{v} \boxed{\div} \boxed{(-i \cdot 5)} \boxed{=}$
 $\boxed{2\text{nd}} \boxed{[i]} \boxed{5} \boxed{)} \boxed{=}$ 0 $\boxed{\text{STO}} \boxed{\text{►}} \boxed{\text{n1}}$

4. Using **solve(n1,v)** will return "false" since it is valid for real solutions only. To get a complex solution, enter **cSolve()** as shown in screen 2.

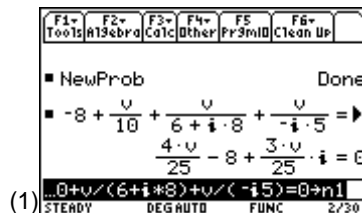
5. Phasors are expressed as a magnitude at an angle, $M\angle\theta$.
There are a couple of ways to obtain this form.

The first way is to use the functions **abs()** (top of screen 3) and **angle()** (middle of screen 3). In this example, $\boxed{\blacklozenge}$ $\boxed{[\approx]}$ is used to get the approximate values for the second **angle()** command (bottom of screen 3).

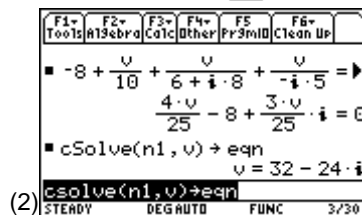
This shows that the phasor form of the voltage is $40\angle-36.87$ in **Degree** mode.

The second approach is to put the TI-89 in **Polar** mode.
Press $\boxed{\text{MODE}}$ and select **Complex Format 3:Polar** (screen 4).

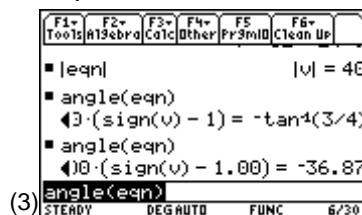
6. Using **cSolve()** (and with $\boxed{\blacklozenge}$ $\boxed{[\approx]}$ for a second approximate solution) gives the same results in the polar mode as **abs()** and **angle()** in the rectangular mode, as shown in the bottom two lines of screen 5.



- (1) **Note:** The usual imaginary number j used in electrical engineering is entered as i which is $\boxed{2\text{nd}} \boxed{[i]}$.

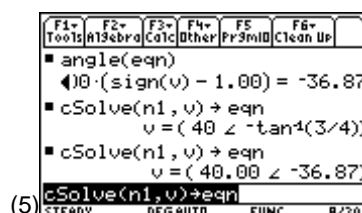


- (2) **Note:** To enter **cSolve()**, press $\boxed{\text{F2}}$ **A:Complex 1:cSolve()** or $\boxed{\text{CATALOG}}$ **cSolve()**.



- (3) **Note:** To see the values on the right end of a solution line in the history section, press $\boxed{\text{◀}}$ to get to the line and then press $\boxed{\text{▶}}$ to move to the right.

Note: Press $\boxed{\text{MODE}}$ to switch to degree mode if it isn't already set. If in radian mode the angle would have been given in radians.



Topic 14: Graphing Frequency Response

It is easy to find the voltage across the current source as a function of ω using symbolic expressions. Table 1 shows the variations of phasor circuit elements with radian frequency ω .

Element	Element Equation	Phasor Result
$8\cos 10^5 t$	$v(t) = \text{Re}[V e^{j\omega t}]$	$V = 8\angle 0^\circ$
$10\ \Omega$	$Z_{Ra} = R_a$	$Z_{Ra} = 10\ \Omega$
$6\ \Omega$	$Z_{Rb} = R_b$	$Z_{Rb} = 6\ \Omega$
$80\ \mu\text{H}$	$Z_L = j\omega L$	$Z_L = j\omega 80 \times 10^{-6}\ \Omega$
$2\ \mu\text{F}$	$Z_C = \frac{1}{j\omega C}$	$Z_C = \frac{1}{j\omega 2 \times 10^{-6}}\ \Omega$

Table 1: Frequency Dependence of Phasor Circuit Elements

The nodal equation then becomes

$$-8\angle 0^\circ + \frac{v}{r_a} + \frac{v}{r_b + z_{ell}} + \frac{v}{z_{c1}} = 0$$

Note that **ra**, **rb**, and **zc1** are used because **r1**, **r2**, and **zc** are TI-89 system variables.

1. Switch back to **Rectangular Complex Format** mode and enter the equations as shown in screen 6.

$$(-) 8 + v \div ra + v \div (rb + z_{ell}) + v \div z_{c1} = 0 \text{ STO } n1$$

as shown in screen 6.

2. Define the element values from the table (screens 7 and 8). For convenience, **w** (α **w**) is used instead of ω (α **w**).

(6)

Note: **zell** is used to avoid confusing **z1** with **z1** (**z** followed by a **1**), a reserved name.

(7)

(8)

3. With **cSolve()**, the solution in screen 9 shows that the voltage varies with frequency.

The complete answer is

$$v = \frac{-4000000.00(iw + 75000.00)}{w^2 - 125000.00iw - 10000000000.00}$$

The answer is a bit of a mess. To check it with the previous solution, enter **eqn | w=100000** (screen 10).

It's the same answer as Topic 13, screen 2.

4. To view the variation of the voltage magnitude versus frequency, graph **v** versus **w**. Since the original problem used **w=100,000**, graph from **w=0** to **w=200,000**. Press **▣**[WINDOW] and set **xmin** to 0, **xmax** to 200000, **ymin** to 0, and **ymax** to 50. Press **▣**[Y=] and set **y1** to graph the magnitude of **v** (screen 11).

5. Press **▣**[GRAPH] to see the magnitude graph (screen 12).

6. This graph takes a long time to complete because the “with” substitutions are made over and over again for each pixel. One way to speed it up is to do the “withs” once before graphing and save the result in another variable name which is then graphed.

To do this, press **▢**[HOME] and enter the expression:

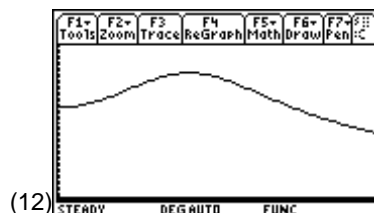
[CATALOG] **abs(v | eqn)** **▢** **[STO>]** **eqn2** as shown in screen 13.

Then press **▣**[Y=], deselect **y1**, and enter **y2** (screen 14).

(9)

(10)

(11)



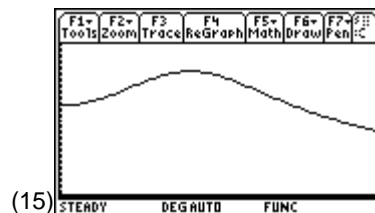
(13)

(14)

Note: Deselect equation **y1(x)** with **▢**[F4].

7. Press \blacklozenge [GRAPH]. The same result as the previous graph appears much more quickly (screen 15).
8. The phase can be graphed defining the phase angle of the voltage on the Home screen as **eqn3** (screen 16).
9. Press \blacklozenge [Y=], deselect **y2**, and enter **eqn3** as plot variable **y3** (screen 17).
10. Press \blacklozenge [WINDOW] and set **ymin** to -90 and **ymax** to 0 (since the calculations have been in the degree mode). **xmin** and **xmax** can remain the same.
11. Press \blacklozenge [GRAPH] to see the phase graph as shown in screen 18.
12. Usually the magnitude and phase plots are shown together. This can be done using the split screen mode. To do this, press [MODE] [F2] \downarrow \odot . Screen 19 presents the Split Screen options.
13. Press [ENTER]. Move down to **Split 2 App**, and select **4:Graph**. Finally, set **Number of Graphs** to 2 (screen 20).
14. Press [ENTER] to view the split screen plots (screen 21).

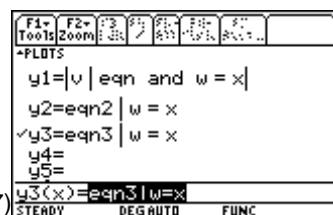
The top graph is the phase plot shown before; the bottom graph contains no data yet.



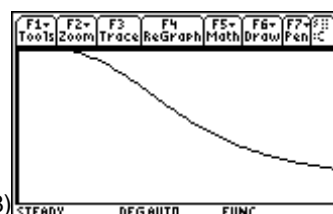
(15)



(16)



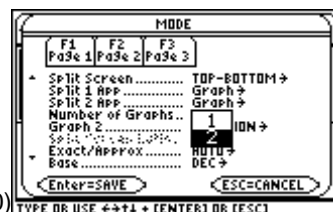
(17)



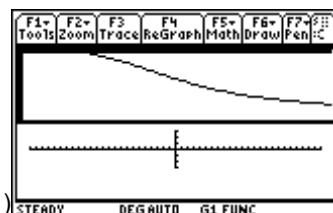
(18)



(19)

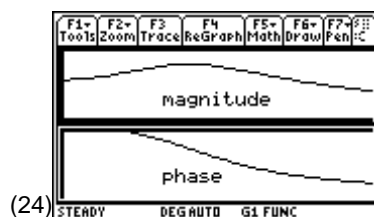
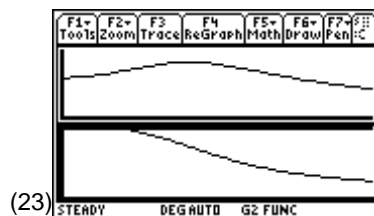
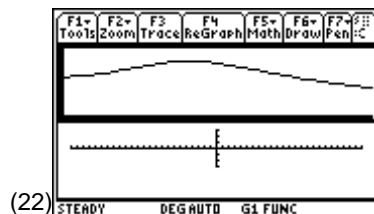


(20)



(21)

15. Convention says the magnitude plot should be on top.
To do this, press \blacklozenge [Y=] and use [F4] to select **y2** and deselect **y3**. Next, press \blacklozenge [WINDOW] and set **ymin** to 0 and **ymax** to 50. Finally, press \blacklozenge [GRAPH] to see the magnitude plot in the upper graph as shown in screen 22.
16. To set up the phase plot in the lower window, change to the other half of the screen by pressing [2nd][\square] and set up the graph as before. The following operations will give the phase plot in the bottom window (screen 23).
 \blacklozenge [Y=], select **y3**, \blacklozenge [WINDOW], set **xmin** to 0, **xmax** to 200000, **ymin** to -90 and **ymax** to 0, and finally, \blacklozenge [GRAPH]
17. You can use [2nd][F7] **7:Text** to add magnitude and phase labels to the graphs. To do this, press [2nd][F7] **7:Text** and position the cursor where the text should start. The characters will appear below and to the right of the crosshairs. Be careful; once a letter is placed it can't be erased except by [2nd][F7] **2:Eraser**.
18. To return to a single screen, press [MODE][F2] and set **Split Screen** to **1:FULL**.



Topic 15: Filter Design Overview

A class of realizable frequency responses for lowpass filters has the form

$$|H(f)|^2 = \frac{1}{1 + \epsilon^2 \Psi^2(f)}$$

where $\Psi(f)$ is a polynomial in f . If

$$\Psi(f) = \left(\frac{f}{f_p} \right)^n$$

the filter is a Butterworth filter. An alternative is to make $\Psi(f) = C_n(f/f_p)$ where C_n is a Chebyshev polynomial, the filter is a Chebyshev filter. The next topic deals with a Butterworth filter, the following topic with a Chebyshev filter.

The design of lowpass filters requires specification of passband and stopband responses often given in dB. $|H(f)|^2$ in dB is calculated as

$$|H(f)|_{\text{dB}}^2 = 10 \log \left(\frac{1}{1 + \epsilon^2 \Psi^2(f)} \right)$$

which becomes

$$|H(f)|_{\text{dB}}^2 = 10 \log \left(\frac{1}{1 + \epsilon^2 \left(\frac{f}{f_p} \right)^{2n}} \right)$$

for Butterworth and

$$|H(f)|_{\text{dB}}^2 = 10 \log \left(\frac{1}{1 + \epsilon^2 C_n^2 \left(\frac{f}{f_p} \right)} \right)$$

for Chebyshev.

Topic 16: Butterworth Filter

The performance specifications of a filter are often given in graphical form as shown in Figure 3. The design of a Butterworth filter with these performance specifications is described here.

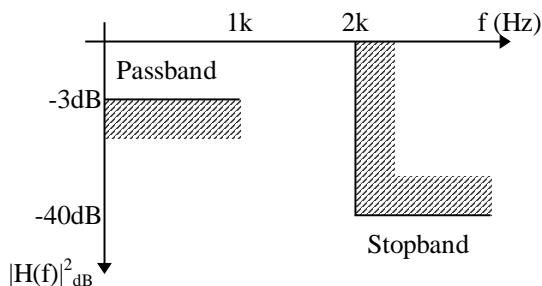


Figure 3. Filter design specifications for a Butterworth filter

Suppose a filter with the maximum passband ripple is -3 dB, and the passband edge is at $f_p=1\text{kHz}$ is to be designed. Additionally, the stopband gain is to be no more than -40 dB with a stopband edge at $f_s=2\text{kHz}$.

- From the Home screen, press \blacktriangleright 1 [2nd] [F6] 2:NewProb to reset the TI-89 to a known state. Then enter the Butterworth equation as shown in screen 25.

10 [X] [CATALOG] log(1 [÷] (1 [÷] eps [^] 2 [X] (f [÷] fp) [^] 2
[2n] [)] [)] [)] [STO] butter

- Press [APPS] 9:Numeric Solver exp=butter [ENTER] (screen 26).

- Note that the Numeric Solver listed each of the variables for values to be entered. Find the value of **eps** by entering the data for the passband edge with a -3 dB response at 1000 Hz. **n** is unknown but at the passband edge all values give the same result, so for now enter 1 for **n** as shown in screen 27.

- Place the cursor on the **eps** line and press [F2] to solve for eps. After a second or two, the screen shows **eps** is about 1 (screen 28).

- Now, find the order of the filter by setting the stopband edge response (**exp**) to -40 dB and **f** to 2000 (screen 29).

- Solve for **n** by placing the cursor on **n** and pressing [F2]. After a couple of seconds the solution of **n**=6.6 is shown, as in screen 30.

- Since **n** must be an integer, set **n** to the next larger integer value of 7 and solve for **exp** to find the stopband gain for this value of **n** (screen 31).

With a 7th-order Butterworth filter the stopband gain is -42 dB, a little better than the minimum needed.

(25)

(26)

(27)

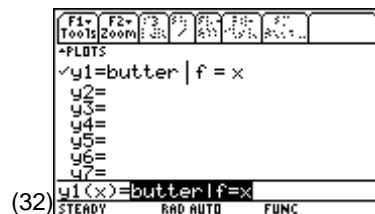
(28)

(29)

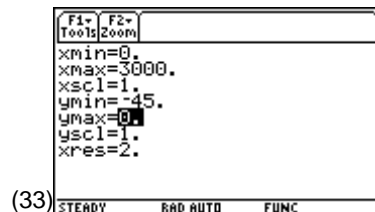
(30)

(31)

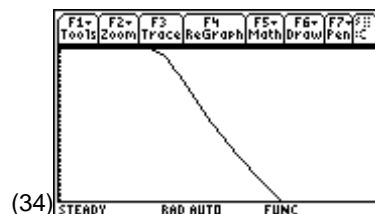
8. Now, plot the Butterworth equation to see the frequency response. To do this, press \diamond [Y=] and enter **butter** with **f** replaced by **x** (screen 32).



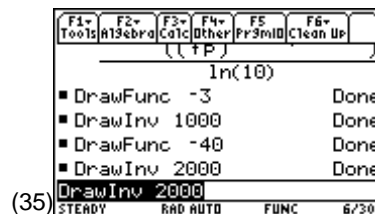
9. Since the stopband edge of the filter is 2 kHz, plot **x** from 0 to 3000. The stopband value is -40 dB, so plot **y** from -45 to 0. Enter these values in the Window Editor (\diamond [WINDOW]) as shown in screen 33.



10. Press [MODE] to be sure the **Graph** mode is set to **FUNCTION**. Then press \diamond [GRAPH] and wait a few seconds to see screen 34.

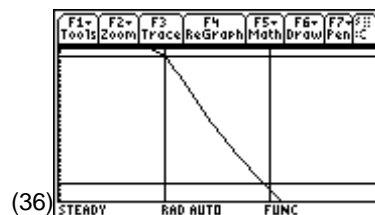


The response in the passband looks very flat, which is correct for Butterworth, but are the passband and stopband edges in the right places? These can be checked graphically by pressing [2nd] [F6] **2:DrawFunc** to draw horizontal lines at -3 and -40 dB and [2nd] [F6] **3:DrawInv** to draw vertical lines at 1000 and 2000 Hz (screen 35).



11. Press \diamond [GRAPH] to plot the results (screen 36).

The curve passes through the -3 dB point at 1000 Hz and passes below the -40 dB point at 2000 Hz. The filter meets the required specifications.



Topic 17: Chebyshev Filter

This section shows how to design a Chebyshev lowpass filter with the same specifications as discussed in Topic 16 and shown in Figure 4.

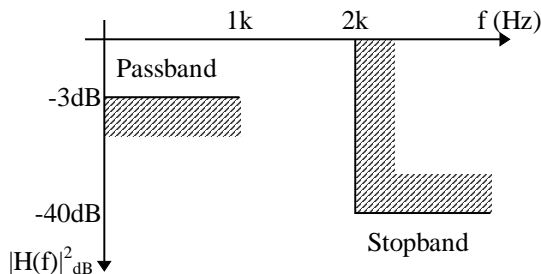


Figure 4. Filter design specifications for a Chebyshev filter

The Chebyshev equations are

$$C_n\left(\frac{f}{f_p}\right) = \cos\left(n \cos^{-1}\left(\frac{f}{f_p}\right)\right) \quad \left|\frac{f}{f_p}\right| < 1$$

$$C_n\left(\frac{f}{f_p}\right) = \cosh\left(n \cosh^{-1}\left(\frac{f}{f_p}\right)\right) \quad \left|\frac{f}{f_p}\right| > 1$$

where n is the order of the polynomial.

Therefore H_{dB} is

$$|H(f)|_{dB}^2 = 10 \log \left(\frac{1}{1 + \epsilon^2 C_n^2\left(\frac{f}{f_p}\right)} \right)$$

Follow these steps to enter these three equations.

1. From the Home screen, clear **f**, **fp**, **eps**, and **n** using **DelVar**.
2. Enter C_n for $|f/f_p| < 1$ as shown in screen 37.

`[2nd] [COS] n [×] [♦] [COS⁻¹] f [÷] fp [)] [)] [STO] cheb1`

Note: **DelVar** can be entered by pressing `[F4]4:DelVar`.

(37)

F1	F2	F3	F4	F5	F6
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
■ DrawInv 2000 Done ■ DelVar f, fp, eps, n Done ■ $\cos\left(n \cdot \cos^{-1}\left(\frac{f}{f_p}\right)\right) \rightarrow \text{cheb1}$ $\cos\left(-\cos^{-1}\left(\frac{f}{f_p}\right) \cdot n\right)$ $\cos(n * \cos^{-1}(f/f_p)) \rightarrow \text{cheb1}$					
STEADY RAD AUTO FUNC B/30					

3. Enter C_n for $|f/f_p| > 1$ as shown in screen 38.

CATALOG **cosh**(**n** × **CATALOG** **cosh**⁻¹(**f** ÷ **f_p**) **STO**►
cheb2

4. Define **cheb** to be **cheb1** for $|f/f_p| < 1$ and **cheb2** for $|f/f_p| > 1$. This is done using the “when” function as shown in screen 39.

CATALOG **when**(**CATALOG** **abs**(**f** ÷ **f_p**) **2nd** [**>**] **1** , **cheb2**
cheb1 **STO**► **cheb**

5. Define **hdb** to be $|H(f)|_{\text{dB}}$ as shown in screen 40.

10 **CATALOG** **log**(**1** ÷ (**1** + **eps** ^ **2** × **cheb** ^ **2**))
STO► **hdb**

6. To design the filter, use the Numeric Solver. Press **APPS** **9:Numeric Solver** **exp=hdb** **ENTER** (screen 41).

7. Proceed as with Butterworth. Find **eps** for -3dB at the passband edge by entering the values as shown in screen 42. The result is the same as with Butterworth.

8. Now find **n** for -40 dB at 2000 Hz. It takes a few seconds to find the order of the filter (screen 43).

9. To meet the design specifications, **n** must be 5. Enter 5 and calculate the response. See what the gain is (screen 44).

This filter exceeds the design specifications by more than 10 dB with a lower order than the Butterworth.

(38)

$$\cos\left(-\cosh^{-1}\left(\frac{f}{f_p}\right) \cdot n\right)$$

$$\cosh\left(n \cdot \cosh^{-1}\left(\frac{f}{f_p}\right)\right) \rightarrow \text{cheb2}$$

$$\cosh\left(\cosh^{-1}\left(\frac{f}{f_p}\right) \cdot n\right)$$

$$\cosh(n \cdot \cosh^{-1}(f/f_p)) \rightarrow \text{cheb2}$$

(39)

$$\begin{cases} \text{cheb2}, & \frac{f}{f_p} > 1 \\ \text{cheb1}, & \text{else} \end{cases} \rightarrow \text{cheb}$$

$$\frac{f}{f_p} > 1 \rightarrow \begin{cases} \text{cheb2}, & \frac{f}{f_p} > 1 \\ \text{cheb1}, & \text{else} \end{cases}$$

$$\frac{f}{f_p} > 1, \text{cheb2}, \text{cheb1} \rightarrow \text{cheb}$$

(40)

$$10 \cdot \ln\left(\frac{1}{\text{eps}^2 \cdot \begin{cases} \text{cheb2}, & \frac{f}{f_p} > 1 \\ \text{cheb1}, & \text{else} \end{cases}}\right)$$

$$\ln(10)$$

$$\frac{10 \cdot \ln\left(\frac{1}{\text{eps}^2 \cdot \begin{cases} \text{cheb2}, & \frac{f}{f_p} > 1 \\ \text{cheb1}, & \text{else} \end{cases}}\right)}{\ln(10)} \rightarrow \text{hdb}$$

(41)

```

exp=hdb
exp=-42.123841319632
eps=
f=
fp=
n=
bound={-1.E14,1.E14}

```

(42)

```

exp=hdb
exp=-3.
eps=.99762834511099
f=1000.
fp=1000.
n=1.
bound={-1.E14,1.E14}
left-rt=1.E-13

```

(43)

```

exp=hdb
exp=-40.
eps=.99762834511099
f=2000.
fp=1000.
n=4.0248947693522
bound={-1.E14,1.E14}
left-rt=0.

```

(44)

```

exp=hdb
exp=-51.153580310131
eps=.99762834511099
f=2000.
fp=1000.
n=5.
bound={-1.E14,1.E14}
left-rt=0.

```

10. In the Y= Editor, enter the expressions for the two filters to compare them (screen 45). Note that $n=7$ is added to $y1$ so that the Butterworth filter uses the order computed for it instead of using the value of $n=5$ used for the Chebyshev filter.

11. Press \square [GRAPH] to see the graphs (screen 46).

(See Topic 16 for a review of the instructions for a graphical comparison of the filters' performances.)

As expected, the Chebyshev filter has ripples in the passband, but it drops more rapidly in the transition band than the Butterworth.

12. Compare the filters' performances with the lines which represent the passband and stopband design specifications (screen 47). Press 2nd [F6] **2:DrawFunc** to draw horizontal lines at -3 and -40 dB. Press 2nd [F6] **3:DrawInv** to draw vertical lines at 1000 and 2000 Hz.

Both filters show the required -3 dB response at 1000 Hz; both filters exceed the specifications since they are below -40 dB at 2000 Hz.

Topic 18: Logarithmic Frequency Plots

Often frequency responses are plotted on a log frequency scale. Although the TI-89 doesn't directly support log plots, they are easy to do.

1. Return to the Y= Editor and alter the "with" operation to include the logarithmic relation of frequency with x as shown in screen 48.

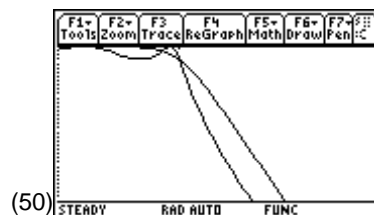
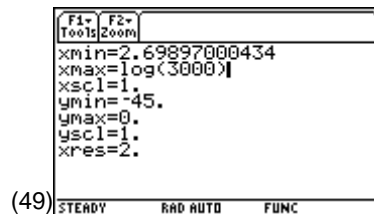
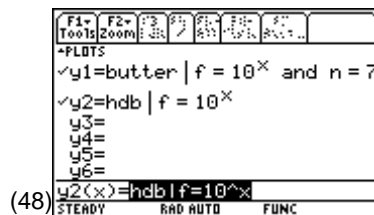
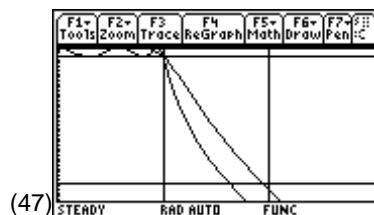
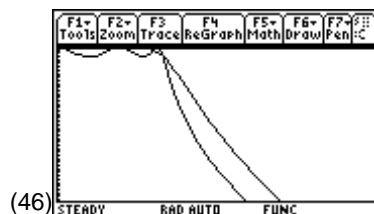
y1: butter \square $f = 10^x$ \square x [CATALOG] and $n = 7$

y2: hdb \square $f = 10^x$ \square x

The values for x are linearly spaced, but the values of 10^x are logarithmically spaced.

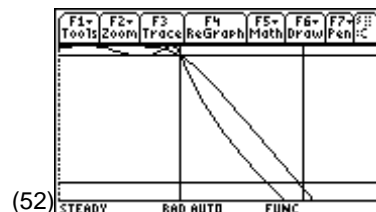
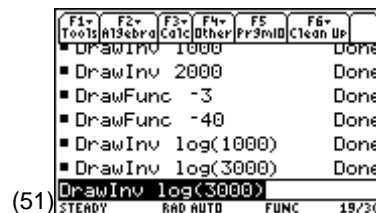
2. Press \square [WINDOW] to adjust the range on x . Graph the functions for $f=500$ and $f=3000$. To do this, enter **log(500)** for $xmin$ and **log(3000)** for $xmax$ as shown in screen 49.

3. Press \square [GRAPH] to display screen 50. The logarithmic plots take on a different appearance than the linear plots of screen 46.



4. The passband and stopband lines are added by using **log()** of the **f** values in **DrawInv** (screen 51). Press **[2nd] [F6] 2:DrawFunc** for -3 and -40 dB. Press **[2nd] [F6] 3:DrawInv** for $\log(1000)$ and $\log(2000)$ Hz.

As expected, the plots in screen 52 show that both filters meet the design specifications.



Tips and Generalizations

Topic 13 show how to enter, solve, and display equations with complex numbers. Multiple equations with multiple complex unknowns can be solved. Also the matrix approach of Topic 3 can be used with complex numbers.

The Numeric Solver works nicely for filter design but can easily solve any equation for an unknown value.

Once a steady-state response is known, the power dissipated by the various elements can be found. Chapter 5 explores this topic further.

Features Used

`[]`, `abs()`, `real()`, `imag()`,
`conj()`, `[f]`, `[a]`, `NewProb`, `[∠]`,
 Program Editor, `[Y=]`,
`exp►list()`

Setup

`[*] 1`, `NewFold power`,
`setMode("Angle", "Degree")`,
`setMode("Complex Format",`
`"Polar")`

Power Engineering

This chapter shows how to handle phasor algebra, complex power, power factor corrections, and unbalanced three-phase calculations using the TI-89.

Topic 19: Phasor Algebra

The sinusoidal, single-frequency voltages and currents of power systems are usually written as phasors — complex numbers in magnitude and phase form. This form is particularly useful in three-phase calculations when phasors must be added or subtracted. For Y-configurations, the line-to-neutral voltages, v_{an} , v_{bn} , and v_{cn} , (often called the phase voltages) are combined to give the line-to-line voltages, v_{ac} , v_{ba} , and v_{cb} as shown in Figure 1.

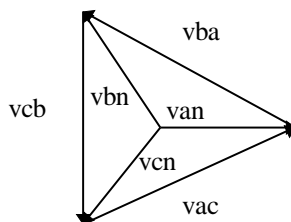


Figure 1. Three-phase line and phase voltages

For a positive phase sequence, the phasor forms of the line to neutral voltages are given as

$$v_{an} = 110$$

$$v_{bn} = v_{an} \angle 120^\circ = 110 \angle 120^\circ$$

$$v_{cn} = v_{an} \angle 240^\circ = 110 \angle 240^\circ = 110 \angle -120^\circ$$

A negative phase sequence gives the opposite signs for the phase angles.

1. Clear the TI-89 by pressing $\boxed{2nd} \boxed{F6}$ **2:NewProb** \boxed{ENTER} .
2. The function **vphase()** uses **van** and the sign of the phase sequence to calculate the phase voltages (see screen 1). Press \boxed{APPS} **7:Program Editor 3:New**, select **2:Function** for **Type**, and enter **vphase** for **Variable**. Then type the function lines as shown in screen 1.

See Tips and Generalizations for shortcuts on entering functions.

```

F1-  F2-  F3-  F4-  F5-  F6-
Tools Control I/O Var Find... Mode
: vphase(van, sqnc)
: Func
: {van, van*(1 < sqnc*120), v
:   an*(1 < sqnc*240)}
: EndFunc

```

(1) POWER DEGAUTO FUNC

Note: To enter \angle , press $\boxed{2nd} \boxed{\angle}$.

3. Calculate the phase voltages for **van**=110 with a positive phase sequence. Return to the Home screen, and use the function **vphase()** with arguments of 110 and 1 as shown in screen 2.

Two entries are displayed in screen 2. The first entry shows the returned answer; the second shows the answer scrolled to display the right side.

```

F1-  F2-  F3-  F4-  F5-  F6-
Tools AT3eBrd Calc Other Pr3mID Clean Up
■ NewProb Done
■ vphase(110, 1)
  {110 (110 < 120) (110 <
■ vphase(110, 1)
  {(110 < 120) (110 < -120)}
vphase(110, 1)
POWER DEGAUTO FUNC 3/30

```

(2)

4. For a negative phase sequence, use arguments of 110 and -1 (screen 3).

If different results are displayed, press \boxed{MODE} and set the modes as shown in the Setup section.

```

F1-  F2-  F3-  F4-  F5-  F6-
Tools AT3eBrd Calc Other Pr3mID Clean Up
■ vphase(110, 1)
  {(110 < 120) (110 < -120)}
■ vphase(110, -1)
  {110 (110 < -120) (110
■ vphase(110, -1)
  {(110 < -120) (110 < 120)}
vphase(110, -1)
POWER DEGAUTO FUNC 5/30

```

(3)

5. Each line-to-line voltage is expressed as the phasor difference of the two adjacent phase voltages, for example, **vac**=**van**-**vcn**. The function **phas2lin()** returns the line-to-line voltages in a list {**vac**, **vba**, **vcb**} for **van** and the sign of the phase sequence.

Enter the function as shown in screen 4. The function **vphase()** is used to calculate the phase voltages which then are used to calculate and return the list of line voltages.

```

F1-  F2-  F3-  F4-  F5-  F6-
Tools Control I/O Var Find... Mode
: phas2lin(van, sqnc)
: Func
: Local phase
: vphase(van, sqnc)→phase
: {phase[1]-phase[3], phase[
: 2]-phase[1], phase[3]-phas
: e[2]}
: EndFunc

```

(4) POWER DEGAUTO FUNC

6. To calculate the line-to-line voltages for a phase voltage **van**=110 with a positive phase sequence, use **phas2lin()** with arguments of 110 and 1 (screen 5). Two entries are displayed again to show the complete answer.

```

F1-  F2-  F3-  F4-  F5-  F6-
Tools AT3eBrd Calc Other Pr3mID Clean Up
■ vphase(110, -1)
  {(110 < -120) (110 < 120)}
■ phas2lin(110, 1)
  {(110·√3 < 30) (110·√3 <
■ phas2lin(110, 1)
  {(√3 < 150) (110·√3 < -90)}
phas2lin(110, 1)
POWER DEGAUTO FUNC 7/30

```

(5)

7. Phase voltages are calculated from line-to-line voltages as the difference of the adjacent line voltages, for example, $v_{an} = (v_{ac} - v_{ba})/3$. The function `lin2phas()` returns the phase voltages given the line-to-line voltages and the sign of the phase sequence.

Enter the function shown in screen 6. The **vac** line to line voltage and phase sequence are used to calculate **van** which is used to calculate the other two phase voltages.

(6)

```

F1 F2 F3 F4 F5 F6
Tools Control I/O Var Find... Mode
: lin2phas(vac, sqnc)
: Func
: Local van
: (vac-vac*(1 ∠ sqnc*120))/
: 3 → van
: (van, van*(1 ∠ sqnc*120), v
: an*(1 ∠ sqnc*240))
: EndFunc
POWER DEGAUTO FUNC

```

8. Screen 7 shows the calculation of the phase voltages for $v_{ac} = 110\sqrt{3}\angle 30^\circ$ and a positive phase sequence.

(7)

```

F1 F2 F3 F4 F5 F6
Tools R13cbra Calc Other Pr3mid Clean Up
: phase2lin(110, 1)
: (110 ∠ 150) (110 ∠ -90)
: lin2phas((110 ∠ 30), 1)
: (110 (110 ∠ 120) (110 ∠
: lin2phas((110 ∠ 30), 1)
: (110 ∠ 120) (110 ∠ -120)
: lin2phas((110 ∠ 30), 1)
POWER DEGAUTO FUNC 9/30

```

Topic 20: Average Power

Instantaneous power is defined as $p(t) = i(t)v(t)$ for real signals where $i(t)$ is the current which flows in the direction of the voltage drop across the element, $v(t)$. $p(t)$ varies with time as the signal varies. On the other hand, the average power of many signals is constant and often a more useful parameter. It is defined as

$$P_{AVG} = \frac{1}{T} \int_{t=t_0}^{t_0+T} v(t)i(t)dt$$

where T is the period of the signal.

1. Set the **Angle** mode to **Radian**.
2. To calculate the average power dissipated in a 1 kΩ resistor with a voltage of $v(t) = 10\sin(2\pi 60t)$ V across it, first calculate the current as shown in screen 8.

Since $i = v/r$, the current is $i(t) = 10\sin(2\pi 60t)$ (bottom of screen 8). The period of this signal is calculated from the relationship $T = 1/f = 1/60$ s.

3. Enter the average power, P_{AVG} , as shown in screen 9.

`[2nd] [f] v(t) i(t) t, 0, 1 ÷ 60 ÷ (1 ÷ 60)`

The average power is calculated as $1/20 = 50$ mw.

(8)

```

F1 F2 F3 F4 F5 F6
Tools R13cbra Calc Other Pr3mid Clean Up
: setMode("angle", "radian")
: "DEGREE"
: 10 ∠ sin(2 ∠ π ∠ 60 ∠ t) → v(t)
: Done
: v(t) → i(t)
: Done
: 1000 ÷ i(t)
: v(t)/1000 → i(t)
POWER RAD AUTO FUNC 12/30

```

(9)

```

F1 F2 F3 F4 F5 F6
Tools R13cbra Calc Other Pr3mid Clean Up
: v(t)
: 1000 ÷ i(t)
: Done
: ∫ 1/60 (v(t) ∠ i(t)) dt
: 1/60
: 1/20
: (t) i(t), t, 0, 1/60 ÷ (1/60)
POWER RAD AUTO FUNC 13/30

```

4. The average power dissipated in a $1\text{ }\mu\text{F}$ capacitor with a voltage of $v(t)=10\sin(2\pi 60t)$ V across it is calculated in a similar way. However, for a capacitor $i(t)=Cdv/dt$. The calculator can do the work of calculating the derivative as well as the power. To do this, enter the three expressions as shown in screen 10.

1 [EE] [(-)] 6 [STO] c
c [2nd] [d] v [(] t [)] , t [)] [STO] i [(] t [)]

and

[2nd] [] v [(] t [)] i [(] t [)] , t [)] 0 , 1 [÷] 60 [)] ÷ [(] 1 [÷] 60 [)]

The average power dissipated in a capacitor is always zero!

5. The integral calculation of power is valid when the voltage and current are not so nicely related. For example, consider an unusual device that has a periodic pulse train voltage across it with one cycle defined with the “when” function as shown on the top of screen 11.

[CATALOG] when(t [2nd] [<] 0.01 [,] 10 [,] 2 [) [STO] v [(] t [)]

This device also has a sinusoidal current given by $i(t)=5\sin(2\pi 50t+\pi/4)$, as shown on the bottom of screen 11. These expressions are entered as voltage and current.

6. Note that the two signals must be periodic with the same period $T=0.02$ seconds. To graph the two signals over one period, enter $v(x)$ and $i(x)$ in the Y= Editor (screen 12).

7. Enter the graphing parameters in the Window Editor screen as shown in screen 13.

8. Press [♦] [GRAPH] to display the results in the graph in screen 14.

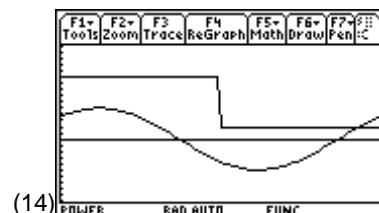
Note: The integral entered before can be copied to the entry line by moving the cursor into the history section with [◀], highlighting the integral, and pressing [ENTER].

(10)

(11)

(12)

(13)



9. The average power is found by using the same integral over the period of 0.02 seconds. Return to the Home screen, and enter this as shown in screen 15.

$$[2\text{nd}] [f] v(t) \times i(t) \int_0^{0.02} \div 0.02$$

Therefore, the average power dissipated in a device with the square wave voltage and sinusoidal current is 9.00 W.

Topic 21: Complex Power

For steady-state, sinusoidal signals with $v(t)=v_o \sin \omega t$ and $i(t)=i_o \sin(\omega t-\theta)$, the average power is calculated with the integral as before where $T=2\pi/\omega$.

1. Enter $v(t)$ and $i(t)$ as shown in screen 16.
2. Find the average power as shown in screen 17.

$$[\alpha] w \div [2] [2\text{nd}] [\pi] \times [2\text{nd}] [f] v(t) \times i(t) \int_0^{2\pi} \div [\alpha] w$$

(15)

$$5 \cdot \sin\left(2 \cdot \pi \cdot 50 \cdot t + \frac{\pi}{4}\right) \cdot i(t) \int_0^{0.02} \div 0.02 = 9.00$$

(16)

$$v_o \cdot \sin(\omega \cdot t) \rightarrow v(t)$$

$$i_o \cdot \sin(\omega \cdot t - \theta) \rightarrow i(t)$$

Note: To enter ω , press α w .

(17)

$$\frac{\omega}{2 \cdot \pi} \cdot \int_0^{2 \cdot \pi} (v(t) \cdot i(t)) dt \div \frac{i_o \cdot v_o \cdot \cos(\theta)}{2} = 9.00$$

This form provides the basis for the common expression for average power

$$P_{\text{AVG}} = \frac{v_o}{\sqrt{2}} \frac{i_o}{\sqrt{2}} \cos \theta$$

where θ is the angle by which the current lags the voltage. The terms $(v_o/\sqrt{2})$ and $(i_o/\sqrt{2})$ are known as the root-mean-square (abbreviated as RMS) voltage and current, respectively. RMS quantities are defined as the square root of the mean (or average) of the quantity squared.

$$x_{\text{rms}} = \sqrt{\frac{\int_0^T x^2(t) dt}{T}}$$

3. Calculate the RMS voltage of $v(t)=v_o \sin(\omega t)$ as shown in screen 18.

$$[2\text{nd}] [\sqrt{}] \alpha w \div [2] [2\text{nd}] [\pi] \times [2\text{nd}] [f] v(t) \int_0^{2\pi} \div \alpha w$$

The result shows that the RMS value of any sinusoidal signal is $\frac{\sqrt{2}}{2}$ times its peak magnitude.

(18)

$$\frac{\omega}{2 \cdot \pi} \cdot \int_0^{2 \cdot \pi} (v(t)^2) dt \div \frac{v_o^2 \cdot \sqrt{2}}{2} = 9.00$$

When sinusoidal voltage and current are written in phasor form, $v_o \angle 0$ and $i_o \angle -\theta$, the average power is given as $P_{\text{AVG}} = \text{Real}(v_o \cdot \text{conj}(i_o \angle -\theta)) / 2$.

4. Calculate the average power for $v(t) = v_o \sin(\omega t)$ and $i(t) = i_o \sin(\omega t - \theta)$ using the phasor method.

First enter the voltage phasor as shown in screen 19.

$\boxed{\text{v}_o \div \boxed{2\text{nd}} \boxed{\sqrt{}} \boxed{2} \boxed{)} \boxed{2\text{nd}} \boxed{\angle} \boxed{0} \boxed{)} \boxed{\text{STO}} \boxed{\text{vphasor}}$

5. Enter the current phasor (screen 20).

$\boxed{\text{i}_o \div \boxed{2\text{nd}} \boxed{\sqrt{}} \boxed{2} \boxed{)} \boxed{2\text{nd}} \boxed{\angle} \boxed{(-)} \boxed{\blacklozenge} \boxed{\theta} \boxed{)} \boxed{\text{STO}} \boxed{\text{iphasor}}$

6. Calculate the average power (screen 21).

$\boxed{\text{CATALOG}} \boxed{\text{real}(\text{vphasor} \times \boxed{\text{CATALOG}} \boxed{\text{conj}(\text{iphasor} \boxed{)} \boxed{)})$

The results are identical with the time-domain averaging.

This leads to the generalized concept of complex power. The complex power, S , is defined as $S = P + jQ$ where the real part P is identical to average power and is expressed in watts, the imaginary part Q is known as reactive power expressed in VARs (volt-amperes reactive), and the complex power S has units of VA (volt-amperes). S is calculated by $S = v_o \angle 0 \cdot \text{conj}(i_o \angle -\theta)$. The average power is $P = \text{real}(S)$; the reactive power is $Q = \text{imag}(S)$.

7. Find the reactive power as shown in screen 22.

$\boxed{\text{CATALOG}} \boxed{\text{imag}(\text{vphasor} \times \boxed{\text{CATALOG}} \boxed{\text{conj}(\text{iphasor} \boxed{)} \boxed{)})$

8. Calculation of the complex power for a load of $z_1 = 2 - j3 \, \Omega$ with a current $\text{iphasor1} = 20 \text{ A rms}$ proceeds as follows.

- a. First, enter the values of z_1 and iphasor1 (screen 23).

(19)

$$\frac{v_o}{\sqrt{2}} \angle 0 \rightarrow \text{vphasor}$$

(20)

$$\frac{i_o}{\sqrt{2}} \angle -\theta \rightarrow \text{iphasor}$$

(21)

$$\frac{\text{real}(\text{vphasor} \cdot \text{conj}(\text{iphasor}))}{2}$$

(22)

$$\frac{\text{imag}(\text{vphasor} \cdot \text{conj}(\text{iphasor}))}{2}$$

(23)

$$2 - j3 \rightarrow z_1$$

$$20 \rightarrow \text{iphasor1}$$

$$z_1 \cdot \text{iphasor1}$$

- b. The voltage of the load is $v_{\text{phasor1}} = i_{\text{phasor1}} \cdot z_{z1}$ and the complex power is $S1 = v_{\text{phasor1}} \cdot \text{conj}(i_{\text{phasor1}})$ (screen 24).

`[] vphasor1 [alpha] [x] [CATALOG] conj(iphasor1 [] []`
`[STO] s1 [2nd] [:] s1 [CATALOG] ►Rect`

The calculation gives $s1 = 800 - j1200$ VA.

(24)

Note: To enter $j = \sqrt{-1}$, press `[2nd] [i]`.

Note: Two or more commands can be entered on one line using the colon (`[2nd] [:]`) to separate them.

9. Calculation of the complex power associated with a load given by $z_{z2} = 3 + j4 \Omega$ with a voltage of $v_{\text{phasor2}} = 110$ Vrms, applied across it proceeds as follows.

- a. Press `[MODE]` and change the **Complex Format** mode to **RECTANGULAR**.
- b. First enter z_{z2} and v_{phasor2} (screen 25).
- c. The rms load current is $v_{\text{phasor2}} / z_{z2} = i_{\text{phasor2}}$; the complex power is $S2 = v_{\text{phasor2}} \cdot \text{conj}(i_{\text{phasor2}})$.

Enter the complex power as shown in screen 26.

`vphasor2 [x] [CATALOG] conj(iphasor2 [] [STO] s2`

The complex power is calculated as $s2 = 1452 + j1936$ VA. The average power which does work or produces heat in the 3Ω resistive part of the load is 1452 W. The reactive power which represents the rate of change of stored energy in the $j4 \Omega$ reactive part of the load is 1936 VARs.

(25)

Note: `setMode("Complex Format", "Rectangular")` was used since the rest of the examples are best displayed in rectangular mode. This can be set with `[MODE]` to minimize the typing.

(26)

Topic 22: Power Factor

Complex power S is supplied to a load. The real portion of the power, P , is available to do work or produce heat; the imaginary portion of the power, Q , is unusable. The fraction of complex power which is available to do work is given by the power factor

$$\text{pf} = \frac{\text{Real Power}}{\text{Complex Power}} = \frac{P}{S} = \cos \theta$$

For positive phase angles θ , the power factor is called leading; for negative phase angles, lagging. Resistive loads have a unity power factor, that is, $\text{pf} = 1$; reactive loads have a zero power factor. The angle θ is the same angle as the impedance phase angle. Since $Y = 1/Z$, the admittance phase angle is the negative of the impedance phase angle and can be used to calculate the power factor as well.

The complex power for the load $z_{z1} = 2 - j3 \Omega$ in Topic 21 is $s1 = 800 - j1200$ VARs.

- Find the power factor, pf1 (screen 27).

The power factor of **pf1** = $\frac{2 \cdot \sqrt{13}}{13}$ is lagging since it is a capacitive load, that is, the reactive component is negative.

- The complex power for the load $zz2=3+j4 \Omega$ in Topic 21 is $s2=1452+j1936$ VARs.

Screen 28 shows the power factor of **pf2** = 3/5 is leading for the inductive load.

(27)

F1→	F2→	F3→	F4→	F5	F6→
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
ZZZ					
$66/5 - 88/5 \cdot i$					
vphaser2·conj(iphasor2) →					
$1452 + 1936 \cdot i$					
real(s1) → pf1					
$ s1 $					
$\frac{2 \cdot \sqrt{13}}{13}$					
real(s1)/abs(s1)+pf1					
POWER	RAD AUTO	FUNC	30/30		

(28)

F1→	F2→	F3→	F4→	F5	F6→
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
$1452 + 1936 \cdot i$					
real(s1) → pf1					
$ s1 $					
$\frac{2 \cdot \sqrt{13}}{13}$					
real(s2) → pf2					
$ s2 $					
$3/5$					
real(s2)/abs(s2)+pf2					
POWER	RAD AUTO	FUNC	30/30		

Topic 23: Power Factor Correction Using Impedances

Reduction of the reactive power improves the power factor which means that more of the power generated by the electric utility can be sold. Therefore, rate incentives are offered for industrial users to improve or “correct” their power factor. Most large users have a leading power factor due to the inductive nature of motors. A leading power factor can be “corrected,” that is, brought closer to unity, by adding capacitors in parallel. The capacitive susceptance, $j2\pi fC$, cancels part of the inductive susceptance of the load, $-jB_L$, and makes the angle θ smaller and the $\text{pf}=\cos\theta$ closer to unity. See Figure 2.

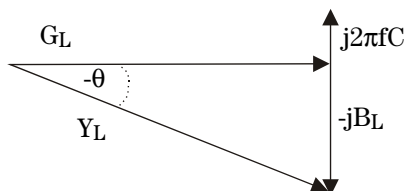


Figure 2. Admittance triangle

The procedure for calculating the amount of parallel capacitance needed to correct the power factor to unity for the inductive load $zz2=3+j4 \Omega = 5 \angle 53.1^\circ \Omega$ (Topic 21) follows. This load has $\text{pf2}=\cos(53.1^\circ)=3/5=0.6$ (Topic 22).

- Convert the impedance to admittance, $yy2=1/zz2= 3/25 - j4/25 = 0.12-j0.16$ S (top of screen 29).
- The added parallel capacitive susceptance must cancel the inductive susceptance so that $j(0.16)=j2\pi(60)C$ which gives $C=424 \mu\text{F}$ (bottom of screen 29).

(29)

F1→	F2→	F3→	F4→	F5	F6→
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
$\frac{1}{zz2} \rightarrow yy2$					
$3/25 - 4/25 \cdot i$					
$-imag(yy2)$					
$2 \cdot \pi \cdot 60$					
$\frac{1}{750 \cdot \pi}$					
$-imag(yy2)$					
$2 \cdot \pi \cdot 60$					
$4.24 \cdot 10^{-4}$					
$-imag(yy2)/(2\pi 60)$					
POWER	RAD AUTO	FUNC	30/30		

Note: Calculate the floating-point result by pressing $\square \approx$.

Unity power factor is often prohibitively expensive to achieve so the rates are set to offer incentive for partial correction.

Repeat the last example for the load $zz2=3+j4 \Omega$ but with a final power factor of $\text{pf}=0.9$ leading. The load conductance does not change with the addition of the parallel capacitor. To obtain the specified power factor, the added capacitive susceptance must result in a leading $\text{pf}=0.9=\cos\theta$ so that $\tan\theta=\tan(\cos^{-1}(0.9)) = -0.484$.

With the addition of the parallel capacitor, the total susceptance of the load is given as $b_{tot} = \text{imag}(y_{y2}) + 2\pi 60c$.

The ratio of $\text{imag}(s)/\text{real}(s) = (\text{imag}(y_{y2}) + 2\pi 60c) / \text{real}(y_{y2}) = \tan\theta = \tan(\cos^{-1}(0.9))$.

Find the desired value of capacitance as shown in screen 30.

`CATALOG solve((CATALOG imag(yy2) + 2 [2nd] [π] 60c) ÷`
`CATALOG real(yy2) = (-) [2nd] [TAN] * [COS-1] 0.9)) , c)`

The result is $C = 270 \mu\text{F}$, somewhat less than that needed for unity power factor and therefore less expensive.

(30)

Topic 24: Power Factor Correction Using Power Triangle

Alternatively, power factor calculations can be made in terms of complex power as well, but these calculations require the voltage or current.

To recalculate the parallel capacitance needed to bring the power factor to 0.9 leading when the complex power is $S = 1452 + j1936 \text{ VA}$, recall that $P = \text{real}(S)$ and $Q = \text{imag}(S)$ (top of screen 31).

Some negative reactive power is introduced by the added parallel capacitance

$$\text{pf} = 0.9 = P/S = P/(P + j(Q - Q_{\text{CAP}})).$$

1. Enter this as shown in screen 31.

`CATALOG solve(0.9 = p2 ÷ (CATALOG abs(p2 + [2nd] [i]`
`[q2 - qcap])) , qcap)`

The complete answer is $q_{\text{cap}} = 2639.24$ or $q_{\text{cap}} = 1232.76$

(31)

2. Enter the second solution to obtain

$$Q_{\text{CAP}} = 1232.8 = 2\pi 60 C |v|^2 = 2\pi 60 C (|v_{\text{phasor2}}|)^2.$$

`CATALOG exp>list([2nd] [ANS] , qcap) [STO] cs`

and

`CATALOG solve(cs [2nd] [i] 1 [2nd] [i] = 2 [2nd] [π] 60c ×`
`CATALOG abs(vphasor2) ^ 2 , c)`

Screen 32 shows this leads to $c = 270 \mu\text{F}$, as before.

Note that the second solution $cs[2] = 2639.2$ gives a power factor of 0.9, but lagging. Since this requires a capacitor twice as large, it is not an economical solution.

(32)

Note: `exp>list(ans(1),qcap) [STO] cs` is used to convert the results of the first solve into a list which is stored in **cs**. **cs[1]** uses the first solution to solve for **c**.

Topic 25: Y-Δ and Δ-Y Transformations

Many circuits are simplified by transforming a circuit from a Y configuration to a Δ or a Δ configuration to a Y as shown in Figure 3. The transformation is particularly useful in simplifying circuits for later series-parallel combinations.

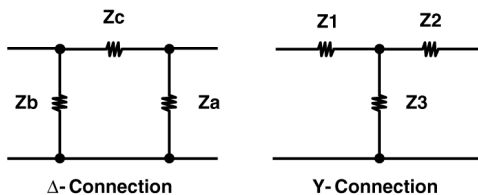


Figure 3. Δ and Y Configurations

The transformations are expressed as

$$\begin{aligned} Z_1 &= \frac{Z_b Z_c}{Z_a + Z_b + Z_c} & Z_a &= \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1} \\ Z_2 &= \frac{Z_c Z_a}{Z_a + Z_b + Z_c} & Z_b &= \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2} \\ Z_3 &= \frac{Z_a Z_b}{Z_a + Z_b + Z_c} & Z_c &= \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3} \end{aligned}$$

Since there are multiple inputs and multiple results, functions work well to make these transformations.

1. Press **[APPS]** **7:Program Editor 3:New** and define the two functions, **delta2y()** and **y2delta()**, to implement these equations (screens 33 and 34).

(33)

```

F1=  F2=  F3=  F4=  F5=  F6=
Tools Control I/O Var Find... Mode
:delta2y(z)
:Func
:Local den,za,zb,zzc
:z[1]→za:z[2]→zb:z[3]→zzc
:za+zb+zzc→den
: (zb*zzc,zzc*za,za*zb)/den
:EndFunc
POWER RAD AUTO FUNC
  
```

(34)

```

F1=  F2=  F3=  F4=  F5=  F6=
Tools Control I/O Var Find... Mode
:y2delta(z)
:Func
:Local num
:z[1]*z[2]+z[2]*z[3]+z[3]*
:z[1]→num
:(num/(z[1]),num/(z[2]),nu
:m/(z[3]))
:EndFunc
POWER RAD AUTO FUNC
  
```

2. Return to the Home screen, and use the **delta2y()** function to calculate the Y-configuration elements for a delta-configured circuit with $z_a\Omega = 3+j$, $z_b=5+j5\Omega$, and $z_c=2-j4\Omega$ as shown in the middle of screen 35.

delta2y **[]** **[2nd]** **[i]** **3** **+** **[2nd]** **[i]** **5** **+** **[2nd]** **[i]** **5** **,** **2** **-** **[2nd]** **[i]** **4** **[2nd]** **[i]** **[]**

The **delta2y()** function gives the results $z_1=2.69-j1.54\Omega$, $z_2=0.77-j1.15\Omega$, and $z_3=1.35+j1.73\Omega$ (middle of screen 35).

3. The inverse transformation, **y2delta()**, returns the original impedance values (bottom of screen 35).

(35)

```

F1=  F2=  F3=  F4=  F5=  F6=
Tools Algebra Calc Other Pr3mID Clean Up
■ solve[cs[1]=2·π·60·C·|v|
c=2.70E-4
■ delta2y({3.+i 5+i.5
{2.69-1.54.i .77-1.15}
■ y2delta({2.6923076923077
{3.00+1.00.i 5.00+5.0
y2delta(ans(1))
POWER RAD AUTO FUNC 30/30
  
```

Topic 26: Unbalanced Three-Phase Systems

Solving balanced three-phase Y-systems is quite simple. Since all phases have the same sources and loads, their voltages and currents differ from each other only by 120° . The solution for a single phase is just shifted by $\pm 120^\circ$ to get the solutions for the other phases. Although the solutions are more complicated for unbalanced three-phase Y-systems, they are easy with the TI-89's matrix operations.

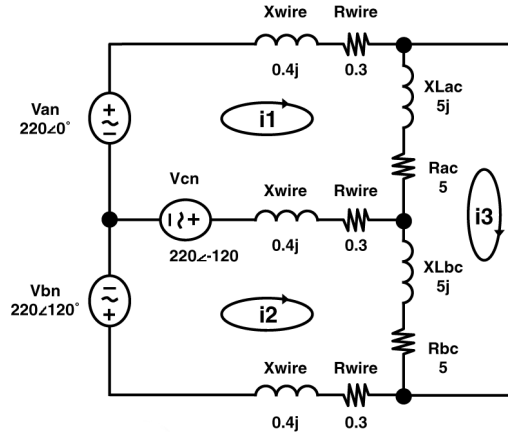


Figure 4. Unbalanced, three-phase circuit

The three-phase system of Figure 4 has a short-circuit across one phase. The currents can be calculated by using Kirchhoff's voltage law to write three mesh equations for the circuit.

$$i1(5.6 + j5.8) - i2(0.3 + j0.4) - i3(5.0 + j5.0) = 220 - (220\angle -120)$$

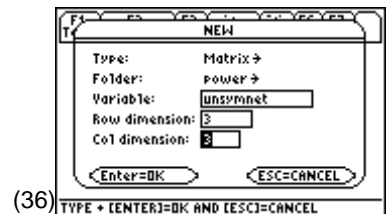
$$-i1(0.3 + j0.4) + i2(5.6 + j5.8) - i3(5.0 + j5.0) = (220\angle 120) + (220\angle -120)$$

$$-i1(5 + j5) - i2(5 + j5) + i3(10 + j10) = 0$$

In matrix form, the equations are

$$\begin{bmatrix} 5.6 + j5.8 & -.3 - j.4 & -5 - j5 \\ -.3 - j.4 & 5.6 + j5.8 & -5 - j5 \\ -5 - j5 & -5 - j5 & 10 + j10 \end{bmatrix} \begin{bmatrix} i1 \\ i2 \\ i3 \end{bmatrix} = \begin{bmatrix} 220 - (220\angle -120) \\ (220\angle 120) + (220\angle -120) \\ 0 \end{bmatrix}$$

1. Press **[MODE]**. Then set **Angle** mode to **DEGREE** and **Complex Format** mode to **POLAR**.
2. Create a new matrix variable. Press **[APPS]** **6:Data/Matrix Editor 3:New**. Select **2:Matrix** for **Type**, name the variable **unsymnet**, and set the row and column dimensions to 3 (screen 36). Press **[ENTER]** to display the matrix.



3. Enter the elements from the matrix equation above. Note that you can enter the values in rectangular form. They will automatically be displayed in polar form (screen 37). For j, press $\boxed{2\text{nd}} \boxed{[i]}$.

4. To enter the 3x1 source matrix on the right hand side of the equation, create a new matrix by pressing $\boxed{F1}$ **3:New** (screen 38). Select **2:Matrix** as the type, name the variable **source**, and set the row dimension to 3 and column dimension to 1. Press $\boxed{\text{ENTER}}$.

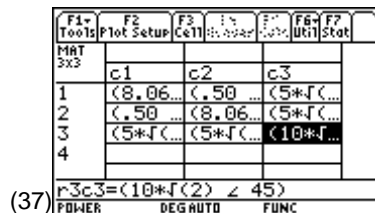
5. Enter the values from the right hand side of the matrix equation in the same manner (screen 39).

6. Once the matrices are entered, matrix math is all that's needed to find the solution for the current matrix, ii. Return to the Home screen, and enter this as shown in screen 40.

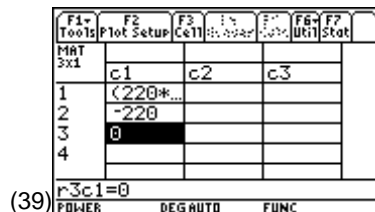
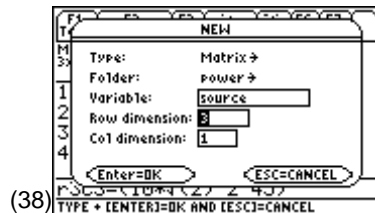
$\text{unsymnet} \wedge \boxed{(-)} \boxed{1} \boxed{)} \times \text{source} \boxed{\text{STO}} \boxed{\triangleright} \text{ii}$

7. The load voltage across the short circuit is zero. The load voltage across the zac is $\text{vac}=(i1-i3)(5+j5)$ (screen 41).

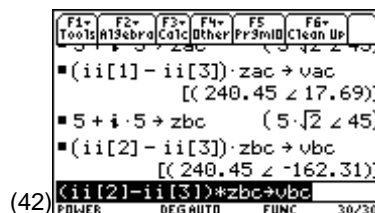
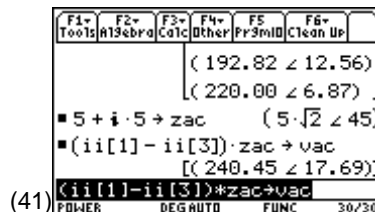
8. The load voltage across zbc is $\text{vbc}=(i2-i3)(5+j5)$. Enter the expressions as shown in screen 42. As expected, the two non-zero voltages are the same magnitude, but 180° out of phase.



Note: Press $\boxed{\blacktriangledown} \boxed{1}$ and set the column width to 6 to get the display above.



Note: When the first element of a complex number is preceded by a minus sign it must be entered with $\boxed{(-)}$. A minus sign on a following element is entered as $\boxed{-}$.



Tips and Generalizations

There are many shortcuts for entering functions. Check under the **F2**, **F3**, and **F4** menus while in the Program Editor. For example, **Local** can be entered by pressing **F4** **3:Local**.

The string **phase** is entered often in the examples in this chapter. To save typing, enter it once and just after pressing **e**, press and hold **↑** while pressing **Ⓢ** five times (once for each letter in **phase**).

Finally, release the **↑** key and press **Ⓢ** [COPY]. This will save a copy of the highlighted letters so that they can be quickly entered by pressing **Ⓢ** [PASTE].

The TI-89 can easily handle Greek symbols such as α and ω . However, they take four keystrokes each to enter (**Ⓢ** **Ⓢ** **[alpha]** **a** and **Ⓢ** **Ⓢ** **[alpha]** **w**). If convenience is important, simply use the English equivalents (**a**, **w**) which take only two keystrokes (**[alpha]** **a** or **[alpha]** **w**) or fewer if [a-lock] is already set.

The concept of building a simple function to do repetitive tasks was introduced. Don't underestimate the power of this method. For example, the whole power factor correction example could be written as a function that takes various circuit parameters as an input and returns the capacitor value needed to correct the power factor. Think of the time savings!

Another analysis technique involves the Laplace transform and the s-domain. Chapter 6 shows how the symbolic capabilities of the TI-89 make it the ideal tool for dealing with all of those s's.

Features Used

[GRAPH], solve(), expand(),
getDenom(), zeros(),
NewProb, [Y=], getNum(),
factor(), [1], 3D graph, abs(),
[WINDOW], cFactor(),
NewData, cZeros(), real(),
imag()

Setup

◆ 1, NewFold laplace

Laplace Analysis: The s-domain

This chapter demonstrates the utility of symbolic algebra by using the Laplace transform to solve a second-order circuit. The method requires that the circuit be converted from the time-domain to the s-domain and then solved for $V(s)$. The voltage, $v(t)$, of a sourceless, parallel, RLC circuit with initial conditions is found through the Laplace transform method. Then the solution, $v(t)$, is graphed.

This chapter also shows how to find and plot the poles and zeros of a circuit's transfer function $H(s)$ to gain insight to the frequency response.

Topic 27: RLC Circuit

Given the circuit shown in Figure 1, find $v(t)$ for $t > 0$ when $v(0) = 4$ V and $i(0) = 1$ A.

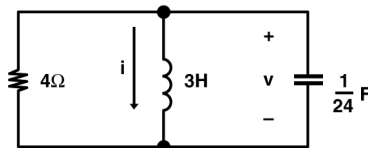


Figure 1. Simple parallel RLC circuit

Convert the components to their s-domain equivalents. Remember, the time-domain components map to their s-domain counterparts as shown in Figure 2.

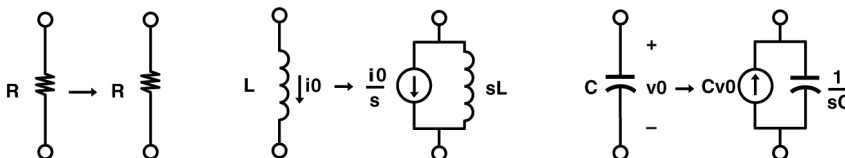


Figure 2. Time-domain to s-domain mappings

Note that the initial voltage and current transform into equivalent sources in the s-domain. The circuit in the s-domain is shown in Figure 3.

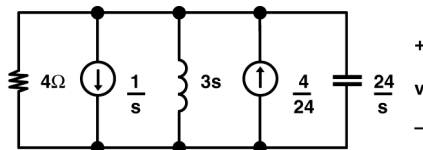


Figure 3. s-domain equivalent of the circuit in Figure 1

Using Kirchhoff's current law to sum the currents out of the top node, the equation is

$$\frac{v}{4} + \frac{v}{3s} + \frac{v}{24} = \frac{4}{24} - \frac{1}{s} \rightarrow n1$$

1. Clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.

2. Enter the equation above (screen 1).

$$v \div 4 + v \div (3s) + v \div (24 \div s) = 4 \div 24 - 1 \div s \text{ STO } n1$$

(1)

3. The s-domain voltage is found with **solve(n1,v)** as shown in screen 2.

$$\text{[CATALOG] solve(n1, v) STO eqn}$$

(2)

4. Enter **expand(eqn)** to put **eqn** in a form for easy calculation of the inverse Laplace transform via a table lookup (screen 3).

This must be an overdamped circuit since there are two real poles. The answer should contain two decaying exponents. From a Laplace transform table, the solution is

$$v(t) = 20e^{-4t} - 16e^{-2t} \quad t \geq 0$$

This answer is in the expected mathematical form. How does $v(t)$ appear as a function of time?

(3)

5. To obtain a graph, press \diamond [Y=] and enter the expression for $v(t)$ as shown in screen 4. Note that x is substituted for t using the “with” operator, $\boxed{\text{I}}$, since the Y= Editor requires equations to be expressed as functions of x .

$$20 \times \diamond [e^x] (-) 4t \boxed{\text{I}} - 16 \times \diamond [e^x] (-) 2t \boxed{\text{I}} \boxed{\text{I}} t = x$$

6. Now press $\boxed{\text{F2}}$ 6:ZoomStd to see the graph (screen 5).

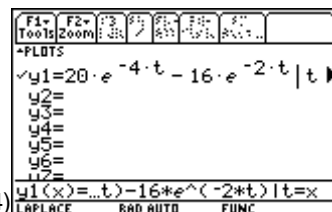
It appears to be a typical overdamped response!

7. To zoom in for a closer look, press \diamond [WINDOW] and set the range of x to be 0 to 4 (screen 6).

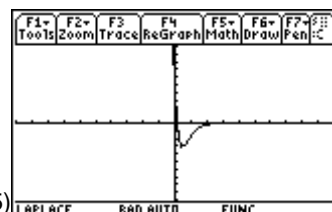
8. Now, press \diamond [GRAPH] to see the graph of $v(t)$ as shown in screen 7.

9. Press $\boxed{\text{F1}}$ 9:Format and specify ON for Grid and Labels (screens 8 and 9).

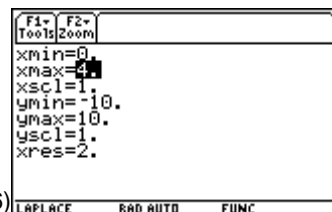
Note: x is substituted for t using the **with operator**, $\boxed{\text{I}}$, since the Y= Editor requires equations to be expressed as functions of x .



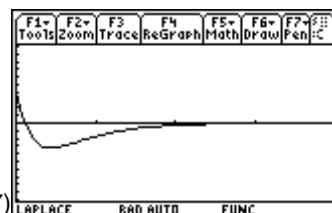
(4)



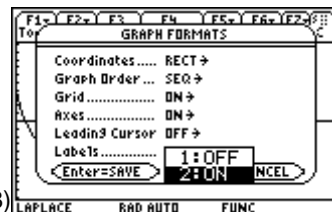
(5)



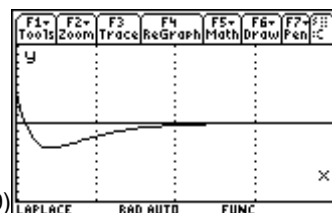
(6)



(7)



(8)



(9)

Topic 28: Critical Damping

Given the circuit of Topic 27, change the value of the resistor so that the circuit will be critically damped.

Assign the resistor value as r (see Figure 4) and let the TI-89 compute the node voltage $v(s)$ in terms of r .

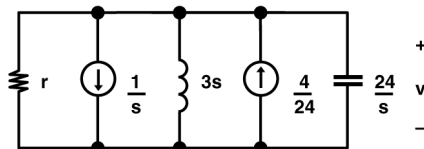


Figure 4. Circuit of Figure 3 with the $4\ \Omega$ resistor changed to r

The nodal equation in the s -domain is

$$\frac{v}{r} + \frac{v}{3s} + \frac{v}{\frac{24}{s}} = \frac{4}{24} - \frac{1}{s} \rightarrow n1$$

- Return to the Home screen, and enter this as shown in screen 10.

$$v \div r + v \div (3s) + v \div (24/s) = 4/24 - 1/s \rightarrow n1$$

- Solve for the node voltage with **solve(n1,v) → eqn** (screen 11).

- For critical damping, the time constants of the two exponentials of $v(s)$ must be real and equal. To determine this condition, the two roots of the denominator of $v(s)$ are found and set equal, and the resulting equation is solved for the required value of r . Get the denominator with **getDenom()** as shown in screen 12.

$$\text{CATALOG } \text{getDenom}(v | \text{eqn}) \rightarrow \text{eqn2}$$

- Solve for values of s which are the roots of the denominator using the **zeros()** command as shown in screen 13.

$$\text{CATALOG } \text{zeros}(\text{eqn2}, s) \rightarrow z$$

(10)

(11)

(12)

(13)

5. Set the two roots equal to each other and solve for r as shown in screen 14.

CATALOG **solve**(**z** [2nd] [C] 1 [2nd] [1] [=] **z** [2nd] [C] 2 [2nd] [1] [,] **r** [1])

Since negative resistances are not physically possible, the answer must be $r = 3\sqrt{2}$.

6. To get the floating point approximation, press \diamond [\approx] as shown in screen 15.

So $r = 4.2$ will give critical damping.

(14)

zeros(eqn2, s) \rightarrow z
 $\left\{ \begin{array}{l} -2 \cdot \left(\sqrt{-2 \cdot (r^2 - 18)} + 6 \right) \\ r \end{array} \right\}$
 solve(z[1] = z[2], r)
 $r = 3 \cdot \sqrt{2}$ or $r = -3 \cdot \sqrt{2}$
solve(z[1]=z[2],r)
 LAPLACE RAD AUTO FUNC 9/30

(15)

Tools Al3ebra Calc Other Pr3mID Clean Up
 $\left\{ \begin{array}{l} -2 \cdot \left(\sqrt{-2 \cdot (r^2 - 18)} + 6 \right) \\ r \end{array} \right\}$
 solve(z[1] = z[2], r)
 $r = 3 \cdot \sqrt{2}$ or $r = -3 \cdot \sqrt{2}$
 solve(z[1] = z[2], r)
 $r = 4.24$ or $r = -4.24$
solve(z[1]=z[2],r)
 LAPLACE RAD AUTO FUNC 10/30

Topic 29: Poles and Zeros in the Complex Plane

Given that

$$H(s) = \frac{s^4 + 14s^3 + 74s^2 + 200s + 400}{s^4 + 10s^3 + 49s^2 + 100s}$$

find and plot the poles and zeros.

1. Enter $h(s)$ as shown in screen 16.

$(s^4 + 14s^3 + 74s^2 + 200s + 400) \div (s^4 + 10s^3 + 49s^2 + 100s) \rightarrow h$

2. A quick way to see the poles and zeros is to factor $h(s)$ as shown in screen 17.

CATALOG **factor**(**h**)

(16)

F1 Tools Al3ebra Calc Other Pr3mID Clean Up
 $\frac{s^4 + 14 \cdot s^3 + 74 \cdot s^2 + 200 \cdot s + 400}{s^4 + 10 \cdot s^3 + 49 \cdot s^2 + 100}$
 $\frac{s^4 + 14 \cdot s^3 + 74 \cdot s^2 + 200 \cdot s + 400}{s \cdot (s^3 + 10 \cdot s^2 + 49 \cdot s + 10)}$
 $\rightarrow h$
 LAPLACE RAD AUTO FUNC 11/30

Note: To enter **factor**(, press **F2** **2:factor**(.

(17)

F1 Tools Al3ebra Calc Other Pr3mID Clean Up
 $s \cdot (s^3 + 10 \cdot s^2 + 49 \cdot s + 10)$
factor(h)
 $(s^2 + 2 \cdot s + 10) \cdot (s^2 + 12 \cdot s + 25)$
 $s \cdot (s + 4) \cdot (s^2 + 6 \cdot s + 25)$
factor(h)
 LAPLACE RAD AUTO FUNC 12/30

3. However, since **factor**() doesn't give complex factors, use **cFactor**() to get more information about $h(s)$ (screen 18).

CATALOG **cFactor**(**h** [1] **s** [1])

Press \odot \triangleright to see the rest of the terms of $h(s)$. The complete answer is

$$\frac{(s - 2(-3 + i))(s - (-1 + 3i))(s + 1 + 3i)(s + 2(3 + i))}{s(s + 4)(s - (-3 + 4i))(s + 3 + 4i)}$$

(18)

F1 Tools Al3ebra Calc Other Pr3mID Clean Up
 $(s^2 + 2 \cdot s + 10) \cdot (s^2 + 12 \cdot s + 25)$
 $s \cdot (s + 4) \cdot (s^2 + 6 \cdot s + 25)$
cFactor(h, s)
 $(s - 2 \cdot (-3 + i)) \cdot (s - (-1 + 3 \cdot i))$
 $s \cdot (s + 4) \cdot (s - (-3 + 4 \cdot i))$
cFactor(h,s)
 LAPLACE RAD AUTO FUNC 13/30

4. **getNum()** and **getDenom()** (screen 19) give the numerator and denominator, respectively.

CATALOG **getNum(h)** **STO** num

CATALOG **getDenom(h)** **STO** denom

The TI-89 automatically expands these terms, so **cFactor()** must be used again if you want to see the factors.

5. Once the numerator and denominator are separated, the zeros and poles are found by using the **cZeros()** command (screens 20 and 21).

CATALOG **cZeros(num, s)** **STO** zero

CATALOG **cZeros(denom, s)** **STO** pole

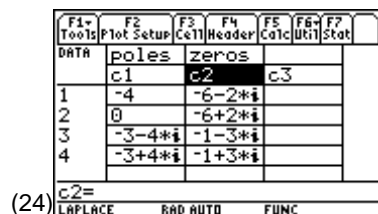
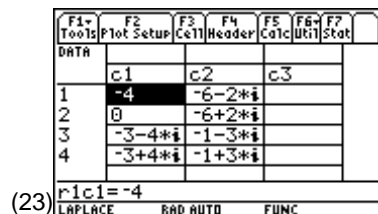
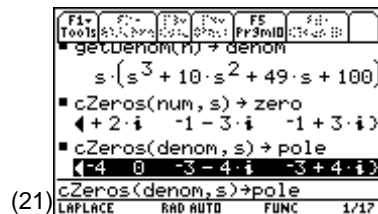
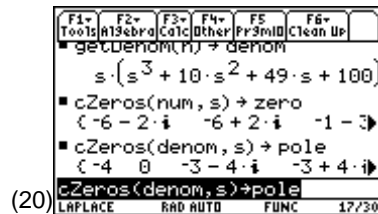
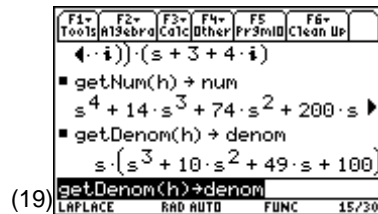
Plotting the poles and zeros takes a few steps.

- a. Store the two lists in a data object called **pz** (screen 22).

- b. **pz** can't be displayed in the Home screen, but it can be edited by pressing **APPS** **6:Data/Matrix Editor 2:Open** (screen 23).

- c. The first column lists the poles; the second column lists the zeros. To help remember this, add labels to each of the columns by pressing **⊖ ⊖** and typing **poles** **ENTER** followed by **⊕ ⊖** and typing **zeros** **ENTER** (screen 24).

Note: To enter **getNum()** and **getDenom()** press **F2** **B:Extract**, then **1:getNum()** or **2:getDenom()**.



The real part of each pole (or zero) provides the x-component and the imaginary part, the y-component in the complex plane.

- d. To separate the poles into their real and imaginary parts, first press \blacktriangleright and type **real(c1)** $\boxed{\text{ENTER}}$. This makes column **c3** the real part of column **c1**.
- e. Then press $\blacktriangleright \ominus$ **imag(c1)** $\boxed{\text{ENTER}}$ to make column **c4** the imaginary part of **c1** (screen 25).
- f. Repeat this process for the zeros making column **c5** the real part of **c2** ($\blacktriangleright \ominus$ **real(c2)** $\boxed{\text{ENTER}}$) and column **c6** the imaginary part of **c2** ($\blacktriangleright \ominus$ **imag(c2)** $\boxed{\text{ENTER}}$). Note that the screen scrolls to reveal **c5** and **c6** (screen 26).

- g. To plot the data, press $\boxed{\text{F2}}$ $\boxed{\text{F1}}$ and fill in the required data as shown in screen 27. Press $\boxed{\text{ENTER}}$.

This will plot the real part of the poles (**c3**) versus the imaginary part of the poles (**c4**) as a cross.

- h. Press $\ominus \boxed{\text{F1}}$ to set **Plot 2** to plot the zeros with boxes (screen 28). Press $\boxed{\text{ENTER}}$.

- i. Press \blacklozenge $\boxed{\text{WINDOW}}$ to set the plot ranges (screen 29). Turn **OFF Grid** and **Labels** with \blacklozenge 1. Turn off the previous graph with \blacklozenge $\boxed{1}$.

- j. Finally, press \blacklozenge $\boxed{\text{GRAPH}}$ to see the poles and zeros graphed in the complex plane (screen 30). This representation is usually called the pole/zero constellation.

F1	F2	F3	F4	F5	F6	F7
Tools	Plot Setup	Cell Header	Calc	Util	Stat	
DATA	poles	zeros				
	c1	c2	c3	c4		
1	-4	-6-2...	-4	0		
2	0	-6+2...	0	0		
3	-3-4...	-1-3...	-3	-4		
4	-3+4...	-1+3...	-3	4		

(25) r1c1=-4

Note: Press \blacklozenge $\boxed{1}$ and select a cell width of 5 to see four columns.

F1	F2	F3	F4	F5	F6	F7
Tools	Plot Setup	Cell Header	Calc	Util	Stat	
DATA	c3	c4	c5	c6		
1	-4	0	-6	-2		
2	0	-6	2			
3	-3	-4	-1	-3		
4	-3	4	-1	3		

(26) r1c6=-2

laplace\pz Plot 1

Plot Type..... Scatter \blacktriangleright

Mark..... Cross \blacktriangleright

X..... c3

Y..... c4

Freq and Categories? NO \blacktriangleright

Enter=SAVE ESC=CANCEL

TYPE * [ENTER]=OK AND [ESC]=CANCEL

(27)

laplace\pz Plot 2

Plot Type..... Scatter \blacktriangleright

Mark..... Box \blacktriangleright

X..... c5

Y..... c6

Freq and Categories? NO \blacktriangleright

Enter=SAVE ESC=CANCEL

TYPE * [ENTER]=OK AND [ESC]=CANCEL

(28)

F1 F2

Tools Zoom

xmin=-10.

xmax=1.

xsc1=1.

ymin=-5.

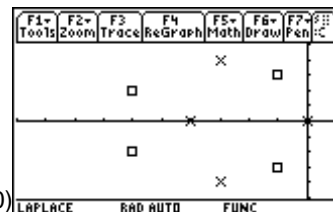
ymax=5.

ysc1=1.

yres=2.

LAPLACE RAD AUTO FUNC

(29)



(30)

Topic 30: Frequency Response

The frequency response that corresponds to the pole/zero constellation in Topic 29 is graphed by noting that

$$|H(j\omega)| = |H(s)|_{s=j\omega}$$

1. To do this, press **HOME** and enter the equation as shown in screen 31.

CATALOG **abs**(**h**) **|** **s** = **i** **w** **STO** **eqn**

2. Enter **eqn** as the function **y1(x)** (screen 32).

eqn **|** **w** = **x** **STO** **y1** (**x**)

3. Press **◆** **[Y=]** to verify this. Be sure to deselect plots 1 and 2 in the Y= Editor using **F4** (screen 33).

4. Press **◆** **[WINDOW]** to set the correct graphing parameters in the Window Editor (screen 34).

5. Press **◆** **[GRAPH]** to display the graph of frequency response (screen 35).

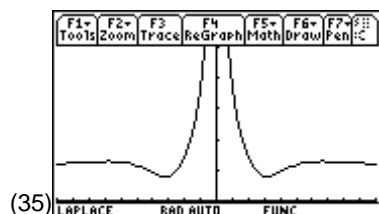
Notice that the effects of the pole farthest from the axis can be seen as slight rises near the left and right sides of the graph. The zeros are causing the dips around $x = \pm 3$, and the pole at the origin is causing the large peak in the middle.

(31)

(32)

(33)

(34)



Topic 31: 3D Poles and Zeros

A different perspective of $H(s)$ is gained from a 3D graph where the z-axis represents the magnitude of $H(s)$.

- To do so, press **MODE** \rightarrow and select **5:3D** (screen 36). Press **ENTER**.

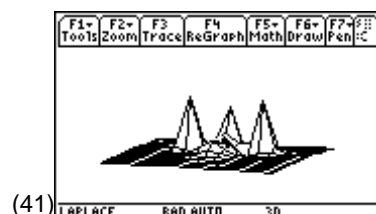
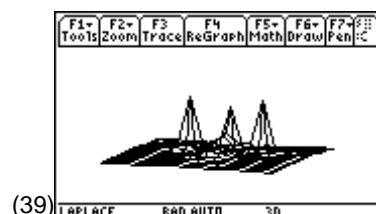
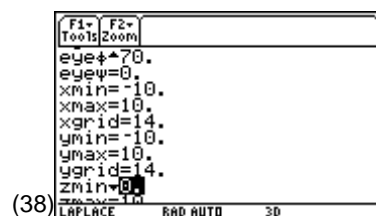
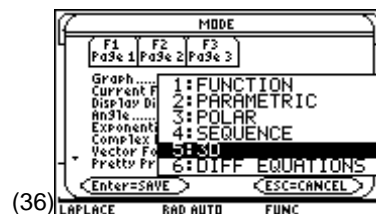
- Press \diamond **[Y=]** and enter the function to be graphed (screen 37).

CATALOG **abs**(**h** **|** **s** **=** **x** **+** **2nd** **[i]** ***** **y** **)**

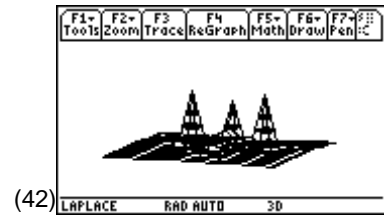
- Press \diamond **[WINDOW]** and set the **x**, **y**, and **z** scales (screen 38). Note that these are the default values, except **zmin** has been set to 0.

- Finally, press \diamond **[GRAPH]** (screen 39). It will take a few minutes for the graph to display. Once the graph is complete, press \diamond **[I]** and select **AXES** and turn **ON** the **Labels**.

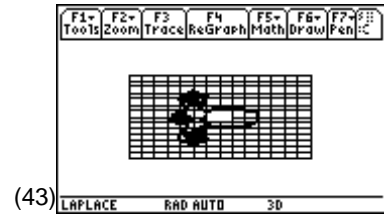
- The three poles are clearly visible. Things to try:
Press **[X]**, **[Y]**, or **[Z]** to look down the corresponding axis.
Use the cursor controls (\leftarrow \rightarrow \uparrow \downarrow) to spin the graph.
Press **[0]** to return to the original view.
- Press \diamond **[I]** and change the **Style** to **HIDDEN SURFACE** (screens 40 and 41).



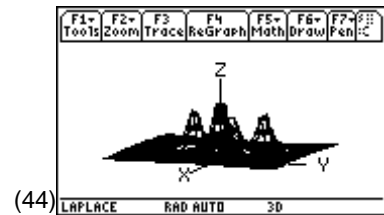
7. Press \blacklozenge \boxed{I} and change the **Style** to **WIRE AND CONTOUR** to see contours highlighted on the graph (screen 42). This will take a few minutes to recalculate.



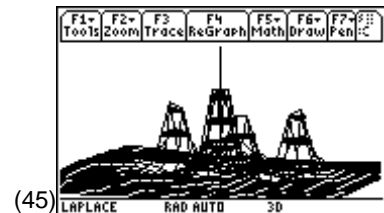
8. Press \boxed{Z} while in **WIRE AND CONTOUR** mode to view the contours from above (screen 43). Press $\boxed{0}$ to return to the original view.



9. Press \blacklozenge \boxed{WINDOW} , set **xgrid** and **ygrid** to larger values (25 in this case), and press \blacklozenge \boxed{GRAPH} to get a smoother graph (screen 44). This also takes a few minutes to recalculate.



10. To zoom in (screen 45), press the $\boxed{\times}$ key (the multiplication key, not the letter x).



Tips and Generalizations

The TI-89's symbolic math capability makes it a good choice for manipulating equations in the s-domain. The key step to plotting on the s-plane (real vs. imaginary) is to use the "with" operator (\boxed{I}) to replace **s** with **x + [i] y**. Although plotting $|H(s)|$ is most common, the TI-89 can just as easily plot the angle of $H(s)$ by entering

angle(h \boxed{I} **s** \boxed{I} **s** $\boxed{=}$ **x** $\boxed{+}$ **[i]** **y** \boxed{I} **).**

Although these examples solved for a single node problem with only one equation, **v(s)**, more complex circuits with more nodes (and therefore more equations) also can be solved.

The TI-89 assisted the conversion from the s-domain to the time-domain by doing the partial fraction expansion. Chapter 7 shows how to find a system's response by staying in the time-domain and using convolution.

Features Used

[\int], when(), NewProb,
[ANS]

Setup

♦1
NewFold conv

Convolution This chapter shows the convolution of two functions. To simplify the details, the functions are finite, piecewise, and continuous.

Topic 32: The Convolution Integral

Given a linear, time-invariant system with an impulse response of

$$h(t) = 2e^{\frac{-t}{2}} \text{ for } t > 0$$

$$h(t) = 0 \text{ for } t \leq 0$$

find the output $y(t)$ for the input function $x(t) = 1$ for $1 < t < 3$ and zero elsewhere. $y(t)$ is found by solving the convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(\alpha)x(t-\alpha)d\alpha$$

Topic 33: Piecewise Convolution

Both $x(t)$ and $h(t)$ are piecewise, continuous functions. That is, they are continuous everywhere within sub-ranges and discontinuous only at the boundaries between subranges. As such, they can be entered using the **when** function.

1. Clear the TI-89 by pressing [2nd] [F6] 2:NewProb [ENTER].
2. Enter the piecewise function for $x_1(t)$ as shown in screen 1.

[CATALOG] when(1 [2nd] [<] t [CATALOG] and t [2nd] [<] 3 [,] 1 [,] 0
[] [STO] x1 [] t []

The **when** function says $x_1(t)$ has the value 1 for $1 < t < 3$ and the value 0 for all other values of t . $x_1(t)$ is used instead of x since the TI-89 uses x when graphing.



3. Enter the piecewise function for $h(t)$ as shown in screen 2.

CATALOG when(t [2nd] [>] 0 [] 2 [♦] [e^x] [(-)] 1 [÷] 2 t [] , 0 []
 STO► h [] t []

To be sure the functions are entered correctly, graph them. This book follows the standard electrical engineering convention of writing these as functions of time, t . The TI-89, however, displays graphs as functions of x .

4. Enter $h(t)$ and $x_1(t)$ in the Y= Editor as functions of x as shown in screen 3.
5. Set the plot ranges in the Window Editor as shown in screen 4.
6. Press [♦] [I] and set **Grid** to **ON**.

7. Press [♦] [GRAPH] to graph the functions (screen 5).

If you enter the convolution integral from Topic 32, an error message is displayed as in screen 6. Therefore, the piecewise integral must be divided into sub-ranges “by hand.”

1. First, graph $x(t-\alpha)$ versus α .

To do this, pick a value for t , such as $t=0$, and enter it on the Home screen (screen 7).

$$h(t) = \begin{cases} x_1(t), & 1 < t \text{ and } t < 3 \\ 0, & \text{else} \end{cases}$$

$$y_1 = h(x)$$

$$y_2 = x_1(x)$$

$$x_{min} = -4$$

$$x_{max} = 4$$

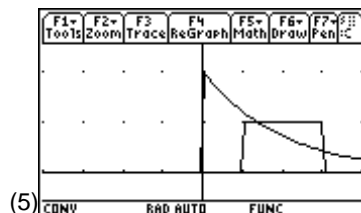
$$x_{sc1} = 1$$

$$y_{min} = -.5$$

$$y_{max} = 2.5$$

$$y_{sc1} = 1$$

$$x_{res} = 2$$



$$\int_{-\infty}^{\infty} h(\alpha) \cdot x_1(t - \alpha) d\alpha$$

Error: First argument of

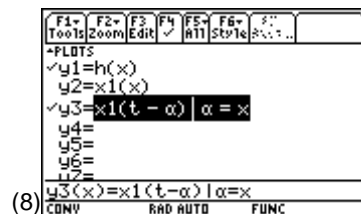
Note: To enter the integral press [2nd] [f]. To enter α press [♦] [alpha] a. (Save keystrokes by entering “a” instead of “ α .”)

$$\int_{-\infty}^{\infty} h(\alpha) \cdot x_1(t - \alpha) d\alpha$$

Error: First argument of

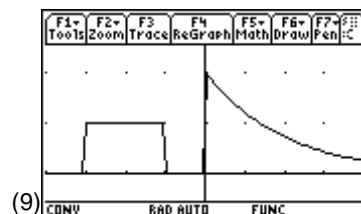
$t = 0$

- Then define the functions in the Y= Editor as shown in screen 8. Notice that $y_2(x)$ is deselected (use $\boxed{F4}$) and $y_1(x)$ and $y_3(x)$ are selected.

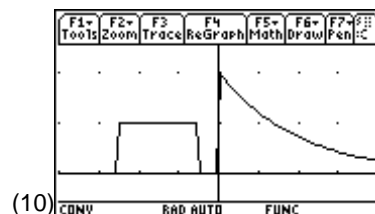


- Press $\boxed{\blacklozenge}$ [GRAPH].

$y_3(x)$ graphs a version of $x_1(t)$ that is “flipped” about the y-axis. The graph for $t=0$ is shown in screen 9. $x_1(t)$ is flipped so that its edges are at -3 and -1. For each value of $t>0$ $x_1(t)$ is positioned further to the right.



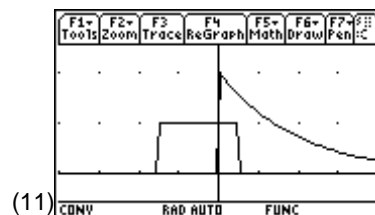
- Return to the Home screen and set $t=0.5$. Press $\boxed{\blacklozenge}$ [GRAPH] to see the result (screen 10). Notice that for $t=0.5$, $x_1(t)$ is closer to $h(t)$.



- Continue to consider the convolution integral for various ranges of t . The following ranges are chosen so that the integrals are easy to define.

Try a value of $t \leq 1$. From screen 10, the product of $x_1(t-\alpha)h(\alpha)$ is 0 since there are no values of t where both functions are non-zero. Therefore, $y(t) = 0$ for $t \leq 1$.

- Use a value of t such that $1 < t < 3$. Set t to 1.5, and graph the functions (screen 11).



For these values of t , the product of $x_1(t-\alpha)h(\alpha)$ is non-zero over a range where there is some overlap between the two functions. The convolution integral is now

$$y(t) = \int_{-\infty}^{\infty} h(\alpha)x(t-\alpha)d\alpha$$

$$= \int_0^{t-1} 2e^{-\frac{\alpha}{2}}(1)d\alpha$$

7. On the Home screen, use **DelVar** to delete the variable t before doing the integral since it was previously set to 1.5. Then enter the integral as shown in screen 12.

$\boxed{2\text{nd}} \boxed{[f]}$ $\boxed{2} \boxed{[e^x]}$ $\boxed{(-)}$ $\boxed{1} \boxed{[div]}$ $\boxed{2} \boxed{[e^x]}$ $\boxed{[alpha]}$ \boxed{a} $\boxed{[x]}$ $\boxed{1}$ $\boxed{[e^x]}$ $\boxed{[alpha]}$ \boxed{a} $\boxed{0}$ $\boxed{[t]}$ $\boxed{-}$ $\boxed{1}$ $\boxed{[]}$

8. Once the integral is calculated, store the result in $y_4(x)$ as shown in screen 13 so that it can be graphed in Topic 34.

$\boxed{2\text{nd}} \boxed{[ANS]}$ $\boxed{[STO]}$ $\boxed{y_4}$ $\boxed{[x]}$ $\boxed{[]}$

Therefore, $y(t)$ has the value shown in screen 12 for a range of values of t . The graphs show that the overlap starts when the $t-1$ edge of $x_1(t)$ passed $t=0$. Therefore, when $t-1>0$, or when $t>1$, this form of $y(t)$ is valid. However, when the $t-3$ edge of $x_1(t)$ passes $t=0$, the integral takes on a different form. This form of $y(t)$ is valid when $t-3<0$, or $t<3$. Therefore, this graph is valid for $1<t<3$.

9. Now use the range $t>3$. In this range, $x_1(t)$ lies completely within $h(t)$.

Set $t = 3.5$ on the Home screen. Then graph $y_1(x)$ and $y_3(x)$ as shown in screen 14.

For this range, the integral is

$$y(t) = \int_{-\infty}^{\infty} h(\alpha)x(t-\alpha)d\alpha$$

$$= \int_{t-3}^{t-1} 2e^{-\frac{\alpha}{2}}(1)d\alpha$$

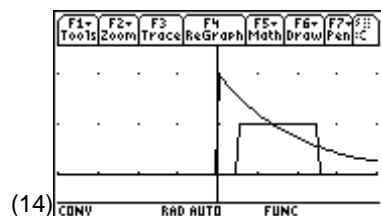
which is like the previous integral except the lower limit is changed.

(12)

The screen shows the 'DelVar' menu with 't' selected. Below, the integral $\int_0^{t-1} (2 \cdot e^{-1/2 \cdot \alpha} \cdot 1) d\alpha$ is entered. The result is displayed as $4 \cdot e^{-\frac{t}{2}} \cdot \left(e^{\frac{t}{2}} - \sqrt{e} \right)$. The bottom status bar shows 'CONV', 'RAD AUTO', 'FUNC', and '9/30'.

(13)

The screen shows the same integral result as in screen 12. Below, the expression $4 \cdot e^{-\frac{t}{2}} \cdot \left(e^{\frac{t}{2}} - \sqrt{e} \right) \rightarrow y_4(x)$ is entered. The bottom status bar shows 'CONV', 'RAD AUTO', 'FUNC', and '10/30'.



10. On the Home screen, use **DelVar** to delete **t** and then enter the integral as shown in screen 15.

$\boxed{2\text{nd}} \boxed{f} \boxed{2} \boxed{\diamond} \boxed{e^x} \boxed{(-)} \boxed{1} \boxed{\div} \boxed{2} \boxed{\times} \boxed{\diamond} \boxed{[\text{alpha} a]} \boxed{\times} \boxed{1} \boxed{\diamond} \boxed{[\text{alpha} a]} \boxed{,} \boxed{t-3} \boxed{,} \boxed{t-1} \boxed{)} \boxed{}$

11. Store the results in the variable **y5(x)** as shown in screen 16 for plotting in Topic 34.

This is valid for $t-3 > 0$, or $t > 3$.

(15)

DelVar t Done
 $\int_{t-3}^{t-1} (2 \cdot e^{-1/2 \cdot \alpha} \cdot 1) d\alpha$
 $4 \cdot (e-1) \cdot e^{1/2 - \frac{t}{2}}$
 $\rightarrow *e^{(-1/2 * \alpha)} * 1, \alpha, t-3, t-1 \rightarrow$
 CONV RAD AUTO FUNC 13/30

(16)

$4 \cdot (e-1) \cdot e^{1/2 - \frac{t}{2}}$
 $\rightarrow 4 \cdot (e-1) \cdot e^{1/2 - \frac{t}{2}} \rightarrow y5(x)$
 Done
 ans(1) → y5(x)
 CONV RAD AUTO FUNC 14/30

Topic 34: Graphing Piecewise Convolution Results

The output $y(t)$ is given in three different pieces.

$$y(t) = 0 \quad \text{for } t \leq 1$$

$$y(t) = 4e^{-t/2}(e^{t/2} - e^{1/2}) \quad \text{for } 1 < t < 3$$

$$y(t) = 4(e-1)e^{1/2-t/2} \quad \text{for } t > 3$$

1. Combine these using the **when()** function as shown in screen 17.

$\boxed{\text{CATALOG}} \boxed{\text{when}} \boxed{(} \boxed{t} \boxed{2\text{nd}} \boxed{<} \boxed{=} \boxed{1} \boxed{,} \boxed{0} \boxed{,} \boxed{\text{CATALOG}} \boxed{\text{when}} \boxed{(} \boxed{1} \boxed{2\text{nd}} \boxed{<} \boxed{t} \boxed{\text{CATALOG}} \boxed{\text{and}} \boxed{t} \boxed{2\text{nd}} \boxed{<} \boxed{3} \boxed{,} \boxed{y4} \boxed{(} \boxed{x} \boxed{)} \boxed{,} \boxed{y5} \boxed{(} \boxed{x} \boxed{)} \boxed{)} \boxed{)} \boxed{\text{STO}} \boxed{\rightarrow} \boxed{yy}$

(17)

$\begin{cases} 0, t \leq 1 \\ y4(x), 1 < t \text{ and } t < 3, e1 \\ y5(x), \text{else} \end{cases}$
 $\begin{cases} 0, t \leq 1 \\ y4(x), 1 < t \text{ and } t < 3, e1 \\ y5(x), \text{else} \end{cases}$
 $\rightarrow \text{and } t < 3, y4(x), y5(x) \rightarrow y4$
 CONV RAD AUTO FUNC 15/30

2. In preparation for graphing **yy**, change all the **t**'s in **yy** to **x**'s and save **yy** in **y6(x)** as in screen 18.

(18)

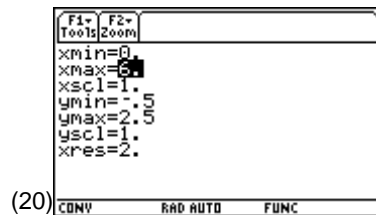
$\begin{cases} y4(x), 1 < t \text{ and } t < 3, e1 \\ y5(x), \text{else} \end{cases}$
 $\begin{cases} 0, t \leq 1 \\ y4(x), 1 < t \text{ and } t < 3, e1 \\ y5(x), \text{else} \end{cases}$
 $\rightarrow yy \mid t = x \rightarrow y6(x)$
 Done
 $yy \mid t = x \rightarrow y6(x)$
 CONV RAD AUTO FUNC 16/30

3. Press $\boxed{\diamond} \boxed{[Y=]}$. Use $\boxed{F4}$ to deselect **y1(x)**, **y3(x)**, **y4(x)**, and **y5(x)** and select **y2(x)** and **y6(x)** (screen 19).

(19)

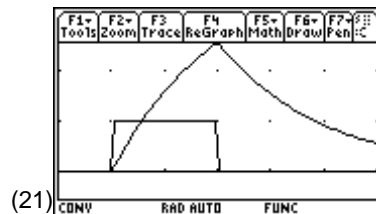
$y4 = 4 \cdot e^{-\frac{t}{2}} \cdot \left(e^{\frac{t}{2}} - e^{1/2} \right)$
 $y5 = 4 \cdot (e-1) \cdot e^{1/2 - \frac{t}{2}}$
 $\checkmark y6 = yy \mid t = x$
 $y6(x) = yy \mid t = x$
 CONV RAD AUTO FUNC

4. In the Window Editor, change the plot range on **x** so that **xmin** is 0 and **xmax** is 6 as shown in screen 20.



5. Press \square [GRAPH] to see a graph of the convolution integral as shown in screen 21.

The effects of the system on the input pulse $x_1(t)$ are clearly seen in screen 21. The input pulse is amplified and “smeared” or broadened as it passes through the system.



Tips and Generalizations

The **when()** function is a powerful feature of the TI-89 that allows piecewise functions to be manipulated easily. Here, the three pieces of the solution to a piecewise convolution were combined into a single function (**yy**), allowing it to be graphed as if it were a single continuous function. The **when()** function can be used anytime a piecewise function is needed.

Sometimes a new function is built by defining pieces over different time intervals. Other times it is better to define a function by adding sinusoids of different frequencies. In Chapter 8, the TI-89 will be used to find the Fourier series coefficients of a signal and reconstruct that signal from some of the coefficients.

Features Used

[f], [1], limit(), Σ , [Y=],
NewProb, DelVar,
[WINDOW], [GRAPH],
[ANS]

Setup

1

NewFold fourier

Fourier Series This chapter shows how to compute and graph the complex Fourier Series coefficients for a square wave.

Topic 35: Square Wave: Computing the Coefficients

The TI-89 can easily sum the Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{-j2\pi k f_0 t}$$

and evaluate the complex Fourier coefficients defined by the integral

$$c_k = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) e^{-j2\pi k f_0 t} dt \quad \text{where } f_0 = \frac{1}{T_0}$$

Suppose $x(t)$ is a square wave as shown in Figure 1.

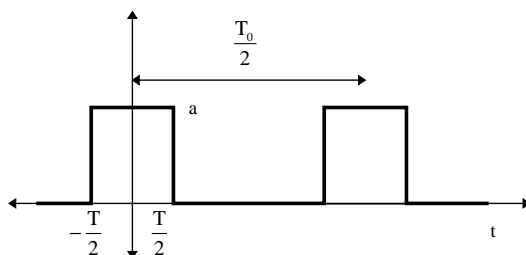


Figure 1. Periodic pulse train

For this example, the complex coefficient becomes

$$c_k = \frac{1}{T_0} \int_{-\frac{T}{2}}^{\frac{T}{2}} a e^{-j2\pi k f_0 t} dt$$

1. Clear the TI-89 by pressing $\boxed{2\text{nd}} \boxed{F6}$ **2:NewProb** $\boxed{\text{ENTER}}$.
2. Three variable substitutions are needed before entering the expression for the complex coefficient. T_0 is entered as **t00** (**t0** is reserved), T is entered as **tt** (**t** and **T** are the same on the TI-89), and j is $[i]$ which is entered as $\boxed{2\text{nd}} \boxed{[i]}$. Before entering the equation, any previous values of the variables used must be deleted as shown in screen 1.

$\boxed{\text{CATALOG}}$ **DelVar** **t00** $\boxed{,}$ **tt** $\boxed{,}$ **f0**

3. Enter the integral (screen 2).

$1 \div \boxed{t00} \boxed{2\text{nd}} \boxed{[j]} \boxed{a} \boxed{\blacklozenge} \boxed{[e^x]} \boxed{(-)} \boxed{2\text{nd}} \boxed{[i]} \boxed{2} \boxed{2\text{nd}} \boxed{[\pi]} \boxed{k} \boxed{\times} \boxed{f0} \boxed{\times} \boxed{t} \boxed{)} \boxed{t} \boxed{,} \boxed{(-)} \boxed{tt} \boxed{\div} \boxed{2} \boxed{,} \boxed{tt} \boxed{\div} \boxed{2} \boxed{)} \boxed{\text{STO}} \boxed{\blacktriangleright} \boxed{\text{tmp}}$

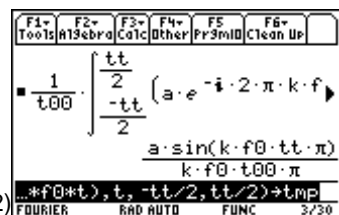
4. Next plot the coefficients. To do this, pick values for **a**, **f0** (which sets **t00** also), and **tt**. Try the values shown in screen 3.
5. In this example, **a** is set to 1 and **f0** to 1000 Hz. With **tt** set to **t00/2**, the duty cycle is $\frac{1}{2}$ so the square wave will be "on" half the time. Display the value of **tmp** (top of screen 4).
6. Now save the formula for the coefficient in a function called **c(k)** as shown in screen 4.

$\boxed{2\text{nd}} \boxed{[\text{ANS}]} \boxed{\text{STO}} \blacktriangleright \boxed{\text{c}} \boxed{[]} \boxed{k} \boxed{[]}$

Using the answer from the integral ensures that the value stored in **c(k)** is the result of the integral, not the integral itself. If the integral is saved, it is reevaluated every time a coefficient is computed. With **c(k)** stored as a function, the integral is evaluated once and the resulting formula is used each time a coefficient value is needed.



(1)

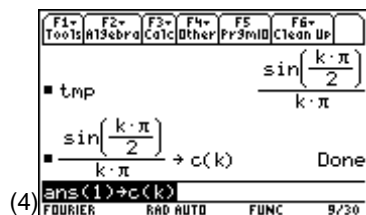


(2)

Note: If a Domain Error message appears, try switching to radian angle mode by pressing $\boxed{\text{MODE}}$ and selecting **RADIAN**.



(3)



(4)

Topic 36: Square Wave: Constructing the Wave from the Coefficients

The original signal can be rebuilt from the coefficients by using

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{-j2\pi k f_0 t}$$

1. To do this, return to the Home screen and enter the expression as shown in screen 11.

`CATALOG` Σ (`cc` (`k`) \cdot `[e-x]` `2nd` `[i]` `2` `2nd` `[π]` `k` \times `f0` \times `t`)

Notice that the TI-89 applied Euler's Identity to terms of the form $e^{i\theta} + e^{-i\theta}$ to get $2\cos(\theta)$.

The complete output is

$$\frac{-2\cos(6000\pi t)}{3\pi} + \frac{2\cos(2000\pi t)}{\pi} + 1/2$$

2. Save the result as **y1(x)**, screen 12.

`2nd` `[ANS]` `[]` `t` `=` `x` `STO>` `y1` (`[]` `x`)

3. Switch the **Graph** mode to **FUNCTION** by pressing `MODE` \rightarrow **1:FUNCTION** `[ENTER]`. Then press `[Y=]` to verify that the equation is entered in the Y= Editor (screen 13).

4. Press `[WINDOW]` and set the plot range as shown in screen 14.

5. Press `[GRAPH]` to see the graph of **x(t)** as shown in screen 15.

It's not quite a square wave, but it's not too bad for using only 5 non-zero coefficients. Recall that **c(-3)**, **c(-1)**, **c(0)**, **c(1)** and **c(3)** are non-zero; **c(-2)=c(2)=0**.

6. To get a more accurate representation, include more coefficients of the series. On the Home screen, change the summation range to -5 to 5 as shown in screen 16.

`CATALOG` Σ (`cc` (`k`) \cdot `[e-x]` `2nd` `[i]` `2` `2nd` `[π]` `k` \times `f0` \times `t`)

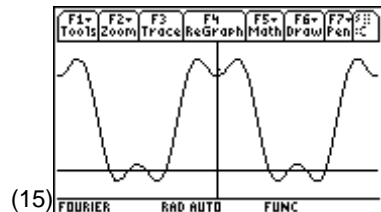
`[]` `[]` `k` `[]` `(-)` `5` `[]` `5` `[]`

(11)

(12)

(13)

(14)



(16)

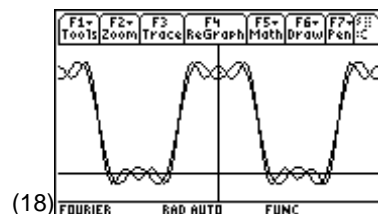
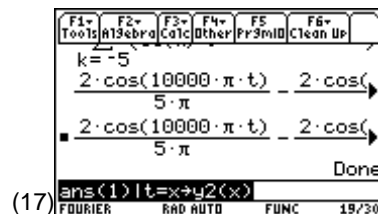
The output for the 5-term series is

$$\frac{2 \cos(10000\pi t)}{5\pi} - \frac{2 \cos(6000\pi t)}{3\pi} + \frac{2 \cos(2000\pi t)}{\pi} + 1/2.$$

7. Save the result in **y2(x)** as shown in screen 17.

2nd **[ANS]** **[]** **t** **[]** **x** **[STO]** **y2** **[]** **x** **[]**

8. Press **[]** **[GRAPH]**. Screen 18 compares the resulting graph of the sum for **k=-5** to 5 with the original graph for the sum of **k=-3** to 3. The new result more closely represents a square wave because the sum more closely represents a square wave as the number of terms increases. With an infinite number of terms, the sum exactly represents the square wave.



Tips and Generalizations

The TI-89 can easily find the Fourier Series coefficients in closed form for many periodic signals. In this chapter, **c(k)** could be expressed as a simple equation. More complex signals may not have a closed form solution. In these cases, use numeric integration (**nlnt**) to find each of the coefficients.

Chapter 9 adds a new dimension by showing how the TI-89 can manipulate vectors.

Features Used

[\angle], [MODE], ►Rect, Cylind, ►Sphere, unitV(), dotP(), crossP(), norm(), Func, [1], \odot , [COPY], [PASTE], NewProb, \approx

Setup

► 1, NewFold emag

Vectors This chapter describes how to use the TI-89 for coordinate system transformations, vector algebra, and vector component transformations. The functions **rec2cyl()**, **cyl2rec()**, **rec2sph()**, and **sph2rec()** are created in this chapter for vector component transformations and are used in Chapters 10 and 11.

Topic 37: Coordinate Systems and Coordinate Transformations

Points in 3-D space can be entered in rectangular, cylindrical, and spherical coordinate systems. The TI-89 does this with position vectors, which are vectors that point from the origin to the coordinates of the point in space. On the TI-89, each position vector is represented by the coordinates of its endpoint—(x,y,z) in rectangular, (r,θ,z) in cylindrical, or (ρ,φ,θ) in spherical coordinates.

The TI-89 notation differs in two ways from the standard form of (ρ,φ,z) for cylindrical and (r,θ,φ) for spherical coordinates used in most electrical engineering texts. The coordinates are related as $\rho_{STD}=r_{TI}$, $r_{STD}=\rho_{TI}$, $\phi_{STD}=\theta_{TI}$, and $\theta_{STD}=\phi_{TI}$. This difference in angle notation appears to interchange the angles. The two vector forms are related as $[\rho_{STD}, \angle\phi_{STD}, z]_{STD}=[r_{TI}, \angle\theta_{TI}, z]_{TI}$ for cylindrical coordinates and $[r_{STD}, \angle\theta_{STD}, \angle\phi_{STD}]_{STD}=[\rho_{TI}, \angle\phi_{TI}, \angle\theta_{TI}]_{TI}$ for spherical coordinates. Check the Guidebook to be sure of the definition of these variables. The coordinate transformations of this section use the TI-89 form.

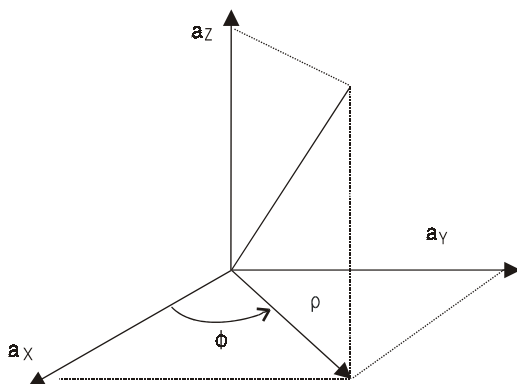


Figure 1a. Cylindrical coordinates

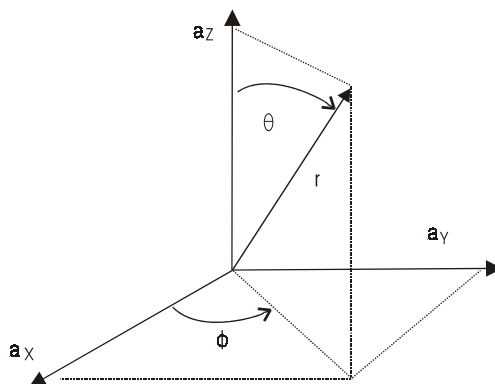


Figure 1b. Spherical coordinates

The TI-89 defines vectors as either a 1x3 row matrix or a 3x1 column matrix. The *vector entry mode* is determined solely by the format of the vector elements and establishes the coordinate system in which the vector is interpreted, but not necessarily displayed.

- For rectangular coordinates, the vector is entered as three scalar expressions, for example, $[1, \cos(x+\sqrt{y}), -2.9]$,
- For cylindrical coordinates, the second entry is entered as an angle with a preceding \angle ($\text{2nd} [\angle]$), for example, $[z^3, \angle 25, \sin(y^*z)]$,
- For spherical coordinates, the second and third entries are entered as angles, for example, $[1, \angle \pi/4, \angle (\theta-3)]$.

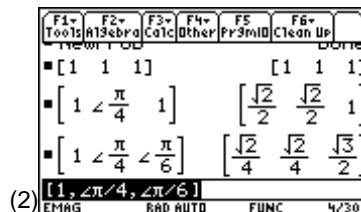
Regardless of the entry mode, vectors are displayed according to the **Vector Format** mode—rectangular, cylindrical, or spherical.

- Clear the TI-89 by pressing $\text{2nd} [\text{F6}] \text{2:NewProb} [\text{ENTER}]$. Press $[\text{MODE}]$ to verify that **Vector Format** is set to **RECTANGULAR** (screen 1).



- On the Home screen, enter the rectangular position vector $[1,1,1]$ as shown in the top of screen 2. Enter the cylindrical position vector $[1, \angle \pi/4, 1]$. Enter the spherical position vector $[1, \angle \pi/4, \angle \pi/6]$.

Note that the forms are all displayed in the **RECTANGULAR** format regardless of the entry mode.



- The TI-89 has three commands that change the display format of these coordinates from one coordinate system to another for the current entry only. The rectangular-to-standard cylindrical coordinate transformation is based upon

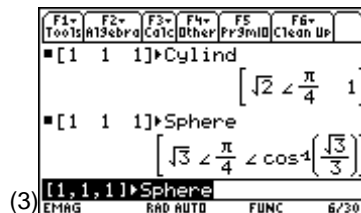
$$\rho = \sqrt{x^2 + y^2} \quad x = \rho \cos \phi$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \quad y = \rho \sin \phi$$

$$z = z$$

Use **Cylind** to display $[1,1,1]$ in cylindrical components as shown in the top of screen 3.

$\text{2nd} [\text{C}] \ 1 \ 1 \ 1 \ \text{2nd} [\text{C}] \ \text{CATALOG} [\text{Cylind}]$



4. The rectangular-to-standard spherical coordinate transformation is based upon

$$r = \sqrt{x^2 + y^2 + z^2} \quad x = r \sin \theta \cos \phi$$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right) \quad y = r \sin \theta \sin \phi$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \quad z = r \cos \theta$$

Use ► **Sphere** to display [1,1,1] in spherical coordinates as shown in the bottom of screen 3.

[2nd] [C] 1 [] 1 [] 1 [2nd] [] [CATALOG] [►] **Sphere**

5. To convert back to original rectangular form, use **ans(1)** to get the previous answer, and then enter ► **Rect** as shown in screen 4.

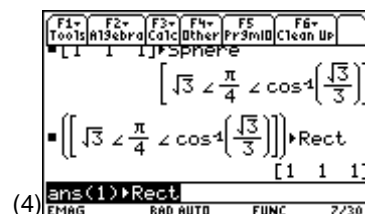
[2nd] [ANS] [CATALOG] [►] **Rect**

6. Set the **Vector Format** mode to **CYLINDRICAL**.

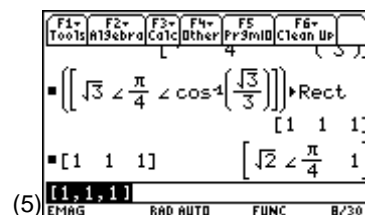
[MODE] [◀] [◀] [◀] [◀] [◀] [◀] [◀] [2:CYLINDRICAL] [ENTER]

7. Enter [1, 1, 1] which is now displayed in cylindrical format in screen 5.

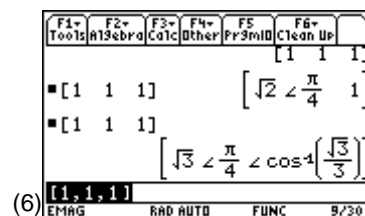
8. Try this for **SPHERICAL Vector Format** mode also as shown in screen 6.



(4)



(5)



(6)

Topic 38: Vector Components

In the common notation, vectors are given as $\mathbf{v} = A_1\mathbf{a}_1 + A_2\mathbf{a}_2 + A_3\mathbf{a}_3$ where all vectors are noted by **bold** symbols, such as \mathbf{v} , and unit vectors in the i th direction as \mathbf{a}_i .

On the TI-89, the magnitudes of the components, A_1 , A_2 , and A_3 , for \mathbf{v} are entered as a vector. A vector, \mathbf{v} , can be represented in any orthogonal coordinate system as long as the unit vectors, \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 form a right-hand coordinate system.

A right-hand coordinate system is one for which the first vector crossed into the second gives the third, the second crossed into the third gives the first, and the third crossed into the first gives the second. Mathematically this is written as $\mathbf{a}_1 \times \mathbf{a}_2 = \mathbf{a}_3$, $\mathbf{a}_2 \times \mathbf{a}_3 = \mathbf{a}_1$, and $\mathbf{a}_3 \times \mathbf{a}_1 = \mathbf{a}_2$. The proper right-hand orders for the standard coordinate systems are $(\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z)$, $(\mathbf{a}_\rho, \mathbf{a}_\phi, \mathbf{a}_z)$, and $(\mathbf{a}_r, \mathbf{a}_\theta, \mathbf{a}_\phi)$.

For operations on vector components, it is best to use the rectangular vector entry and display modes.

1. Set the **Vector Format** mode to **RECTANGULAR**.

2. Enter $\mathbf{v1} = -4\mathbf{a}_x + 3\mathbf{a}_y + 0\mathbf{a}_z$ as shown in screen 7.

$\boxed{2\text{nd}} \boxed{[]} \boxed{(-)} \boxed{4} \boxed{,} \boxed{3} \boxed{,} \boxed{0} \boxed{2\text{nd}} \boxed{[]} \boxed{\text{STO}} \boxed{\triangleright} \boxed{\mathbf{v1}}$

Although $\mathbf{v1}$ was given in rectangular coordinates, this could represent the cylindrical vector

$\mathbf{v1} = -4\mathbf{a}_\rho + 3\mathbf{a}_\phi + 0\mathbf{a}_z$ or the spherical vector $\mathbf{v1} = -4\mathbf{a}_r + 3\mathbf{a}_\theta + 0\mathbf{a}_\phi$ equally well.

F1	F2	F3	F4	F5	F6
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
$\left[\sqrt{3} \angle \frac{\pi}{4} \cos^{-1} \left(\frac{\sqrt{3}}{3} \right) \right]$					
$\begin{aligned} &[-4 \ 3 \ 0] \rightarrow \mathbf{v1} \quad [-4 \ 3 \ 0] \\ &\text{norm}(\mathbf{v1}) \quad 5 \\ &\text{unitU}(\mathbf{v1}) \rightarrow \mathbf{av1} \\ &\quad [-4/5 \ 3/5 \ 0] \end{aligned}$					
(7) $\text{unitU}(\mathbf{v1}) \rightarrow \mathbf{av1}$					
EMAG RAD AUTO FUNC 12/30					

Note: The convention in this book is to save the unit vector of \mathbf{vn} as \mathbf{avn} .

3. Use **norm()** to find the magnitude of $\mathbf{v1}$.

$\boxed{\text{CATALOG}} \boxed{\text{norm}(\mathbf{v1})} \boxed{}$

4. Use **unitV()** to find the unit vector in the direction of $\mathbf{v1}$.

$\boxed{\text{CATALOG}} \boxed{\text{unitV}(\mathbf{v1})} \boxed{ } \boxed{\text{STO}} \boxed{\triangleright} \boxed{\mathbf{av1}}$

Topic 39: Angle between Vectors

Calculate the angle between $\mathbf{v1}$ from Topic 38 and a second vector $\mathbf{v2}$, which extends from (2,-5,4) to (1,1,3).

1. Enter the two endpoints in rectangular coordinates using position vectors of $[2,-5,4] \rightarrow \mathbf{v2a}$ and $[1,1,3] \rightarrow \mathbf{v2b}$, respectively, as shown in screen 8.

F1	F2	F3	F4	F5	F6
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
$\begin{aligned} &[-4 \ 3 \ 0] \rightarrow \mathbf{v1} \quad [-4 \ 3 \ 0] \\ &\text{norm}(\mathbf{v1}) \quad 5 \\ &\text{unitU}(\mathbf{v1}) \rightarrow \mathbf{av1} \\ &\quad [-4/5 \ 3/5 \ 0] \end{aligned}$					
$\begin{aligned} &[2 \ -5 \ 4] \rightarrow \mathbf{v2a} \quad [2 \ -5 \ 4] \\ &[1 \ 1 \ 3] \rightarrow \mathbf{v2b} \quad [1 \ 1 \ 3] \end{aligned}$					
(8) $[1,1,3] \rightarrow \mathbf{v2b}$					
EMAG RAD AUTO FUNC 14/30					

2. $\mathbf{v2}$ is the difference between the two position vectors, see Figure 2. Enter $\mathbf{v2b} - \mathbf{v2a} \rightarrow \mathbf{v2}$ as shown in screen 9.

F1	F2	F3	F4	F5	F6
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
$\begin{aligned} &[2 \ -5 \ 4] \rightarrow \mathbf{v2a} \quad [2 \ -5 \ 4] \\ &[1 \ 1 \ 3] \rightarrow \mathbf{v2b} \quad [1 \ 1 \ 3] \\ &\mathbf{v2b} - \mathbf{v2a} \rightarrow \mathbf{v2} \quad [-1 \ 6 \ -1] \\ &\text{unitU}(\mathbf{v2}) \rightarrow \mathbf{av2} \\ &\quad \left[\frac{-\sqrt{38}}{38} \ \frac{3\sqrt{38}}{19} \ \frac{-\sqrt{38}}{38} \right] \end{aligned}$					
(9) $\text{unitU}(\mathbf{v2}) \rightarrow \mathbf{av2}$					
EMAG RAD AUTO FUNC 16/30					

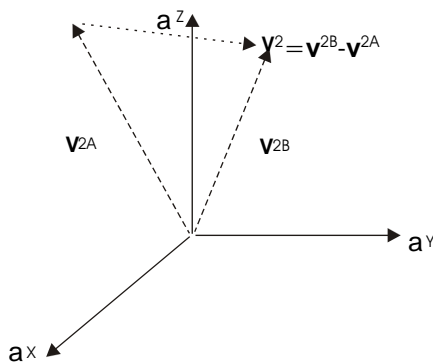


Figure 2. Vector addition

3. Use **unitV()** to find the unit vector in the direction of **v2**.

[CATALOG] **unitV**(**v2**) [] [STO] **av2**

4. Set **Angle** mode to **DEGREE** using [MODE].

5. The angle between two vectors is given by the arccosine of the dot product of their unit vectors, that is, $\cos^{-1}(\text{dotP}(\text{av1}, \text{av2})) \rightarrow \text{angv1v2}$. This is entered as shown in screen 10.

[◀] [COS⁻¹] [CATALOG] **dotP**(**av1** [,] **av2**) [] [] [STO] **angv1v2**

6. Use [◀] [≈] to get a floating point value of about 45° (screen 11).

(10)

F1=	F2=	F3=	F4=	F5=	F6=
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
unitV(v2) → av2					
$\left[\frac{-\sqrt{38}}{38} \quad \frac{3\sqrt{38}}{19} \quad \frac{-\sqrt{38}}{38} \right]$					
cos ⁻¹ (dotP(av1, av2)) → angv1v2					
$\cos^{-1}\left(\frac{11\sqrt{38}}{95}\right)$					
1(dotP(av1, av2)) → angv1v2					
EMAG	DEGR AUTO	FUNC	17/30		

(11)

F1=	F2=	F3=	F4=	F5=	F6=
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
$\left[\frac{38}{38} \quad \frac{19}{19} \quad \frac{38}{38} \right]$					
cos ⁻¹ (dotP(av1, av2)) → angv1v2					
$\cos^{-1}\left(\frac{11\sqrt{38}}{95}\right)$					
44.46					
1(dotP(av1, av2)) → angv1v2					
EMAG	DEGR AUTO	FUNC	18/30		

Topic 40: Parallel and Perpendicular Vectors

Other common vector operations are to find the vector components of **v1** that are parallel and perpendicular to **v2**.

Every vector can be decomposed into two orthogonal vector components so that **v1** is the vector sum of two components, one parallel and one perpendicular, **v1par** and **v1perp** as in Figure 3.

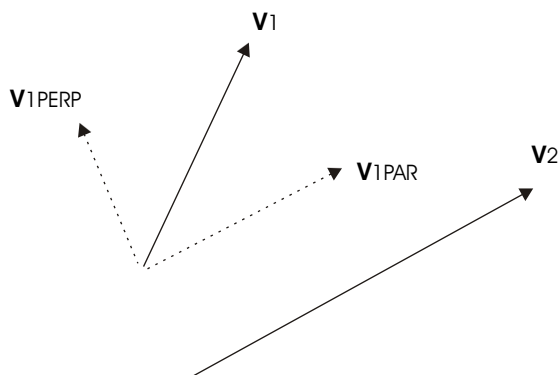


Figure 3. Parallel and perpendicular components

1. The magnitude of the component of **v1** that is parallel to **v2** is the dot product of **v1** and **av2** as shown in screen 12.

[CATALOG] **dotP**(**v1** [,] **av2**) [] [STO] **v1parmag**

2. The parallel component is the magnitude multiplied by the unit vector **av2** (screen 12).

v1parmag [×] **av2** [STO] **v1par**

(12)

F1=	F2=	F3=	F4=	F5=	F6=
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
44.46					
dotP(v1, av2) → v1parmag					
$\frac{11\sqrt{38}}{19}$					
v1parmag · av2 → v1par					
$\left[-11/19 \quad 66/19 \quad -11/19 \right]$					
v1parmag · av2 → v1par					
EMAG	DEGR AUTO	FUNC	20/30		

3. Find **v1perp** as **v1-v1par**→ **v1perp** as shown in screen 13.

4. An alternate method of finding **v1perp** begins by calculating the vector perpendicular to the plane containing **v1** and **v2** using the **crossP()** command as shown in screen 14.

CATALOG **crossP**(**v1** , **v2**) **STO**→ **v3**

5. The cross product **crossP(v3,v2)**→**v4** is perpendicular to **v2** and so is in the direction of **v1perp**. Therefore, the dot product **dotP(v1,av4)** gives the magnitude of the perpendicular component of **v1**. The vector form of the component of **v1** perpendicular to **v2** is calculated by **unitV(crossP(v2,v3))** as shown in screen 14.

CATALOG **unitV**(**CATALOG** **crossP**(**v2** , **v3**)) **STO**→ **aperp**

6. Calculate the dot product of **dotP(v1,aperp)** **aperp**.

CATALOG **dotP**(**v1** , **aperp**) **×** **aperp** **STO**→ **v1perp**

Screen 15 shows this result agrees with the earlier calculation of **v1perp** in screen 13.

(13)

F1→	F2→	F3→	F4→	F5→	F6→
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
11-√38					
19					
v1parmag·av2→v1par					
[-11/19 66/19 -11/19]					
v1-v1par→v1perp					
[-65/19 -9/19 11/19]					
v1-v1par→v1perp					
EMAG	DEGRUTO	FUNC	21/30		

(14)

F1→	F2→	F3→	F4→	F5→	F6→
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
[-65/19 -9/19 11/19]					
crossP(v1,v2)→v3					
[-3 -4 -21]					
unitV(crossP(v2,v3))→aperp					
[-65·√4427 4427 -9·√4427 4427 11]					
unitV(crossP(v2,v3))→aperp					
EMAG	DEGRUTO	FUNC	23/30		

(15)

F1→	F2→	F3→	F4→	F5→	F6→
Tools	Algebra	Calc	Other	Pr3mID	Clean Up
[-3 -4 -21]					
unitV(crossP(v2,v3))→aperp					
[-65·√4427 4427 -9·√4427 4427 11]					
dotP(v1,aperp)·aperp→v1perp					
[-65/19 -9/19 11/19]					
tP(v1,aperp)aperp→v1perp					
EMAG	DEGRUTO	FUNC	24/30		

Topic 41: Rectangular to Cylindrical Vector Transformation

Vector components in one system are transformed to another according to the geometrical relationships between the two systems.

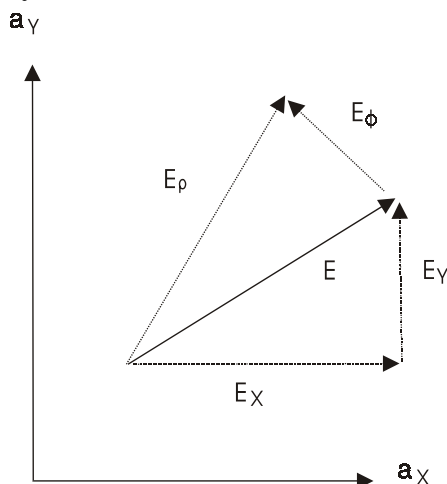


Figure 4. Rectangular and cylindrical components of a vector

The general procedure for transforming any vector is to find its components in the other system using the dot product as $E_i = \mathbf{E} \cdot \mathbf{a}_i$. The rectangular to standard cylindrical transformation of vector components is based on

$$E_\rho = E_X \cos \phi + E_Y \sin \phi$$

$$E_\phi = -E_X \sin \phi + E_Y \cos \phi$$

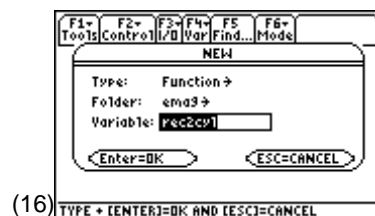
$$E_Z = E_Z$$

These equations are given in matrix form as

$$\begin{bmatrix} E_\rho \\ E_\phi \\ E_Z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_X \\ E_Y \\ E_Z \end{bmatrix}$$

However, it is time consuming to enter the matrix repeatedly for each transformation, particularly since it depends upon the vector components and the coordinates of the point. Functions are an ideal feature of the TI-89 for this use since they can be used in expressions and return results for different input values.

1. Press **[APPS]** **7:Program Editor 3:New**.
2. Select **2:Function** for **Type** and select **emag** for **Folder**.
3. Type **rec2cyl** for **Variable** and press **[ENTER]** **[ENTER]** to display a template for the new function, as shown in screens 16 and 17.



(16) TYPE = [ENTER]=OK AND [ESC]=CANCEL



(17) EMAG DEGAUTO FUNC

4. Input data is provided to functions through the arguments of the function. For the **rec2cyl** function, the arguments are the rectangular components of the vector (**vec**) and the coordinates of the point at which the transformation is to be evaluated (**pt**). Enter the arguments in the parentheses as **vec** and **pt** (screen 18).



(18) EMAG DEGAUTO FUNC

5. Enter the instructions for the function.

Local rho,x,y

Defines **rho**, **x**, and **y** as local variables. The function arguments as well as all variables listed in the **Local** instruction exist only within the function and are erased upon the completion of the function.

```
pt[1,1]->x
pt[1,2]->y
```

Gets local variables **x** and **y** from the input vector **pt**.

Note: To enter T , press **[2nd]** **[CHAR]** **2:Math 9:** T .

$$\sqrt{(x^2+y^2)} \rightarrow \rho$$

Computes local variable **rho** based on **x** and **y**.

$$([x/\rho, y/\rho, 0][-y/\rho, x/\rho, 0][0, 0, 1])^T \text{vec}^T$$

Computes $\cos\phi = x/\rho$ and $\sin\phi = y/\rho$, forms the transformation matrix, and performs matrix multiplication. **vec** is transposed to a 3x1 column matrix in order to be multiplied by the transformation matrix. The results are transposed to be put back into the row matrix format used earlier.

6. Return to the Home screen, and use the **rec2cyl** function to transform the rectangular vector components [1,1,1] to cylindrical components at the point [-1,2,3].

rec2cyl [] [2nd] [] 1 [] 1 [] 1 [2nd] [] [] [2nd] [] [] (-) 1 [] 2 [] 3 [2nd] [] []

7. Press [ENTER] to observe the exact result (top of screen 19).

$$\left[\frac{\sqrt{5}}{5} \quad \frac{-3\sqrt{5}}{5} \quad 1 \right]$$

8. Press [◀][≈] to observe the floating point result (bottom of screen 19).

[.45 -1.34 1.00]

This result shows the cylindrical components $[E_\rho \ E_\phi \ E_z]$ of the original rectangular vector.

Topic 42: Cylindrical to Rectangular Vector Transformation

Standard cylindrical components are transformed to rectangular components by

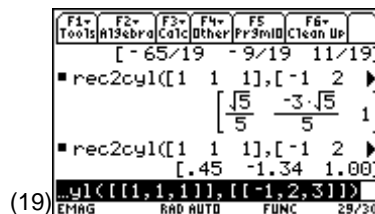
$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_\rho \\ E_\phi \\ E_z \end{bmatrix}$$

Rather than enter this transformation matrix, the **rec2cyl** function in Topic 41 can be copied to a new function and then edited by making use of the matrix relationship

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \text{Cyl} \\ \text{to} \\ \text{Rec} \end{bmatrix} \begin{bmatrix} E_\rho \\ E_\phi \\ E_z \end{bmatrix} = \begin{bmatrix} \text{Cyl} \\ \text{to} \\ \text{Rec} \end{bmatrix} \begin{bmatrix} \text{Rec} \\ \text{to} \\ \text{Cyl} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

which leads to

$$\begin{bmatrix} \text{Cyl} \\ \text{to} \\ \text{Rec} \end{bmatrix} = \begin{bmatrix} \text{Rec} \\ \text{to} \\ \text{Cyl} \end{bmatrix}^{-1}$$



1. Press **[APPS]** **7:Program Editor 1:Current** to display **rec2cyl**.
2. Use \downarrow to move to the beginning of the third line. Hold down \uparrow and press $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$ to highlight the instructions to be copied as shown in screen 20.
3. To copy the highlighted instructions, press \blacklozenge **[COPY]**.
4. Press **[F1]** **3:New** to open a new function. Name the function **cyl2rec** (screen 21).
5. Enter the arguments of **vec** and **pt** as before (screen 22).
6. Move the cursor to the third line and press \blacklozenge **[PASTE]** to paste the copied instructions to **cyl2rec** (screen 23).
7. The transformation from cylindrical to rectangular components uses the inverse of the original matrix. Edit the last instruction by inserting \wedge $(\)$ $(\)$ **1** $(\)$ as highlighted in screen 24.
8. To verify that it works, return to the Home screen. Use **ans(1)** as the first argument of **cyl2rec** to transform the cylindrical components from Topic 41 back to rectangular components (screen 25).

(20)

```

F1+ F2+ F3+ F4+ F5+ F6+
Tools Control I/O Var Find... Mode
:rec2cyl(vec,pt)
:Func
:Local rho,x,y
:pt[1,1]→x
:pt[1,2]→y
:√(x^2+y^2)→rho
:((x/rho,y/rho,0)I-y/rho,
x/rho,0)I(0,0,1)→vec1)I
:EndFunc
  
```

EMAG DEG AUTO FUNC

(21)

```

F1+ F2+ F3+ F4+ F5+ F6+
Tools Control I/O Var Find... Mode
NEW
Type: Function→
Folder: ema3→
Variable: cyl2rec
Enter=OK ESC=CANCEL
:EndFunc
  
```

TYPE + [ENTER]=OK AND [ESC]=CANCEL

(22)

```

F1+ F2+ F3+ F4+ F5+ F6+
Tools Control I/O Var Find... Mode
:cyl2rec(vec,pt)
:Func
:EndFunc
  
```

EMAG DEG AUTO FUNC

(23)

```

F1+ F2+ F3+ F4+ F5+ F6+
Tools Control I/O Var Find... Mode
:cyl2rec(vec,pt)
:Func
:Local rho,x,y
:pt[1,1]→x
:pt[1,2]→y
:√(x^2+y^2)→rho
:((x/rho,y/rho,0)I-y/rho,
x/rho,0)I(0,0,1)→vec1)I
:EndFunc
  
```

EMAG DEG AUTO FUNC

(24)

```

F1+ F2+ F3+ F4+ F5+ F6+
Tools Control I/O Var Find... Mode
:cyl2rec(vec,pt)
:Func
:Local rho,x,y
:pt[1,1]→x
:pt[1,2]→y
:√(x^2+y^2)→rho
:((x/rho,y/rho,0)I-y/rho,
x/rho,0)I(0,0,1)→vec1)I
:EndFunc
  
```

EMAG DEG AUTO FUNC

(25)

```

F1+ F2+ F3+ F4+ F5+ F6+
Tools H13ebra Calc Other Pr3mID Clean Up
[√5 -3√5 1]
rec2cyl([1 1 1],[ -1 2
[.45 -1.34 1.00]
cyl2rec([.44721359549995
[1.00 1.00 1.00]
cyl2rec(ans(1),[-1,2,3])
  
```

EMAG DEG AUTO FUNC 27/30

Topic 43: Rectangular to Spherical Vector Transformation

Transformation from rectangular to standard spherical coordinates is based on

$$\begin{bmatrix} E_r \\ E_\theta \\ E_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \sin \phi & \cos \theta \cos \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

A function **rec2sph** is constructed in a manner similar to **rec2cyl** (Topic 41).

1. Press **[APPS]** **7:Program Editor 3:New** and name the function **rec2sph**.
2. Enter the function lines shown below.

```
:rec2sph(vec,pt)
:Func
:Local r,ra,x,y,z
:pt[1,1]->x:pt[1,2]->y:pt[1,3]->z
:√(x^2+y^2)->ra:√(ra^2+z^2)->r
:([x/r,y/r,z/r][x*z/r/ra,y*z/r/ra,-ra/r][-y/ra,x/ra,0]]*vec^T)
:EndFunc
```

3. Return to the Home screen and use **rec2sph** to transform the rectangular components [1,1,1] to spherical components at the point [-1,2,3].

rec2sph **(** **[** **2nd** **[** **1** **,** **1** **,** **1** **]** **,** **[** **(-)** **1** **,** **2** **,** **3** **]** **)**

The exact result shown in screen 26 is

$$\begin{bmatrix} \frac{2\sqrt{14}}{7} & \frac{-\sqrt{70}}{35} & \frac{-3\sqrt{5}}{5} \end{bmatrix}$$



Note: The function **rec2sph** gives undefined results for $x=y=0$. A more robust function could be written using the **when()** command.

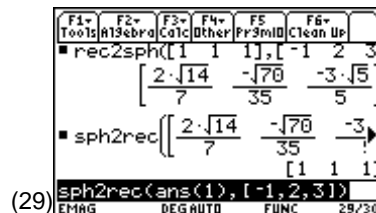
Topic 44: Spherical to Rectangular Vector Transformation

The standard spherical to rectangular transformation **sph2rec** uses the matrix inverse as in Topic 42 and transforms the spherical components back to the original rectangular components.

Screen 27 shows **sph2rec** with the instructions copied from **rec2sph**. Follow the procedure from Topic 42.

Screen 28 shows how to edit the function for the matrix inverse.

Screen 29 shows the spherical components transformed back to the rectangular components in Topic 43.



Tips and Generalizations

Mathematicians and engineers have chosen different “standard” ways of representing vectors. The TI-89 follows the mathematicians’ standards. The functions developed here always follow the engineers’ standards.

All of the transformations in this chapter work for symbolic entries as well, although they often lead to quite complicated results.

The real excitement begins when Chapter 10 builds on this chapter and introduces vector calculus using the TI-89.

Features Used

right(), product(), /, *, list►mat(), mod(), For...EndFor, norm(), unitV(), [f], ans(1), Func, NewProb, comDenom(), unitV(), [d], [≈], If...Elseif...Endif

Setup

◀ 1, setFold(emag)

Vector Calculus

This chapter describes how to use the TI-89 for differential and integral calculus. The differential operations illustrated include gradient, divergence, curl, and Laplacian. Line and surface integrals also are included. The functions **cordchk()**, **grad()**, **div()**, **curl()**, and **lap()** are created in this chapter, and the functions **rec2sph()** and **cyl2rec()** from *Chapter 9: Vectors* are used.

Topic 45: Gradient

The gradient is a differential vector operation which gives the magnitude and direction of the greatest rate of change of a scalar potential. Calculation forms are usually given in rectangular, cylindrical, and spherical coordinates. However, the single form

$$\text{grad } v = \nabla v = \frac{\partial v}{h_1 \partial x_1} \mathbf{a}_1 + \frac{\partial v}{h_2 \partial x_2} \mathbf{a}_2 + \frac{\partial v}{h_3 \partial x_3} \mathbf{a}_3$$

can serve for all three coordinate systems where x_i is the i th variable and \mathbf{a}_i is the unit vector associated with the i th variable. h_i is called the metric for the i th variable; it is multiplied by angular variables to calculate length in the angular direction. The table below shows these elements for the three coordinate systems.

	x_i	h_i	\mathbf{a}_i
Rectangular	x, y, z	$1, 1, 1$	$\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$
Cylindrical	ρ, ϕ, z	$1, \rho, 1$	$\mathbf{a}_\rho, \mathbf{a}_\phi, \mathbf{a}_z$
Spherical	r, θ, ϕ	$1, r, r \sin \theta$	$\mathbf{a}_r, \mathbf{a}_\theta, \mathbf{a}_\phi$

Table 1. Variables, metrics, and unit vectors

The similar form for the gradient definition in the three coordinate systems means that the same instructions can be used for all of the coordinate systems. Once the coordinate system has been selected, the variables and metrics for that coordinate system can be applied to the derivatives of the potential.

These instructions are included in the function called **grad**. The arguments for this function are the desired coordinate system and the mathematical form of the potential. Specification of the coordinate system determines the variables and metrics used in **grad**. Since other vector operations also require variable and metric selection, a separate function, **cordchk**, is created for this.

1. Clear the TI-89 by pressing $\boxed{2\text{nd}} \boxed{F6}$ **2:NewProb** $\boxed{\text{ENTER}}$.
2. Press $\boxed{\text{APPS}}$ **7:Program Editor 3:New** and select **Type: Function**.
3. Name the **Variable: cordchk**.
4. Enter the instructions for **cordchk** listed below.

```
:cordchk(cord)
:Func
:If string(cord)="rec" Then
: {x,y,z,1,1,1}
:ElseIf string(cord)="cyl" Then
: {p,phi,z,1,p,1}
:ElseIf string(cord)="sph" Then
: {r,theta,phi,1,r,r*sin(theta)}
:EndIf
:EndFunc
```

cordchk accepts coordinate arguments of **rec**, **cyl**, or **sph** from which it returns a list of the form $\{x_1, x_2, x_3, h_1, h_2, h_3\}$.

5. Create a new function named **grad**.
6. Enter the instructions for **grad** as shown in screen 1.

grad accepts the coordinate argument of **rec**, **cyl**, or **sph** and a symbolic form of the potential argument expressed in the variables of the chosen coordinate system. The results of the **cordchk** function are stored as a local variable, **var**. The elements of **var** are used to calculate the vector components of the gradient. The resulting vector represents the three components of the gradient in the coordinate system of the calculation. The order of the components is (x,y,z), (p,phi,z), or (r,theta,phi).

7. Return to the Home screen and calculate the gradient of $v=10x\sin(y)\exp(e^{-5z})$ as shown in screen 2.

grad $\boxed{[}$ **rec** $\boxed{,}$ **10x** $\boxed{\times}$ $\boxed{2\text{nd}} \boxed{\text{SIN}}$ **y** $\boxed{[}$ $\boxed{\blacklozenge}$ $\boxed{[e^x]}$ $\boxed{(-)}$ **5z** $\boxed{]}$ $\boxed{]}$

The complete answer is

$$[10\sin(y)E^{-5z} \quad 10x \cos(y)E^{-5z} \quad -50x \sin(y)E^{-5z}].$$

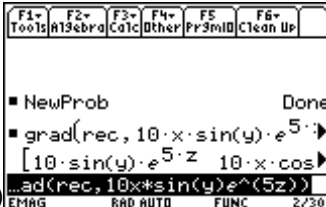
Note: To enter p , press $\boxed{\blacklozenge} \boxed{[}$ α **r**;
to enter θ , press $\boxed{\blacklozenge} \boxed{[}$ θ ; and to enter ϕ , press $\boxed{\blacklozenge} \boxed{[}$ α **f**.

(1)



```
F1- F2- F3- F4- F5- F6-
Tools Control I/O Var Find... Mode
:grad(cord,pot)
:Func
:Local var
:cordchk(cord)+var
: [{d(pot,var[1])/(var[4]),
:d(pot,var[2])/(var[5]),d(
: pot,var[3])/(var[6])}]
:EndFunc
```

(2)



```
F1- F2- F3- F4- F5- F6-
Tools Algebra Calc Other PrgmID Clean Up
■ NewProb Done
■ grad(rec,10xsin(y)·e^5z)
[10sin(y)·e^5z 10x·cos
...ad(rec,10xsin(y)·e^(5z))
EMAG RAD AUTO FUNC 2/30
```

8. Find the gradient of $v = z \cos(\phi) / \rho$ as shown in screen 3.

grad (**cyl** , **z** **×** **2nd** **[COS]** **♦** (**[alpha]** **f**) **÷** **♦** (**[alpha]** **r**)

The answer is

$$\left[\frac{-\cos(\phi)z}{\rho^2} \quad \frac{-\sin(\phi)z}{\rho^2} \quad \frac{\cos(\phi)}{\rho} \right]$$

(3)

9. Find the gradient of $v = \sin(\theta) \cos(\phi) / r$ as shown in screen 4.

grad (**sph** , **2nd** **[SIN]** **♦** **[θ]**) **2nd** **[COS]** **♦** (**[alpha]** **f**) **÷** **r**)

The answer is

$$\left[\frac{-\sin(\theta) \cos(\phi)}{r^2} \quad \frac{\cos(\theta) \cos(\phi)}{r^2} \quad \frac{-\sin(\phi)}{r^2} \right]$$

(4)

Topic 46: Surface Normal

Since the gradient points in the direction of greatest rate of change of a function, it is perpendicular to a surface on which that function is constant. The unit normal vector to a surface can be found using this property as $\mathbf{a}_n = \nabla f / |\nabla f|$ where f is the function which describes the surface.

Find the unit normal vector to a sphere of radius a . The sphere is described by a function of $f = x^2 + y^2 + z^2 - a^2$.

1. Use the function **grad** from Topic 45 to calculate the gradient.

grad(**rec**, $x^2 + y^2 + z^2 - a^2$)

2. Use **unitV()** to find the unit normal vector (bottom of screen 5).

CATALOG **unitV**(**2nd** **[ANS]**)

The answer is

$$\left[\frac{x}{\sqrt{x^2 + y^2 + z^2}} \quad \frac{y}{\sqrt{x^2 + y^2 + z^2}} \quad \frac{z}{\sqrt{x^2 + y^2 + z^2}} \right]$$

(5)

3. To verify that the unit normal vector to the spherical surface is radial, convert to spherical components using **rec2sph** from Topic 43 as shown in screen 6.

rec2sph ([] 2nd [ANS] [] , 2nd [] x [] y [] z 2nd [])

The complete answer is

$$\left[1 \quad \frac{x^2 z}{\sqrt{x^2 + y^2} (x^2 + y^2 + z^2)} + \frac{\frac{y^2 z}{\sqrt{x^2 + y^2}} - \sqrt{x^2 + y^2} z}{x^2 + y^2 + z^2} \quad 0 \right]$$

in which the terms correspond to the r -, θ -, and ϕ -components, respectively.

4. The second term, although rather complicated, has some common terms. Use **ComDenom()** to simplify it as shown in screen 7.

The TI-89 built-in rules of algebra show that the second term is zero. This agrees with intuition that the perpendicular to a sphere is radial only.

(6)

(7)

Topic 47: Divergence

Divergence of a flux density is the differential vector operation which indicates the net flux emanating from a point. When there are sources of flux at the point, the divergence is positive; when there are sinks, it is negative. The mathematical description of divergence is

$$\text{div} \mathbf{D} = \nabla \cdot \mathbf{D}$$

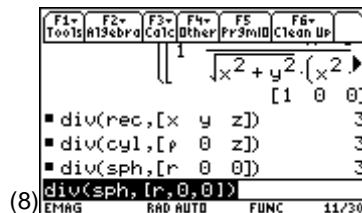
$$= \frac{\frac{\partial(D_1 h_2 h_3)}{\partial x_1} + \frac{\partial(D_2 h_3 h_1)}{\partial x_2} + \frac{\partial(D_3 h_1 h_2)}{\partial x_3}}{h_1 h_2 h_3}$$

where the D_i is the i th component of the flux density vector \mathbf{D} and h_i and x_i are defined as before (Topic 45). These operations are defined in the function **div** shown below.

```
:div(cord,fld
:Func
:Local var,met
:cordchk(cord)>var
:right(var,3)>met
:product(met)*fld./(list▶mat(met))>fld
:(d(fld[1,1],var[1])+d(fld[1,2],v
ar[2])+d(fld[1,3],var[3]))/(product(met))
:EndFunc
```

The function **cordchk** from Topic 45 determines the list of variables and metrics for the chosen coordinate system and stores them in local variable, **var**. The three metrics are extracted from **var** and stored in **met** using **right()**. Scalar multiplication of the field by the product of the metrics dot divided (./) by the vector of the metrics forms the $D_i h_j h_k$ term. The partial derivatives and division by the metric product completes the calculations of **div**.

The position vector from the origin to a point in space is expressed in the three coordinate systems as $[x,y,z]$, $[p,0,z]$, and $[r,0,0]$. The divergence of the position vector should be the same in all three coordinate systems since the coordinate system should not alter the properties of the vector. The calculations shown in screen 8 verify this.



Topic 48: Curl

The curl of a vector field is a measure of its vorticity, which is its tendency to rotate about a point. When the curl of a field is zero everywhere, it is known as a conservative field and an integral around any closed path is zero. The electrostatic field is conservative; the magnetostatic field is not conservative. The mathematical definition of the curl is

$$\text{curl} \mathbf{H} = \nabla \times \mathbf{H} = \frac{\sum_{i=1}^3 \left[\frac{\partial(h_i H_k)}{\partial x_j} - \frac{\partial(h_j H_k)}{\partial x_i} \right] h_i \mathbf{a}_i}{h_1 h_2 h_3}$$

where x_i , \mathbf{a}_i , h_i , and H_i are the variable, unit vector, metric, and vector component of the i th coordinate. (i,j,k) form a right-handed system. Although the form of the curl is somewhat more complicated than previous vector operations, its cyclic nature makes it easy to implement.

The calculations of the curl are implemented in the function **curl**.

1. Define the function **curl** as shown in screens 9 and 10. Notice that it includes the function **cordchk** from Topic 45.



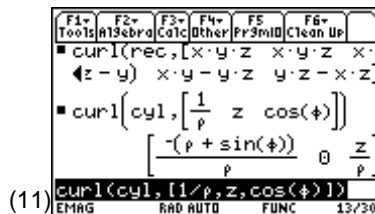
The variables and metrics are stored in local variables **var** and **met**; the elements of **fld** serve as dummy elements to form the local variable **curl**. **n**, **n1**, and **n2** form a cyclical triad used to compute the derivatives of **fld**. Each pass through the **For** loop forms one of the vector components which replaces the dummy elements stored in **curl**.



2. Return to the Home screen, and enter the curl of $[xyz,xyz,xyz]$ as shown in the top of screen 11.

curl (**rec** , **2nd** [**]** **x** **x** **y** **x** **z** , **x** **x** **y** **x** **z** , **x** **x** **y** **x** **z** **2nd** [**]**)

The answer is $[x(z-y) \ xy-yz \ yz-xz]$, as shown at the top of screen 11.



3. Find the curl of $[1/p,z,\cos\phi]$ as shown in the bottom of screen 11.

curl (**cyl** , **2nd** [**]** **1** **÷** **α** **r** , **z** , **2nd** [**]** **COS** **α** **f**) **2nd** [**]**)

4. Find the curl of $[r\cos\theta, \sin\theta, \cos\phi]$ as shown in screen 12.

curl ([sph] , 2nd [r] × 2nd [COS] ♦ [θ]) , 2nd [SIN] ♦
 ([alpha] f) , 2nd [COS] ♦ ([alpha] f) 2nd [1])

The answer is

$$\begin{bmatrix} -\tan\left(\frac{\theta}{2}\right)\cos(\phi) & -\cos(\phi) & \frac{\sin(\theta)r + \sin(\phi)}{r} \end{bmatrix}$$

5. A vector theorem states that the curl of the gradient of a potential is identically zero. The example of a gradient in rectangular coordinates from Topic 45 demonstrates this as $\text{curl}(\text{rec}, \text{grad}(\text{rec}, 10x\sin(y)e^{-5z})) = 0$ (screen 13).

curl ([rec] , grad ([rec] , 10x × 2nd [SIN] y) × ♦ [e^{-x}]
 ([-] 5z))

6. Another vector theorem states that the divergence of any curl is identically zero. This is demonstrated by $\text{div}(\text{cyl}, \text{curl}(\text{cyl}, [1/p, z, \cos(\phi)])) = 0$ (screen 14).

div ([cyl] , curl ([cyl] , 2nd [1] ÷ ♦ ([alpha] r , z ,
 2nd [COS] ♦ ([alpha] f) 2nd [1]))

(12)

(13)

(14)

Topic 49: Laplacian

The behavior of many physical potentials is mathematically described in rectangular coordinates by

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$$

Since this often occurs in Laplace's equation, this is known as the Laplacian. An alternate form of the Laplacian is

$$\text{lapv} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = \nabla \bullet \nabla v = \nabla^2 v$$

From the $\nabla \bullet \nabla v$ term, the method of forming the Laplacian is obvious; it is the divergence of the gradient of the potential v . Although vectors and vector operations are involved, the Laplacian produces a scalar result.

Step 2 requires that along the path, $y=x^2$ and $z=-4$. These substitutions give the first term as $4xydx=4x(x^2) dx$, and the second term as $-3ze^{-x}dy = -3(-4)e^{-x^2} dy$. These explicit substitutions introduce the effects of the path into the integrand. The substitutions have been chosen so that the variables of each integrand are the same as the differential. These substitutions are simplified using the “with” operator, $\boxed{\text{with}}$.

- Now do step 3 on the TI-89 to evaluate the resulting integrals.

$\boxed{2\text{nd}} \boxed{[f]} \boxed{4x} \boxed{\times} \boxed{y} \boxed{[]} \boxed{y} \boxed{=} \boxed{x} \boxed{\wedge} \boxed{2} \boxed{,} \boxed{x} \boxed{,} \boxed{0} \boxed{,} \boxed{2} \boxed{[]} \boxed{-} \boxed{2\text{nd}} \boxed{[f]} \boxed{3z}$
 $\boxed{\blacklozenge} \boxed{[e^x]} \boxed{(-)} \boxed{x} \boxed{[]} \boxed{z} \boxed{=} \boxed{(-)} \boxed{4} \boxed{\text{CATALOG}} \text{ and } \boxed{x} \boxed{=} \boxed{2\text{nd}} \boxed{[\checkmark]} \boxed{y} \boxed{[]}$
 $\boxed{,} \boxed{y} \boxed{,} \boxed{0} \boxed{,} \boxed{4} \boxed{[]}$

The exact value result is $8(5e^2-9)e^{-2}$ (top of screen 18).

- Pressing $\boxed{\blacklozenge} \boxed{[\approx]}$ gives 30.26 in floating point form (bottom of screen 18).

(18)

Example 2

Evaluate

$$\int_{\text{Path}} \mathbf{H} \cdot d\mathbf{l}$$

where $\mathbf{H}=(10/2\pi\rho)\mathbf{a}_\phi$ along the path $\rho=4$, $z=0$, and $0\leq\phi\leq\pi/2$. The natural coordinate system is cylindrical so $\mathbf{H}\cdot d\mathbf{l}=10\rho d\phi/2\pi\rho$. Note that $d\phi$ is multiplied by the metric ρ to get the differential length in the ϕ direction, $\rho d\phi$.

- The integral is entered as shown in screen 19.

$\boxed{2\text{nd}} \boxed{[f]} \boxed{10} \boxed{\blacklozenge} \boxed{[]} \boxed{\alpha} \boxed{r} \boxed{\div} \boxed{[]} \boxed{2} \boxed{2\text{nd}} \boxed{[\pi]} \boxed{\blacklozenge} \boxed{[]} \boxed{\alpha} \boxed{r} \boxed{[]} \boxed{[]}$
 $\boxed{\blacklozenge} \boxed{[]} \boxed{\alpha} \boxed{r} \boxed{=} \boxed{4} \boxed{\text{CATALOG}} \text{ and } \boxed{z} \boxed{=} \boxed{0} \boxed{,} \boxed{\blacklozenge} \boxed{[]} \boxed{\alpha} \boxed{f} \boxed{,} \boxed{0}$
 $\boxed{,} \boxed{2\text{nd}} \boxed{[\pi]} \boxed{\div} \boxed{2} \boxed{[]}$

The result is $5/2$.

- For those not as comfortable with cylindrical coordinate integration, convert the problem to rectangular coordinates. First, transform \mathbf{a}_ϕ to rectangular coordinates using the **cyl2rec** function from Topic 42 (Chapter 9) to obtain $\mathbf{a}_\phi = -y/\sqrt{x^2+y^2}\mathbf{a}_x + x/\sqrt{x^2+y^2}\mathbf{a}_y$ (screen 20).

With the relationship $\rho = \sqrt{x^2 + y^2}$, the dot product of the integrand becomes

$$\mathbf{H} \cdot d\mathbf{l} = \frac{10}{2\pi\sqrt{x^2 + y^2}} \left[\frac{-ydx}{\sqrt{x^2 + y^2}} + \frac{xdy}{\sqrt{x^2 + y^2}} \right]$$

The integration follows the circular path on which $x^2+y^2=16$.

(19)

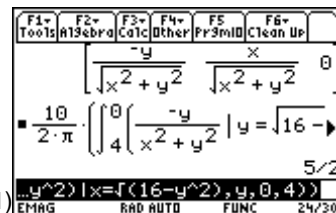
(20)

This is entered as shown in screen 21.

10 ÷ ((2 2nd [π]) (2nd [f] () y ÷ ((x ^ 2 + y ^ 2)
) 1 y = 2nd [√] 16 - x ^ 2) , x , 4 , 0) + 2nd [f]
 x ÷ ((x ^ 2 + y ^ 2) 1 x = 2nd [√] 16 - y ^ 2)
 , y , 0 , 4))

The result is 5/2 as before. Note that x goes from 4 to 0 and y from 0 to 4 as ϕ varies from 0 to $\pi/2$.

(21)



Topic 51: Surface Integrals

The flux passing through a surface is expressed by surface integrals such as

$$\iint_{\text{Area}} \mathbf{J} \cdot d\mathbf{s}$$

where \mathbf{J} is the flux density and $d\mathbf{s}$ is the directed surface element in a specified direction. The evaluation of surface integrals is similar to the three-step process used with line integrals in Topic 50:

1. Evaluate the integrand.
2. Include the effects of the surface.
3. Evaluate the resulting integrals.

Two examples follow which illustrate this process.

Example 1

Find the flux

$$\iint_{\text{Area}} \mathbf{J} \cdot d\mathbf{s}$$

passing through the $x=4$ surface in the $-\mathbf{a}_x$ direction for which $1 \leq y \leq 3$ and $-1 \leq z \leq 1$ where $\mathbf{J} = 10xy^2\mathbf{a}_x$. The integrand when evaluated on the surface is given by

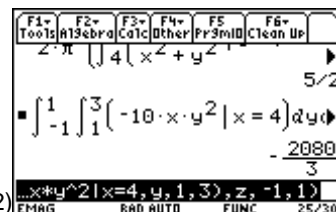
$$\mathbf{J} \cdot d\mathbf{s} = -10xy^2 dydz \Big|_{x=4} = -40y^2 dydz.$$

The resulting integral is entered as shown in screen 22.

2nd [f] 2nd [f] () 10x x y ^ 2 1 x = 4 , y , 1 , 3) , z ,
 () 1 , 1)

The result is $-2080/3$.

(22)



Example 2

The divergence theorem states that the flux out of a closed surface integral is equal to the divergence of the flux density throughout the volume

$$\oiint_A \mathbf{J} \cdot d\mathbf{s} = \iiint_V \nabla \cdot \mathbf{J} dv$$

Calculate the flux out of the entire unit sphere due to $\mathbf{J} = 10xy^2 \mathbf{a}_x$ using the function **div** from Topic 47.

1. Calculate $\text{div} \mathbf{J} = 10y^2$ (screen 23).

$$\text{div} \left(\text{rec} \left(\left[\begin{matrix} 10x & y^2 & 0 \end{matrix} \right] \right) \right)$$

2. Integrate this result throughout the volume of the sphere. Setting the limits of integration with respect to rectangular coordinates is tedious because the volume naturally fits spherical coordinates. Instead, transform the single variable of the integrand to $y = r \sin \theta \sin \phi$ (screen 24).

$$y = r \sin \theta \sin \phi$$

3. Integrate throughout the volume of the sphere using the differential volume of spherical coordinates $dv = r^2 \sin \theta dr d\theta d\phi$ to obtain the total flux of $8\pi/3$ as shown in screen 25.

$$\int_0^{2\pi} \int_0^\pi \int_0^1 10 (\sin \theta)^2 (\sin \phi)^2 r^2 dr d\theta d\phi$$

(23)

(24)

(25)

Tips and Generalizations

WOW! Triple integrals on a pocket calculator, and this is just a warm up. This chapter showed how powerful vector calculus operations can be performed by defining a few simple functions (**grad()**, **div()**, **curl()**, **lap()**). These combined with **with** (**|**) provide a convenient way to do vector calculus.

Chapter 11 shows how these operations can be used to solve typical electromagnetics problems.

Features Used

3-D Graphing,
deSolve(), expand(),
tExpand(), solve(),
limit(), colDim(),
rowDim(), Func, For,
EndFor, newMat(), @n1,
@1, \square , \square

Setup

\square 1, setFold(emag)

Electromagnetics

This chapter describes how to use the TI-89 to solve Laplace's equation for two-dimensional electrostatic problems. The separation of variables method provides exact solutions but is restricted to specific geometries. The relaxation method is applicable to any geometry, but it provides only an approximate solution. This chapter uses the function **lap()** which is defined in *Chapter 10: Vector Calculus*.

Topic 52: Separation of Variables

Electrostatic potentials satisfy Laplace's equation, $\nabla^2 v = 0$, in charge-free regions. A typical two-dimensional problem is shown in Figure 1.

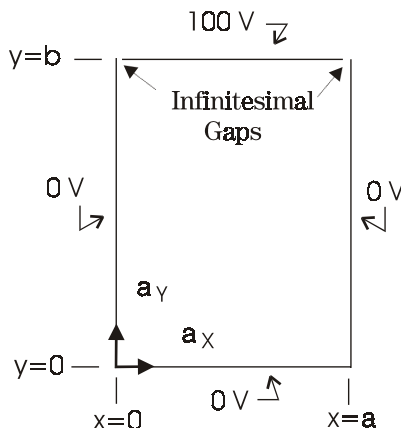


Figure 1. Geometry for 2-D electrostatic problem

In two-dimensional, rectangular geometries, the potential varies with x and y, that is, $v=v(x,y)$. The assumption that v is the product of two functions, each of which depending upon only a single variable, leads to $v=f(x)g(y)$.

1. Clear the TI-89 by pressing \square [2nd] [F6] **2:NewProb** \square [ENTER].

2. Enter the expression for **v** as shown in screen 1.

$f(x) \cdot g(y) \rightarrow v$

3. Applying the Laplacian operator to **v** and dividing the result by **v** leads to

$$\frac{\nabla^2 v}{v} = \frac{\frac{\partial^2 f(x)}{\partial x^2}}{f(x)} + \frac{\frac{\partial^2 g(y)}{\partial y^2}}{g(y)} = 0$$

Enter the expression using the function **lap** from Topic 49 (Chapter 10) as shown in screen 2.

CATALOG **expand(lap(rec(v),v)/v)**

4. Since the first term depends only on **x** and the second only on **y** with their sum equal to zero, both terms must be constant. When the **x**-dependent term is set equal to the constant $-k^2$, the result is the differential equation

$$\frac{\partial^2 f(x)}{\partial x^2} + k^2 f(x) = 0$$

Use **deSolve()** to enter this equation with the boundary condition $f(0)=0$.

CATALOG **deSolve(f(2nd) ['] + k^2 * f = 0**

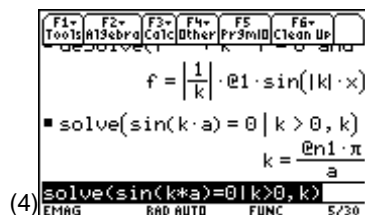
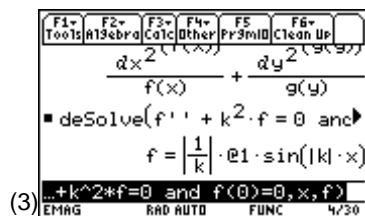
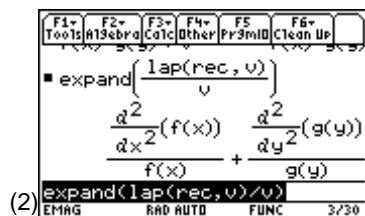
CATALOG **and f(0) = 0, x, f)**

Screen 3 shows the output form is $f(x) = \frac{1}{k} \sin(|k|x)$

times the constant $\frac{1}{k}$. (The constant does not affect the solution.)

5. This equation also equals zero at the other boundary, or $f(a) = \sin(ka) = 0$. Find the allowed values of **k** to be $k = n\pi/a$ as shown in screen 4.

CATALOG **solve(sin(k*a)=0 | k>0, k)**



Note: @1 represents an arbitrary integer. The screen may show @2, @3, and so forth, if **deSolve()** has been executed prior to this.

6. When the first term in the separated form of Laplace's equation equals $-k^2$, the second term in the equation must equal k^2 . This gives a second differential equation

$$\frac{\partial^2 g(y)}{\partial y^2} - k^2 g(y) = \frac{\partial^2 g(y)}{\partial y^2} - \left(\frac{n\pi}{a}\right)^2 g(y) = 0$$

which is entered as shown in screen 5.

CATALOG **deSolve**(**g** **2nd** [**'**] **2nd** [**'**] **-** **k** **^** **2** **x** **g** **=** **0**
CATALOG **and** **g** (**0** **)** **=** **0** **,** **y** **,** **g** **)**

The solution is of the form $g(y) = \left|\frac{1}{k}\right| \sinh(ky)$ times the constant $\left|\frac{1}{k}\right|$.

7. Since $k=n\pi/a$ from the boundary conditions on $f(x)$, $g(y)=\sinh(n\pi y/a)$. But it is not possible for this solution of $g(y)$ to satisfy the upper boundary condition for arbitrary $y=b$ (see top of Figure 1). Since the solutions for each n satisfy Laplace's equation, a linear combination of solutions must satisfy it also; such a solution is

$$v(x, y) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi y}{a}\right)$$

At the upper boundary where $y=b$, this takes the form of a Fourier series in x

$$v(x, b) = 100 = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi b}{a}\right)$$

The constant c_n is determined by usual Fourier techniques. Both sides of the equation are multiplied by $\sin(m\pi x/a)$ and integrated over the range $0 \leq x \leq a$. The left-hand side (LHS) integral becomes

$$\text{LHS} = \int_{x=0}^a 100 \sin\left(\frac{m\pi x}{a}\right) dx = \frac{100a(1 - \cos m\pi)}{m\pi}$$

as shown in screen 6.

2nd [**∫**] **100** **2nd** [**SIN**] **m** **2nd** [**π**] **x** **÷** **a** **)** **,** **x** **,** **0** **,** **a** **)**
STO **→** **lhs**

(5)

Tools | F1 | F2 | F3 | F4 | F5 | F6
 Tools | Algebra | Calc | Other | Fr3rd | Clean Up
 $k = \frac{n\pi}{a}$
 $\text{deSolve}(g'' - k^2 \cdot g = 0 \text{ and } g(0) = 0 \text{ and } g(b) = 0, y, g)$
 $g = \frac{1}{k} \cdot 2 \cdot \sinh(|k| \cdot y)$
 ...-k^2*g=0 and g(0)=0,y,g)
 EMAG RAD AUTO FUNC 6/30

(6)

Tools | F1 | F2 | F3 | F4 | F5 | F6
 Tools | Algebra | Calc | Other | Fr3rd | Clean Up
 $g = \frac{1}{k} \cdot 2 \cdot \sinh(|k| \cdot y)$
 $\int_0^a 100 \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) dx \rightarrow \text{lhs}$
 $\frac{100 \cdot a \cdot (\cos(m \cdot \pi) - 1)}{m \cdot \pi}$
 $\dots \sin(m \cdot \pi \cdot x / a), x, 0, a) \rightarrow \text{lhs}$
 EMAG RAD AUTO FUNC 7/30

8. When $m=2@n2$ is even, $lhs = 0$ (screen 7). When

$$m=2@n2-1 \text{ is odd, } lhs = \frac{200a}{m\pi} \text{ (screen 8).}$$

9. The right hand side integral, given as

$$\int_0^a \sum_{n=1}^{\infty} cn \sinh\left(\frac{n\pi b}{a}\right) \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx$$

requires more careful evaluation. An interchange of the summation and integration leads to a series of integrals, each with a different value of n

$$\sum_{n=1}^{\infty} cn \sinh\left(\frac{n\pi b}{a}\right) \int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx$$

The integral can be calculated directly by the TI-89 as shown in screen 9.

`cn` \times `CATALOG` `sinh` $(n$ `2nd` $[\pi]$ `b` \div `a` `)` `a` \times $($ $($ `m` $+$ `n` `)` \times `CATALOG` `tExpand` $($ `2nd` $[\sin]$ `m` `2nd` $[\pi]$ $-$ `n` `2nd` $[\pi]$ `)` `)` \times `CATALOG` `tExpand` $($ `2nd` $[\sin]$ `m` `2nd` $[\pi]$ $+$ `n` `2nd` $[\pi]$ `)` `)` \times `cn` \div $($ `2` \times $($ `m` $+$ `n` `)` \times $($ `m` $-$ `n` `)` `2nd` $[\pi]$ `)` `)` `n` $=$ `♦` `STO` `n1` `CATALOG` `and` `m` $=$ `♦` `STO` `n2`.

Evaluation of `rhs` for $n \neq m$ requires the `tExpand` command for the trigonometric functions only; in addition $n=@n1$ =integer and $m=@n2$ =integer. Enter the following key strokes for this evaluation.

`CATALOG` `sinh` $(n$ `2nd` $[\pi]$ `b` \div `a` `)` `a` \times $($ $($ `m` $+$ `n` `)` \times `CATALOG` `tExpand` $($ `2nd` $[\sin]$ `m` `2nd` $[\pi]$ $-$ `n` `2nd` $[\pi]$ `)` `)` \times `CATALOG` `tExpand` $($ `2nd` $[\sin]$ `m` `2nd` $[\pi]$ $+$ `n` `2nd` $[\pi]$ `)` `)` \times `cn` \div $($ `2` \times $($ `m` $+$ `n` `)` \times $($ `m` $-$ `n` `)` `2nd` $[\pi]$ `)` `)` `n` $=$ `♦` `STO` `n1` `CATALOG` `and` `m` $=$ `♦` `STO` `n2`.

10. The result shows $RHS=0$ when $n \neq m$ (top of Screen 10).

For the case $n=m$ the evaluation is much simpler as shown in screen 10.

`CATALOG` `limit` $($ `rhs` $,$ `n` $,$ `m` `)` `|` `m` $=$ `♦` `STO` `n1`

(7) `limit` $($ `lhs` $,$ `m` $,$ `2*n1` `)`

(8) `limit` $($ `lhs` $,$ `m` $,$ `2*n1-1` `)`

(9) `*sin` $($ `m*pi*x/a` $,$ `x` $,$ `0` $,$ `a` $)$ `→` `rhs`

(10) `limit` $($ `rhs` $,$ `n` $,$ `m` `)` `|` `m` $=$ `n1`

(11) `limit` $($ `rhs` $,$ `n` $,$ `m` `)` `|` `m` $=$ `n1`

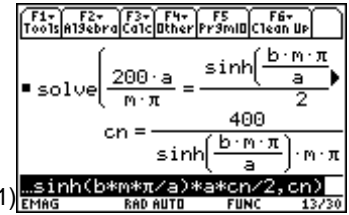
(12) `limit` $($ `rhs` $,$ `n` $,$ `m` `)` `|` `m` $=$ `n1`

(13) `limit` $($ `rhs` $,$ `n` $,$ `m` `)` `|` `m` $=$ `n1`

11. The constant **cn** is determined by equating the evaluations of **lhs** and **rhs** of previous calculations (screen 11).

CATALOG solve(200a ÷ (1 m 2nd [π]) = CATALOG sinh(b
 × m × 2nd [π] ÷ a) × a × cn ÷ 2 , cn)

(11)



12. These results are combined to give the electrostatic potential within the region as

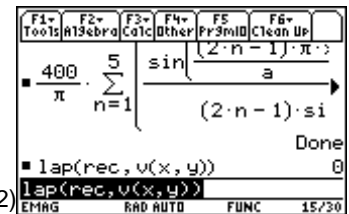
$$v(x,y) = \frac{400}{\pi} \sum_{n=\text{odd}} \frac{\sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi y}{a}\right)}{n \sinh\left(\frac{n\pi b}{a}\right)}$$

This is implemented with a finite number (5) of terms. Due to the TI-89's single step size of 1, the summation index **n** must be replaced by **2n-1** which takes on only odd integer values for consecutive integer values of **n**.

Calculate the electrostatic potential as shown on the top of screen 12.

400 ÷ 2nd [π] CATALOG Σ 4 2nd [SIN] (2n - 1) 2nd [π] x
 ÷ a) CATALOG sinh(2n - 1) 2nd [π] y ÷ a) ÷ (2n - 1) CATALOG sinh(2n - 1) 2nd [π] b ÷ a)
) , n , 1 , 5) STO v (x , y)

(12)



13. To check that this solution satisfies Laplace's equation, use the function **lap** from Topic 49 (screen 12).

lap (rec , v (x , y))

Observe (after awhile) that the result is zero, as it must be!

Topic 53: 3D Potential Graphs

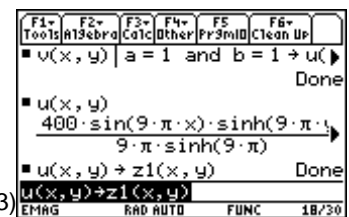
How does **v(x,y)** from Topic 52 vary throughout the region? 3-D graphing provides such a view, but it requires specific values for **a** and **b**.

1. For simplicity, let **a=b=1** as shown in the top of screen 13.

The result is stored as a new function **u(x,y)** by first defining **u(x,y)**.

v (x , y) 1 a = 1 CATALOG and b = 1 STO u (x , y)

(13)



Then verify that the function is correct.

u (x , y)

2. Define the graph function $z1(x,y)=u(x,y)$ so that the plotting is faster than it would be by plotting $v(x,y)|a=1$ and $b=1$ (bottom of screen 13).

$u(x,y) \rightarrow \text{STO} \rightarrow z1(x,y)$

The height z represents the magnitude of the function $v(x,y)$.

3. Press **MODE** and set **Graph** mode to **3D**. Use **WINDOW** to set the window variable values as shown in screens 14 and 15.

$\text{eye}\theta = -120$	$ymin = 0$
$\text{eye}\phi = 75$	$ymax = 1$
$\text{eye}\psi = 0$	$ygrid = 14$
$xmin = 0$	$zmin = 0$
$xmax = 1$	$zmax = 140$
$xgrid = 14$	$ncontour = 5$

4. From the Window Editor, use **GRAPH FORMATS** to set the graph formats as shown in screen 16.

Coordinates: RECT

Axes: AXES

Labels: ON

Style: WIRE AND CONTOUR

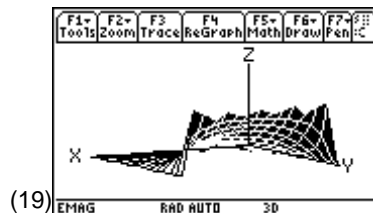
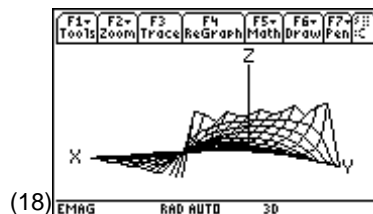
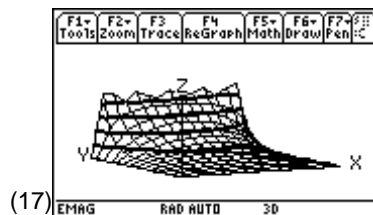
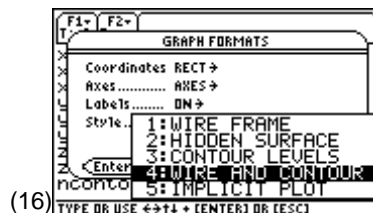
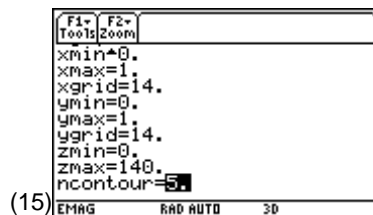
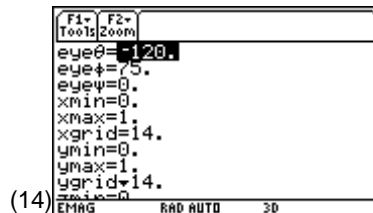
5. Press **GRAPH**. Wait a few minutes for the graph to be calculated. The results will look like screen 17.

The voltage $v(x,y)$ is graphed with equipotential contours every 20 volts. Once the calculations with the contours are made, the contours can be turned on and off.

6. Press **GRAPH** and set **Style** to **WIRE FRAME** to turn off the contours. In the Window Editor, set $\text{eye}\theta = 60$ and press **GRAPH** for a view from "behind" the graph, that is, from the region of $y > 1$ (screen 18).

7. Press **GRAPH** and set **Style** to **HIDDEN SURFACE** to give a different nature to the graph (screen 19).

The variation of the amplitude along the upper edge is due to the limited five-term Fourier Series approximation of the 100 volts. More terms would make this smoother.



8. Press **[Y]** to look along the y-axis directly at these variations from the back of the graph (screen 20).

Use **[X]** to toggle between expanded and normal views (screen 21).

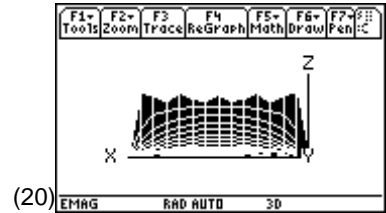
Use **[←]** **[→]** **[↶]** **[↷]** or change values in the Window Editor to change the viewing angle.

9. Return to the original viewing angle by pressing **0**. Then, in the Window Editor, set **eye0=-120** and press **[GRAPH]**. To find the voltage at (x,y) coordinates, press **[F3] Trace** which places a cursor at the center of the x,y space and provides the corresponding z-value. Move the cursor $\pm y$ with **[↶]** **[↷]** and $\pm x$ with **[←]** **[→]**. The voltage at $x=.29$ $y=.79$ is 52.12 V as shown in screen 22.

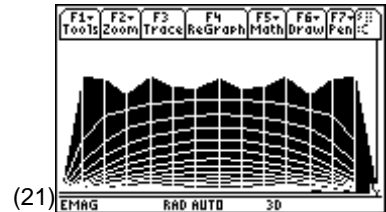
10. To see the voltage at a given position, just type the **x** and **y** coordinates. For example, type **0.25** **[ENTER]** **0.75** **[ENTER]** to see that the value at $x=0.25$ and $y=0.75$ is 43.20 V (screen 23).

11. Press **[♦]** **[1]** and set **Style** to **WIRE AND CONTOUR** to display the equipotential contours on the wire frame.

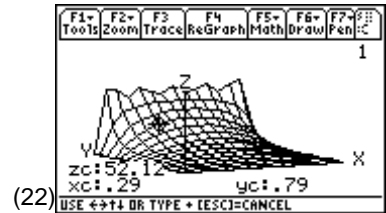
Press **[X]**, **[Y]**, and **[Z]** for views from those axes (screens 24, 25 and 26). The z-axis view (from above the graph) provides the standard view of the equipotentials on the x-y plane. However, **[F2] 5:ZoomSqr** must be used to recompute the graph if the proper ratio of the **x** and **y** dimensions is required.



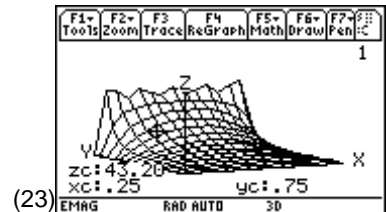
(20)



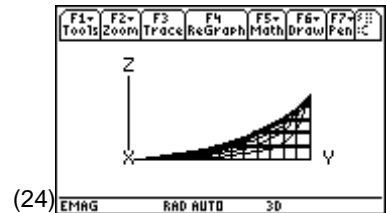
(21)



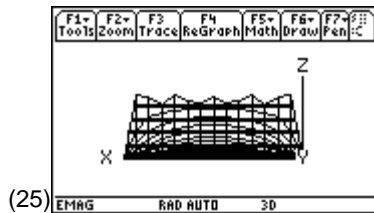
(22)



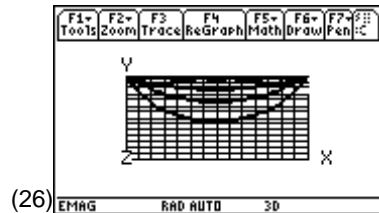
(23)



(24)



(25)



(26)

Topic 54: Relaxation Method

An alternate, but approximate, method which works for all geometries is known as the relaxation method. The procedure is simple: a solution for the voltage $v(x,y)$ is guessed, an iterative set of approximate calculations operates upon this solution, and the “guessed” solution is modified by each successive set of calculations in a way which allows the guessed solution to “relax” to the correct solution.

Electrostatic voltages satisfy Laplace’s equation as $\nabla^2 v = 0$. Finite differences approximate the Laplacian by the central difference form as

$$\begin{aligned}\nabla^2 v &= \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \\ &\approx \frac{v_R + v_L - 2v_o}{\Delta x^2} + \frac{v_T + v_B - 2v_o}{\Delta y^2} \\ &= \frac{v_R + v_L + v_T + v_B - 4v_o}{\Delta x^2} = 0\end{aligned}$$

where $\Delta x = \Delta y$. For non-zero increments, $\Delta x, \Delta y \neq 0$, this equation is solved for the central node voltage as

$$v_o = \frac{v_R + v_L + v_T + v_B}{4}$$

This equation states that the voltage at the center node is equal to the average of the node voltages around it. This equation is valid at all nodes. The solution region is divided into rectangular grids, and the equation for v_o is applied successively to each node. This procedure is repeated until the node voltages approach the correct solution as a limiting value.

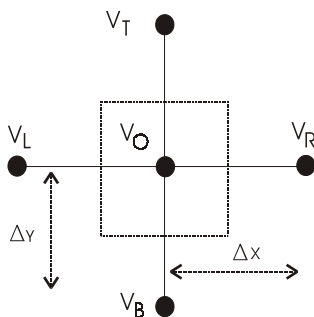


Figure 2. Finite difference cell

The solution region is divided into square grids. Each node is described by two integers, an x-coordinate and a y-coordinate. The x-coordinates are numbered from left to right; the y-coordinates are numbered from bottom to top. The nodes show a great similarity to the elements of a matrix in their geometric arrangement and in their numbering scheme. The storing of node voltages in a matrix provides a convenient and visual display of the voltages. The matrix row-column numbering scheme is used to simply and systematically apply the node voltage calculations to all of the cells. A second matrix is used to identify those boundary nodes at which the voltage is fixed and must not be changed by the calculations.

The TI-89 handles matrices and repetitive calculations very handily. The node voltages are stored in a matrix named **volt**; the geometry data is stored in a matrix named **geo**. Define a function called **relax** to perform the calculations.

```
:relax(v,geo,ntot)
:Func
:Local n,ntot,nrow,ncol
:Local nrowmax,ncolmax
:colDim(v)→ncolmax
:rowDim(v)→nrowmax
:For n,1,ntot
:  For nrow,2,nrowmax-1
:    For ncol,2,ncolmax-1
:      (v[nrow-1,ncol]+v[nrow+1,ncol]+v[nrow,ncol-1]+v[nrow,ncol+1])/4*geo[nrow,ncol]+v[nrow,ncol]*(1-
        geo[nrow,ncol])→v[nrow,ncol]
:    EndFor
:  EndFor
:EndFor
:v
:EndFunc
```

The function **relax** accepts the voltage matrix (**v**), the geometry matrix (**geo**), and the desired number of sets of repeated calculations (**ntot**) as arguments. For simplicity, all interior nodes of the voltage matrix are set to zero as the initial guess. Since the nodes on the edges of the matrix are fixed, the calculations are limited to interior points by the row and column sizes. The single instruction within the nested loops implements the discrete Laplacian at a node. The results of this calculation are multiplied by the corresponding element of the geometry matrix, 0 for nodes that are fixed or 1 for nodes that are modified by calculations. In addition, the nodal voltage is multiplied by $1 - \text{geo}$ so that the original nodal voltage is stored in those nodes which are to remain fixed. This calculated node voltage is stored in **v**, and calculations move to the next node. With the completion of calculations, **v** is displayed on the Home screen.

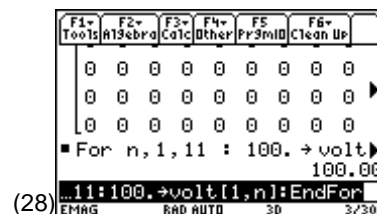
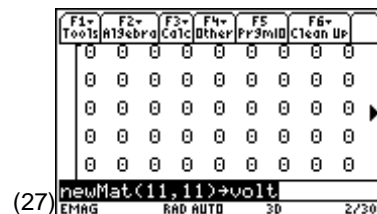
1. Clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
2. For numeric results, the region of screen 27 is divided into 10x10 square grids resulting in 11x11 matrices for the voltage and geometry matrices. On the Home screen, create the voltage matrix filled with zeros (see screen 27).

[CATALOG] newMat(11 [] 11 []) [STO] volt

3. Set the 100 V boundary voltage.

[CATALOG] For n [] 1 [] 11 [2nd] [:] 100 [] [STO] volt [2nd] [] 1 [] n [2nd] [] [2nd] [:] [CATALOG] EndFor

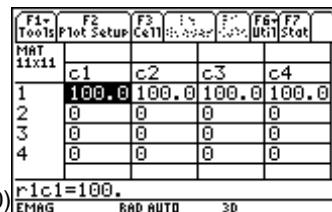
Screen 28 shows that floating point results are returned because of the decimal point in 100.



4. To verify that the matrix has been entered correctly, press **[APPS]** **6:Data/Matrix Editor 2:Open**. Specify **Type** as **Matrix**, **Folder** as **emag**, and **Variable** as **volt** (screens 29 and 30).



(29) TYPE OR USE \leftarrow + [ENTER] OR [ESC]



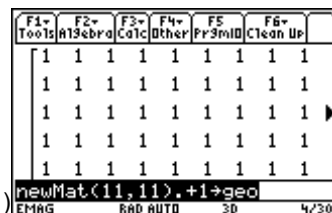
(30) EMAG RAD AUTO 3D

Note: The 100's can be entered directly into this display instead of the entry line **For...EndFor** instruction.

Note: The $\square + \square$ tells the TI-89 to do a dot addition. That is, add 1 to each of the elements in the new, all-zeroes matrix. If the \square is omitted and just the $+$ is used, the 1 will be treated as the identity matrix and then added to the new matrix, which is not wanted.

5. On the Home screen, create the geometry matrix filled with 1's (screen 31).

[CATALOG] **newMat**(11 \square 11 \square \square + \square 1 **[STO]** **geo**

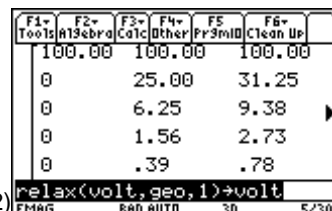


(31) EMAG RAD AUTO 3D 4/30

6. Since the function **relax** automatically excludes the first and last rows, zeros need not be inserted for a rectangular solution region. For non-rectangular regions, zeros must be inserted into **geo** at boundary locations. Do one iteration as shown in screen 32.

relax(\square **volt** \square **geo** \square 1 \square **[STO]** **volt**

The results are stored in **volt** through the command line since **relax** is a function and cannot change any stored variables.

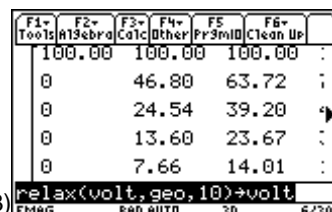


(32) EMAG RAD AUTO 3D 5/30

7. Repeat function **relax** 10 times (screen 33).

relax(\square **volt** \square **geo** \square 10 \square **[STO]** **volt**

It will take a while to get the results.



(33) EMAG RAD AUTO 3D 6/30

8. The matrix can be viewed by pressing **[APPS] 6:Data/Matrix Editor 1:Current** as shown in screen 34.

Since the structure is symmetric, the voltages should show symmetry about **c6** when sufficient iterations have been completed.

(34)

Note: Press **[◀] [1]** and select **5** to show four columns.

9. Since it isn't exactly symmetric, return to the Home screen and repeat the function **relax** five more times as shown in screen 35.

relax **[(]** **volt** **[,]** **geo** **[,]** **5** **[)]** **[STO▶]** **volt**

(35)

10. Return to the Matrix Editor to check the symmetry (screen 36).

It's not exactly there yet, but close enough.

(36)

Topic 55: 3D Graphs of Tabular Data

As with the separation of variables method, a graph of the voltage from Topic 54 is helpful. However, matrix data cannot be graphed directly because it exists only for discrete values of row and column variables. But with the **int()** function, values of the graphing variable within a range can be converted to an integer for which row or column matrix data exists.

1. Create the function **matplot** as shown in screen 37 to convert the data.

The **x,y** coordinates of the data matrix **mat** are converted to integer values that provide the row and column indices. In order that the **y** index begins at the bottom of the matrix, it is reflected about the center row of the matrix.

(37)

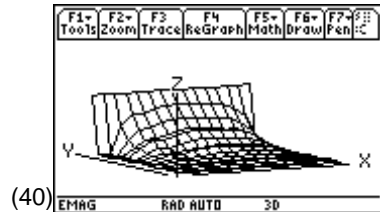
2. Press **[◀] [WINDOW]** and set the window variable values as shown in screen 38 as well as **zmin=0**, **zmax=140**, and **ncoutour=0**. Be sure the TI-89 is in the 3D graph mode.

(38)

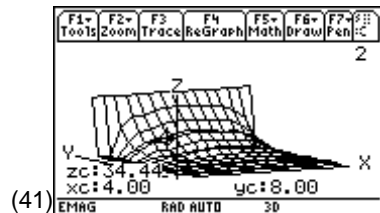
3. Press \diamond [Y=] to display the Y= Editor. Define **z2(x,y)=matplot(x,y,volt)** to graph the relaxation method solution (screen 39). Deselect **z1**.



4. Press \diamond [GRAPH] to see the solution.



5. It looks nearly like the graph with the other method. A comparison with the exact solution of separation of variables using F3 Trace shows that the approximate voltage at $x=1/4 \cdot \text{xmax}=4$ and $y=3/4 \cdot \text{ymax}=8$ is $v=34.44$ V compared to the exact value of 43.20 V (screen 23). This inaccuracy is due to the adjustment of the x- and y-coordinates to take the integer plotting routine into account and the approximate nature of the relaxation method.



Tips and Generalizations

This chapter showed how to solve and display problems using the TI-89 that are normally solved on larger computers. The relaxation method is general enough to solve other configurations by simply changing **volt** and **geo**.

The **matplot()** function can be used to display a 3D graph of any matrix of data. It is faster, though less general, if the matrix to be graphed is explicitly stated in the function rather than passed as a parameter. Screen 42 shows how this is done for the relaxation problem.



Although finding the potential over a 2D surface is useful, sometimes knowing what happens on a line is enough. Chapter 12 looks at some common ways of analyzing transmission lines.

Features Used

`real()`, `limit()`,
`NewProb`, `when()`,
 Numeric Solver

Setup

1, `NewFold tline`
`setMode("Angle",`
`"Degree")`
`setMode("Complex`
`Format", "Polar")`

Transmission Lines

This chapter describes how to calculate the characteristic impedance and phase velocity on transmission lines. Steady state transmission line behavior and simple matching concepts are included also. The functions `reflcoef()`, `lineleng()`, `zin()`, `yin()`, and `vswr()` are created.

Topic 56: Characteristic Impedance

One of the most basic parameters of a transmission line is **zo**, its characteristic impedance. **zo** depends upon the geometry and the material of the transmission line. In this section, **zo** is calculated for four common transmission lines — coaxial, twin-lead, parallel plate, and microstrip. The cross-sections of these lines are shown in Figure 1.

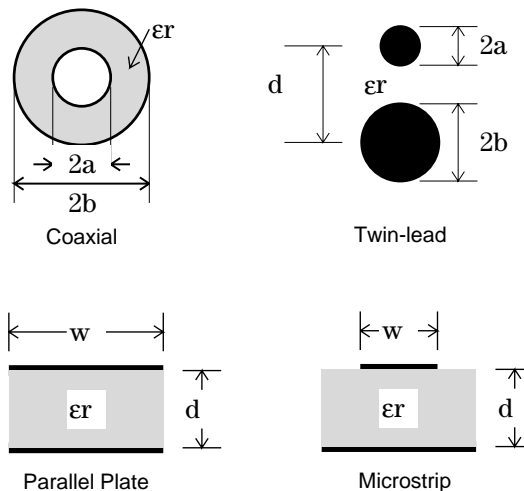


Figure 1. Transmission line cross-sections

Coaxial	$z_0 = \frac{60 \ln\left(\frac{b}{a}\right)}{\sqrt{\epsilon r}}$
Twin-lead	$z_0 = \frac{120 \cosh^{-1}\left(\frac{d}{2\sqrt{ab}}\right)}{\sqrt{\epsilon r}}$
Parallel Plate	$z_0 = \frac{120\pi d}{w\sqrt{\epsilon r}}$
Microstrip	$F = \sqrt{\frac{\epsilon r + 1}{2} + \frac{\epsilon r - 1}{2\sqrt{1 + \frac{12d}{w}}}}$
$w/d \leq 1$	$z_0 = \frac{60 \ln\left(\frac{w}{4d} + \frac{8d}{w}\right)}{F}$
$w/d \geq 1$	$z_0 = \frac{120\pi}{F\left(\frac{2}{3} \ln\left(1.444 + \frac{w}{d}\right) + 1.393 + \frac{w}{d}\right)}$

Table 1. Characteristic impedances

The equations shown in Table 1 are used to calculate z_0 of a transmission line from its geometry and material parameters. However, with the TI-89's numeric solver, any variable can be calculated when the others are known.

Coaxial and Twin-lead

1. Clear the TI-89 by pressing $\boxed{2nd} \boxed{F6} \text{ 2:NewProb } \boxed{ENTER}$.
2. Press $\boxed{APPS} \text{ 9:Numeric Solver}$ to display the Numeric Solver, and enter the equation for **zo** as highlighted in screen 1.

zocoax $\boxed{=}$ $\boxed{60} \boxed{\times} \boxed{CATALOG} \boxed{\ln} \boxed{(} \boxed{b} \boxed{\div} \boxed{a} \boxed{)} \boxed{\div} \boxed{(} \boxed{2nd} \boxed{\sqrt{}} \boxed{\diamond} \boxed{(}$
 $\boxed{\alpha} \boxed{e} \boxed{r} \boxed{)}$

3. Press \boxed{ENTER} or \odot to display the variables in the equation.

Note: To enter ϵ , press $\boxed{\diamond} \boxed{(}$
 $\boxed{\alpha} \boxed{e}$.



Note: The number of digits displayed is independent of the mode settings, since it is a numeric solution.

4. Enter values of $b=.015$, $a=.005$, and $\epsilon r=2$. Then move the cursor to **zocoax**= and press **[F2] Solve** to obtain the solution **zocoax**=46.61... Ω as shown in screen 2.

The ■ symbols on the left side of the screen show which variable was calculated and that the solution is exact since **left-rt=0**.

5. To calculate the value of ϵr which will give **zocoax**=50, enter 50 into **zocoax**, move the cursor to ϵr , and press **[F2]** to display the required value of 1.73... (screen 3).

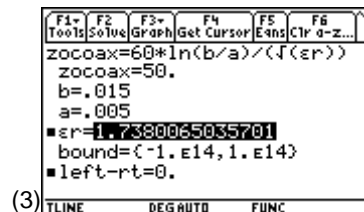
6. In a similar manner, enter the **zo** equation for the twin-lead line by moving to the top of the line.

zotwin = 120 \times **[CATALOG]** **cosh**⁻¹(**d** \div **[]** **2** \times **[2nd]** **[√]** **a** \times **b** **[]** **[]** **[]** **[]** **[]** **[2nd]** **[√]** **[]** **[alpha]** **er** **[]** **[]**)

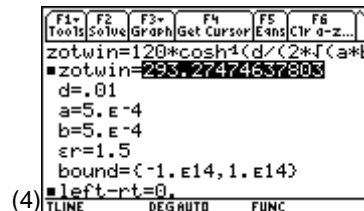
7. Set $d=.01$, $a=b=.0005$, and $\epsilon r=1.5$ and solve for **zotwin**=293.27... Ω (screen 4).
8. These two equations used can be accessed through **[F5]**; the number of "last" equations is eleven (11) by default and is set using **[]** **[1]** **[]** (screen 5).
9. Press **[]** to display both equations. Any equation in this list can be made the active equation by highlighting it and pressing **[ENTER]**; activate **zocoax** (screen 6).
10. Equations can be recalled to the Numeric Solver in this way as long as the number of last equations history is not exceeded. For more permanent storage, press **[F1 2:Save Copy As]**. The equation is placed in the **TLINE** folder. Name it **coax** (if it is named **zocoax**, the equation name will conflict with the variable name) as shown in screen 7.



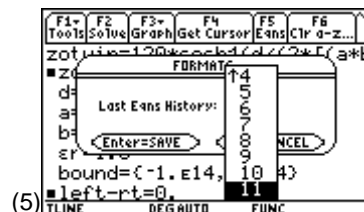
(2)



(3)



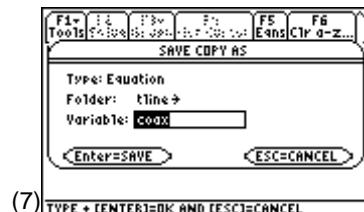
(4)



(5)



(6)



(7)

11. To verify that **coax** has been stored, press **[2nd]** **[VAR-LINK]** and scroll down to the **TLINE** folder to see **coax** and each of the variables **a**, **b**, **zocoax**, and **er** listed (screen 8). Since the variables are listed alphabetically and Roman characters precede Greek, **er** is at the bottom of the folder; scroll down to see it.



Parallel Plate and Microstrip

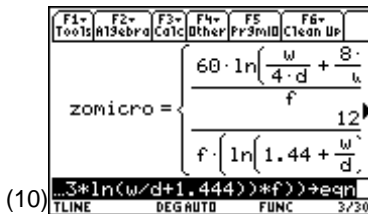
Although the remaining two equations can be entered into the Numeric Solver and saved in the same way, the rather complicated microstrip equation is entered and checked more easily in the Home screen.

1. Return to the Home screen, and clear the variables to be used in the microstrip equation as shown in screen 9.



[CATALOG] **DelVar** **f**, **w**, **[alpha]** **er**, **d**

2. Now enter the **zo** equation (screen 10).



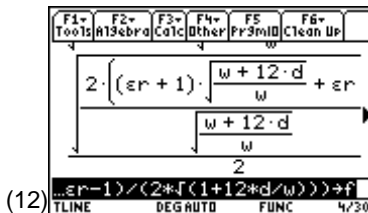
zomicro = **[CATALOG]** **when**(**w** ÷ **d** **[2nd]** **[<]** **=** **1**, **60** × **[2nd]** **[LN]** **8** × **d** ÷ **w** + **w** ÷ **[4]** **d**) ÷ **f**, **120** **[2nd]** **[π]** ÷ **[1]** **[1]** **w** ÷ **d** + **1.393** + **2** ÷ **3** × **[2nd]** **[LN]** **w** ÷ **d** + **1.444** **[STO]** **eqn**

Note how the **when()** command is used to implement the two parts of the **zomicro** equation. The **→eqn** at the end stores the **zomicro** equation so that the Numeric Solver uses it.



3. Press **[APPS]** **9:Numeric Solver** to see **zomicro** in the Numeric Solver.

4. So far **f** has not been defined. Return to the Home screen and define **f** as shown in screen 12.



[2nd] **[√]** **[alpha]** **er** + **1**) ÷ **2** + **[alpha]** **er** - **1**) ÷ **[2]** × **[2nd]** **[√]** **1** + **12** × **d** ÷ **w**)) **[STO]** **f**

To prevent deleting **f** with **NewProb**, be sure to lock it. Alternatively, name it **ff**. See *Do This First*.

5. Display the Numeric Solver and press **[ENTER]**. Enter the values for a microstrip transmission line on a dielectric substrate with thickness **d**=0.00127, **er**=2.2, and a conductor width of **w**=.00391 m. Solving for **zomicro** gives **zomicro**=50.31...Ω (screen 13).



Topic 57: Reflection Coefficient

When sinusoidal generators are used to excite a transmission line, all transient waves have decayed to zero and the line is in steady state. A common steady-state design goal is to match the source impedance to the transmission line input impedance. The input impedance of a transmission line with characteristic impedance z_0 and length d is given by

$$z_{in} = z_0 \frac{1 + \Gamma_L e^{-j4\pi \frac{d}{\lambda}}}{1 - \Gamma_L e^{-j4\pi \frac{d}{\lambda}}}$$

for a frequency with a wavelength of λ . Since this calculation involves complex numbers, creating a function will make the calculations easier.

1. Clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
2. Calculate the reflection coefficient of the load impedance as

$$\Gamma_L = \frac{z_L - z_0}{z_L + z_0}$$

The reflection coefficient is a complex phasor with an amplitude of 1 or less.

Define the function **reflcoef** as shown in screen 14.

Note that **limit()** is used to handle the case of an open circuit with $z_L = \infty$.

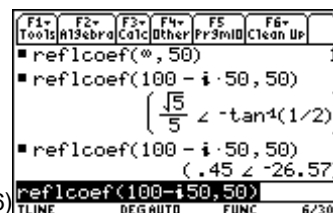
3. Return to the Home screen and use **reflcoef** to calculate the reflection coefficients for real loads of $z_L = 50$, 0, and $\infty \Omega$ on a line with $z_0 = 50 \Omega$ (screen 15).
4. Calculate the coefficients for the complex load of $100 - j50 \Omega$ on a line with $z_0 = 50 \Omega$ (screen 16).
5. Press **[◊] [≈]** to get the floating point value shown at the bottom of screen 16.



(14)



(15)



(16)

The results are 0, -1, +1, and $0.45 \angle -26.57^\circ$. When the load is “matched” to the line, there is no reflected signal; a short circuit reflects the incident signal with opposite polarity; and an open circuit reflects with the same polarity.

Topic 58: Phase Shift

When the load is attached to the end of a length of line, the input reflection coefficient is multiplied by $\exp(-j4\pi d/\lambda)$ which is $1\angle(-720d/\lambda)$ as a phasor in degree form. This term depends only on the line length in terms of wavelength d/λ .

1. Define the function **lineleng** as shown in screen 17.
2. Return to the Home screen and clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
3. Use **lineleng** to calculate the phase shift of a reflection coefficient for line lengths of $d=0$, $1/8$, $1/4$, and $1/2$ wavelengths. Since the line length is given as a fraction of wavelength, $\lambda=1$ (screen 18).

(17) **Note:** To enter λ , press **[α] [L]**. To enter \angle , press **[2nd] [\angle]**. To enter $-$, press **[$-$]**.

Topic 59: Input Impedance/Admittance

The equation for input impedance can be defined as a function.

The input impedance depends upon the line length. For lines with $d=n\lambda/2$, the input impedance equals the load impedance. For loads with $z_l=z_o$, the input impedance is z_o .

1. Press **[MODE]** and set **Complex Format** mode to **RECTANGULAR**.
2. Define the function **zin** as shown in screen 19. **zin** uses **reflcoef** from Topic 57 and **lineleng** from Topic 58.
3. Return to the Home screen, and clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
4. Use **zin** to calculate the input impedance of a line with $z_l=100-j50\ \Omega$, $z_o=50\ \Omega$, and $\lambda=1$. Use $d=.35$, $d=.5$, and $d=1$ (screen 20).
5. Calculate the input impedance for $z_l=50\ \Omega$, $z_o=50\ \Omega$, $d=1$, and $\lambda=1$.

(21) **Note:** The Complex Format mode has been switched to Rectangular so that real and imaginary results are displayed.

- Since connecting elements in parallel with transmission lines is common, admittance is helpful in transmission line calculations.

Define the admittance calculation as a function **yin** which uses the function **zin** (screen 22).

- Return to the Home screen, and clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
- Use **yin** to calculate the input admittance of a **zo=50 Ω** line for **d=0.35m** and **λ=1** for the real values of **zl=0** and **100** (screen 23).
- Calculate the input admittance for the complex values of **zl=j50** and **100-j50**. Use **zo=50 Ω**, **d=0.35m**, and **λ=1m** (screen 24).

(22)

(23)

(24)

Topic 60: VSWR

The reflection coefficient is difficult to measure, so an easily measured alternate parameter is used to describe mismatch, Voltage Standing Ratio (VSWR), given as

$$\text{VSWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

- Define the function **vswr** (screen 25) to implement these calculations. **Vswr** uses the function **reflcoef** from Topic 57.
- Return to the Home screen, and clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
- Calculate the VSWR of loads of 0, 0.01, **j50 Ω**, with **zo=50 Ω** (screen 26).
- Calculate the VSWR of loads of 1000, 50, 100, **100-j50 Ω**.

The results are undefined (**undef**) for short circuits and open circuits.

Use a value of **50** (note the decimal point) in the last entry to get the floating-point value.

VSWR varies from 1 for a matched condition to ∞ for loads of 0, **jX**, or ∞ .

(25)

(26)

(27)

Topic 61: Impedance Matching

A load can be matched to a transmission line, $\Gamma_L=0$, by the addition of parallel circuit elements. One method of matching a load is to insert an additional length of line between the original line and the load as shown in Figure 2. The length of this added line is chosen so that the real part of the input impedance (or admittance) equals the characteristic impedance (or admittance) of the transmission line. Then a parallel element is added to cancel the imaginary part of input admittance resulting in a matched condition. In mathematical terms the match is achieved when

$$\text{real}(y_{in}(z_l, z_0, d, \lambda)) = \text{real}(g_{in} + j b_{in}) = 1/z_0$$

where z_l , z_0 , and λ are fixed and d varies.

The resulting value of susceptance, $j b_{in}$, must be cancelled by a parallel element to achieve the desired match.

Calculate the parameters to match the load $z_l=100-j50$ to a $50\ \Omega$ line.

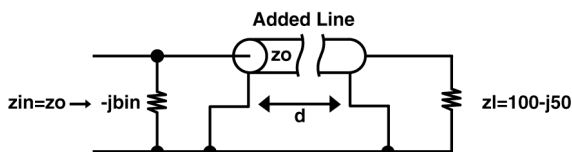


Figure 2. Matching circuit

1. On the Home screen, enter the impedance matching equation as shown in screen 28.

CATALOG real(yin(100 - i 50, 50, d, 1))
= 1 ÷ 50 = 0 STO eqn

The equation is stored in **eqn** so that the Numeric Solver can be used to find the value for **d**.

2. Press **[APPS]** **9: Numeric Solver**. The equation is displayed as shown in screen 29.

3. Press **[ENTER]** **[F2]** to solve for **d** (screen 30).

$d = .125\lambda$ is one solution.

(28)

```

F1 Tools  F2 Algebra  F3 Calc  F4 Other  F5 Pr3mID  F6 Clean Up
NewProb Done
real(yin(100 - i 50, 50, d, 1))
- (4 * (sin(720 * d) + 1) * cos(720 * d))
50 * (4 * sin(720 * d) *
... - i 50, 50, d, 1)) - 1 / 50 = 0 = eqn
TLINE DEGR AUTO FUNC 2/30
  
```

(29)

```

F1 Tools  F2 Solve  F3 Graph  F4 Get Cursor  F5 EAns  F6 Clr d-z...
Enter Equation
eqn: -(4*(sin(720*d)+1))*cos(720*d)
TLINE DEGR AUTO FUNC
  
```

(30)

```

F1 Tools  F2 Solve  F3 Graph  F4 Get Cursor  F5 EAns  F6 Clr d-z...
- (4 * (sin(720 * d) + 1) * cos(720 * d))
d = .124999999999999
bound = (-1. E14, 1. E14)
left-rt = 0.
TLINE DEGR AUTO FUNC
  
```

4. Therefore, $d=.125\lambda$ is the required value for the function y_{in} .

On the Home screen, calculate the parameters for $z_L=100-j50\Omega$, $z_0=50\Omega$, $d=.125\lambda$, and $\lambda=1m$ (screen 31).

$g_{in}=.02=1/50$, and the accompanying susceptance is $j b_{in}=j0.02$. The equality of g and b is merely coincidental.

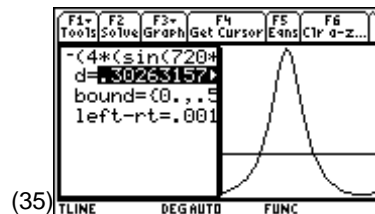
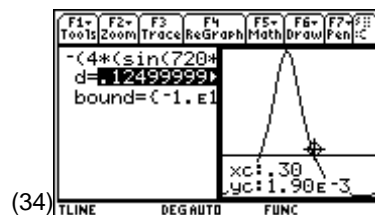
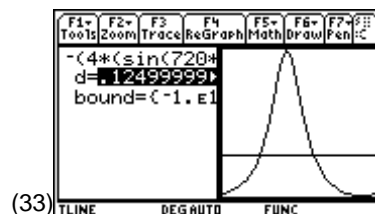
To match this load, a parallel susceptance of $-j0.02$ is needed. This is satisfied by an inductor since $1/2\pi fL=0.02$ or $L=1/0.04\pi f$ where the frequency must be known to determine L .

There are an infinite number of solutions, repeating every $\lambda/2$, that is, $d=.125, .625, 1.125, \dots$. But there are other solutions for d as well.

5. To see these, press \blacklozenge [WINDOW] and set **xmin** to 0 and **xmax** to .5 as shown in screen 32.
6. Press [APPS] 9:Numeric Solver [ENTER] to redisplay the Numeric Solver.
7. Press [F3] 4:ZoomFit to see a graph of the equation on the right of a split screen (screen 33).
8. The Numeric Solver found the first zero; however, the second zero is also a valid solution. To find its value, press the \odot and \ominus keys until the cursor is near the second zero (screen 34).
9. Press [2nd] [=] to switch screens, and then press [F4] Get Cursor (screen 35).
 d now has the x value of the cursor.

(31)

(32)



10. Press **[F2] Solve** to get the second solution (screen 36).

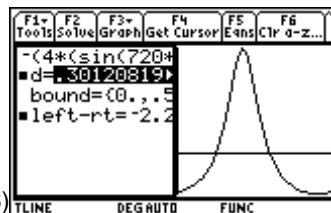
The proper conductance occurs at about $d=.301$.

11. Press **[HOME] [♦] 1** to display a full-sized Home screen.

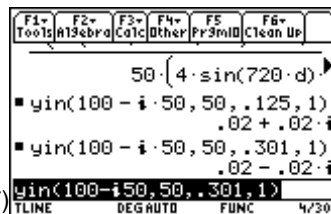
12. Use the function **yin** to calculate the input admittance for $z_l=100-j50\Omega$, $z_o=50\Omega$, length $d=.301\lambda$, and $\lambda=1\text{m}$ (screen 37).

The input admittance for this length is $y_{in}=.020-j.020$.
This can be matched by using a capacitor where $2\pi fC=.02$.

(36)



(37)



Tips and Generalizations

This chapter has again shown the power of the Numeric Solver for finding an unknown in a transcendental equation and plotting the equation versus the unknowns to see if there are multiple solutions. This chapter has also shown that the Solver remembers previous equations, which can be a great time saver.

Finding properties of transmission lines is nice; however, for the ambitious who really want to go far, Chapter 13 on antennas is the way to go.

Features Used

crossP(), dotP(),
real(), conj(), \square ,
NewProb, \square STO \triangleright ,
Polar graphs

Setup

\square 1
NewFold ant
setMode("Complex
Format", "Polar")

Antennas This chapter describes how to perform basic antenna and radiation calculations with the TI-89. Antenna patterns, radiation resistance, radiation integrals, and phased array patterns are included.

Topic 62: Incremental Dipole

The most fundamental antenna is the incremental dipole as pictured in Figure 1.

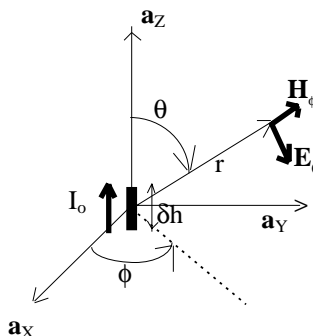


Figure 1. Incremental dipole

For mathematical convenience, the dipole is centered at the origin and aligned with the z-axis. It has a length δh which is much shorter than the wavelength λ , that is, $\delta h \ll \lambda$. It is excited by a sinusoidal current source of angular frequency ω . The resulting current is uniformly distributed along the dipole and has a phasor form of i_0 .

Example 1: Calculating Incremental Dipole Values

The far-zone electric field radiated by the incremental dipole is given by the vector

$$\mathbf{E} = j\eta \frac{i_0 k \delta h \sin \theta}{4\pi} \frac{e^{-jkr}}{r} \mathbf{a}_\theta \text{ V/m}$$

1. Clear the TI-89 by pressing $\boxed{2\text{nd}} \boxed{\text{F6}} \text{2:NewProb} \boxed{\text{ENTER}}$.

Using $\boxed{\text{MODE}}$, set **Complex Format** to **POLAR**.

2. Enter the vector in spherical coordinates as shown in screen 1.

$\boxed{2\text{nd}} \boxed{[]} \boxed{0} \boxed{,} \boxed{2\text{nd}} \boxed{[i]} \text{eta} \boxed{\times} \boxed{\text{io}} \boxed{\times} \boxed{k} \boxed{\times} \boxed{\blacklozenge} \boxed{[]} \boxed{\alpha} \text{dh} \boxed{\times} \boxed{2\text{nd}} \boxed{[\text{SIN}]} \boxed{\blacklozenge} \boxed{[\theta]} \boxed{[]} \boxed{\blacklozenge} \boxed{[e^x]} \boxed{(-)} \boxed{2\text{nd}} \boxed{[i]} \boxed{k} \boxed{\times} \boxed{r} \boxed{[]} \boxed{\div} \boxed{[]} \boxed{4} \boxed{2\text{nd}} \boxed{[\pi]} \boxed{r} \boxed{[]} \boxed{0} \boxed{2\text{nd}} \boxed{[1]} \boxed{\text{STO}} \boxed{\text{eincdip}}$

3. The far-zone magnetic field is

$$\mathbf{H} = j \frac{\text{io} k \delta h \sin \theta}{4\pi} \frac{e^{-jkr}}{r} \mathbf{a}_\phi \text{ A/m}$$

Enter the vector as shown in screen 2.

$\boxed{2\text{nd}} \boxed{[i]} \boxed{0} \boxed{,} \boxed{0} \boxed{,} \boxed{2\text{nd}} \boxed{[i]} \text{io} \boxed{\times} \boxed{k} \boxed{\times} \boxed{\blacklozenge} \boxed{[]} \boxed{\alpha} \text{dh} \boxed{\times} \boxed{2\text{nd}} \boxed{[\text{SIN}]} \boxed{\blacklozenge} \boxed{[\theta]} \boxed{[]} \boxed{\blacklozenge} \boxed{[e^x]} \boxed{(-)} \boxed{2\text{nd}} \boxed{[i]} \boxed{k} \boxed{\times} \boxed{r} \boxed{[]} \boxed{\div} \boxed{[]} \boxed{4} \boxed{2\text{nd}} \boxed{[\pi]} \boxed{r} \boxed{[]} \boxed{2\text{nd}} \boxed{[1]} \boxed{\text{STO}} \boxed{\text{hincdip}}$

4. These fields represent outward propagating, spherical waves with an amplitude that varies with polar angle. The fields decrease as the distance to the antenna is increased. The power density of such a field is given by

$$\mathbf{W} = \frac{\text{Re}(\mathbf{E} \times \mathbf{H}^*)}{2} \text{ w/m}^2$$

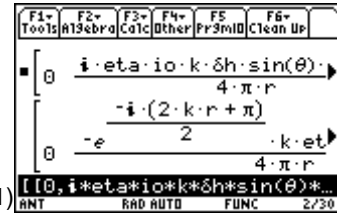
Calculate the power density as shown in screen 3.

$\boxed{\text{CATALOG}} \boxed{\text{real}} \boxed{[]} \boxed{[]} \boxed{[]} \boxed{\text{CATALOG}} \boxed{\text{crossP}} \boxed{[]} \boxed{\text{eincdip}} \boxed{,} \boxed{[]} \boxed{\text{CATALOG}} \boxed{\text{conj}} \boxed{[]} \boxed{\text{hincdip}} \boxed{[]} \boxed{[]} \boxed{[]} \boxed{\div} \boxed{2} \boxed{\text{STO}} \boxed{\text{wincdip}}$

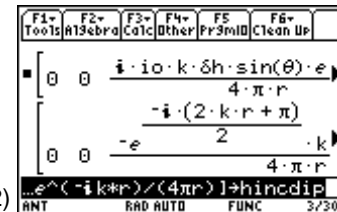
The result is

$$\begin{bmatrix} \frac{k^2 \text{eta} (\text{io})^2 \delta h^2 (\sin(\theta))^2}{32 \pi^2 r^2} & 0 & 0 \end{bmatrix}$$

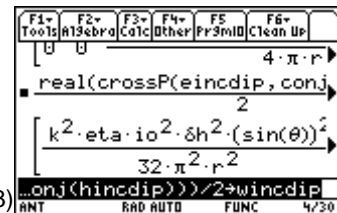
This shows that the power is directed radially outward.



- (1) **Note:** To enter δ , press $\boxed{\blacklozenge} \boxed{[]} \boxed{\alpha} \text{d}$ on the keyboard.



- (2)



- (3)

5. The total power radiated is calculated as

$$P_{\text{RAD}} = \oint_{\text{SPHERE}} \mathbf{W} \cdot d\mathbf{s}$$

Enter this expression as shown in screen 4.

`[2nd] [J] [2nd] [J] [CATALOG] dotP(wincdip , [2nd] [r] ^ 2 ×
[2nd] [sin] [◄] [θ]) , 0 , 0 [2nd] [J]) , [◄] [θ] , 0 , [2nd]
[π]) , [◄] [(alpha f , 0 , 2 [2nd] [π])] STO► pradincd`

Calculation of the radiated power is simplified since there are only radial components of both \mathbf{W} and $d\mathbf{s}$. Note that the differential surface element on the sphere is $r^2 \sin\theta d\theta d\phi$.

6. Substituting of the free space wave impedance as $\eta = 120\pi$ and the wave propagation constant as $k = 2\pi/\lambda$ gives the average power radiated into free space.

Use the “with” operator to substitute the values as shown in screen 5.

`pradincd [1] eta = 120 [2nd] [π] [CATALOG] and k = 2 [2nd] [π]
[÷] [◄] [(alpha I`

7. The current flowing into one side and out of the other side of the dipole is io . From a circuit point of view, the power extracted from the current by radiation is given by

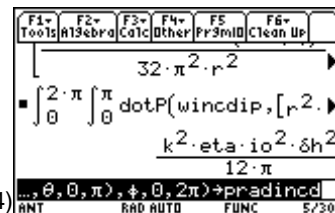
$$P_{\text{RAD}} = \frac{|io|^2 R_{\text{RAD}}}{2}$$

where R_{RAD} represents equivalent resistance to dissipate this power. The radiation resistance is calculated as

$$R_{\text{RAD}} = \frac{2P_{\text{RAD}}}{|io|^2}$$

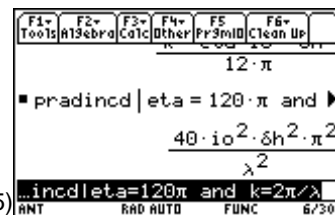
Find the radiation resistance as shown in screen 6.

`2pradincd [÷] io ^ 2 [1] eta = 120 [2nd] [π] [CATALOG] and k =
2 [2nd] [π] [÷] [◄] [(alpha I STO► rradincd`



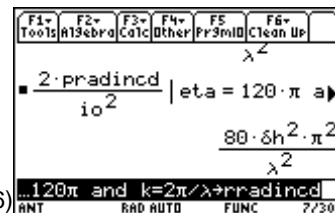
(4)

Note: To enter ϕ , press \blacktriangleleft [(alpha f.



(5)

Note: To enter λ , press \blacktriangleleft [(alpha I.



(6)

4. In the Window Editor, set the window variable values (screen 12) for θ to vary from 0 to 360° (that is $\theta_{\min}=0$ and $\theta_{\max}=360$). Although polar angle θ is defined over 0 to 180° , this range is needed to include both half-planes on which ϕ is constant.

$\theta_{\text{step}}=1$ is used to generate the plot here, but $\theta_{\text{step}}=5$ is faster and good enough in most cases. Since the pattern is normalized to one (division by the $3/2$ factor), the ranges on x and y are ± 1 .

5. Press \blacksquare [GRAPH] to plot the E-plane pattern (screen 13).

This is a distorted view of the pattern due to different scaling on x and y .

6. Use F2 5:ZoomSqr to plot a properly scaled pattern (screen 14).

Since the polar angle θ is measured from the positive z -axis on the dipole and the graphing angle θ is measured from the positive x -axis on the screen, the dipole lies along the x -axis of the pattern.

7. Half-power beamwidth is a parameter used to describe antennas. It is the angular separation of the half-power points of a pattern. Use the trace cursor (F3 and \odot , \odot , 2nd \odot , or 2nd \odot) to display pattern values and angles and find the beamwidth.

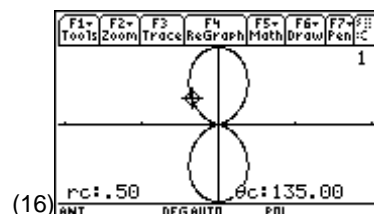
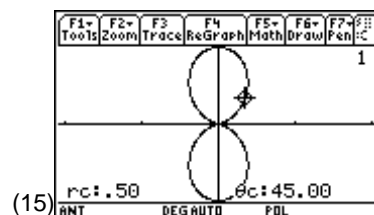
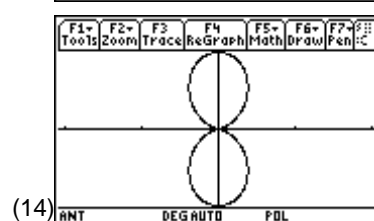
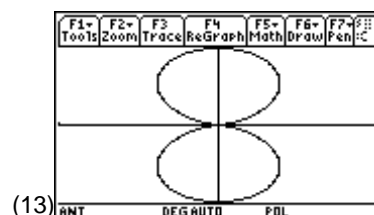
First, use \blacksquare I \odot 2: Polar to set coordinates in the polar mode.

The right-hand half-power point is at $\theta=45^\circ$. Press F3 and 45 [ENTER]. Screen 15 shows that at an angle of 45° the power is 0.5.

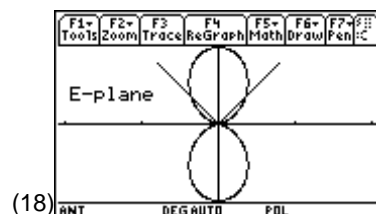
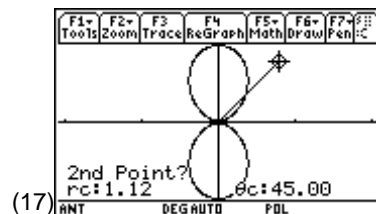
8. The left-hand half-power point is at $\theta=135^\circ$.

Press 135 [ENTER].

So the E-plane beamwidth is $135-45=90^\circ$.



9. The concept of beamwidth can be further emphasized by using **[2nd] [F7] 3:Line**.
10. In response to the on-screen question "1st Point?", position the cursor at the origin and press **[ENTER]**.
11. For "2nd Point?", position the cursor at the angle of the lower half-power point, $\theta=45^\circ$, with $rc=1.12$ and press **[ENTER]** (screen 17).
12. Add a second line at the upper half-power point in the same manner.
13. Press **[ESC]** to exit this mode.
14. Add text to the graph using **[2nd] [F7] 7:Text**. Position the cursor at the starting point (screen 18). If there is an error, use the eraser (**[2nd] [F7] 2:Eraser** and **[ENTER]**), then hold down the **[↑]** key to erase).
15. The H-plane pattern is plotted as a function of ϕ ; however, it has no ϕ dependence so it is a constant and graphs as a circle of radius 1.



Topic 64: Phased Arrays

Phased arrays are commonly used to tailor antenna patterns to a desired shape. When several identical elements are located near each other, they form an array. The pattern of the array is the product of an element factor, a geometric factor, and an array factor. The array factor, AF, for N identical, equi-amplitude radiators located on the z-axis with uniform spacing D, is given by

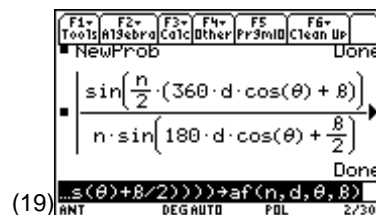
$$AF = \frac{\sin\left(\frac{N}{2}\left(2\pi\frac{D}{\lambda}\cos\theta + \beta\right)\right)}{N\sin\left(\pi\frac{D}{\lambda}\cos\theta + \frac{\beta}{2}\right)}$$

β is the progressive phase shift along the array from one element to the next. An alternate form allows graphing with θ in degrees and expresses radiator spacing, $d=D/\lambda$, in fractions of the wavelength for easy graphing of the array factor

$$AF = \frac{\sin\left(\frac{N}{2}(360d\cos\theta + \beta)\right)}{N\sin\left(180d\cos\theta + \frac{\beta}{2}\right)}$$

1. Clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
2. Enter the array factor expression as shown in screen 19.

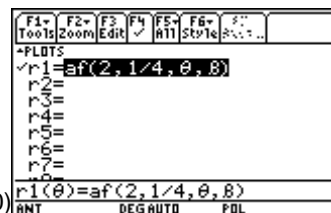
CATALOG **abs** (**[2nd] [SIN]** **n** **÷** **2** **[]** **360d** **×** **[2nd] [COS]** **♦** **[θ]** **[]**
+ **♦** **[]** **alpha** **b** **[]** **÷** **[]** **n** **×** **[2nd] [SIN]** **180d** **×** **[2nd] [COS]**
♦ **[θ]** **[]** **+** **♦** **[]** **alpha** **b** **÷** **2** **[]** **)** **STO>** **af** **[]** **n** **[]** **d**
[] **♦** **[θ]** **[]** **+** **♦** **[]** **alpha** **b** **[]**



3. The beam from an array can be “steered” by varying the phase-shift between elements.

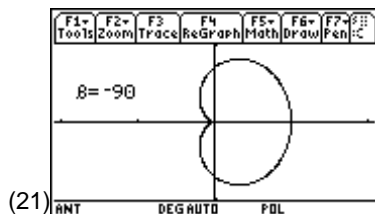
To graph the patterns for a two-element array ($n=2$) with $d=1/4$ spacing for phase shifts of $\beta=-90^\circ$, 0° , 45° , and 90° , define -1 in the Y= Editor as shown in screen 20.

(20)

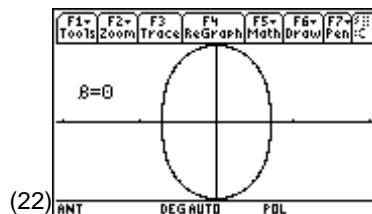


$\text{af}([2], [1] \div [4], [\theta], [\alpha], [\text{alpha}], [b])$

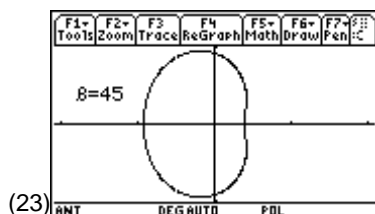
4. On the Home screen, assign the values -90, 0, 45, and 90 to β . Then graph each pattern as shown in screens 21 through 24.



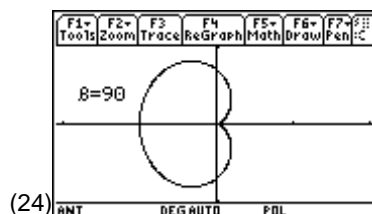
(21)

 $-90 \rightarrow \beta$ 

(22)

 $0 \rightarrow \beta$ 

(23)

 $45 \rightarrow \beta$ 

(24)

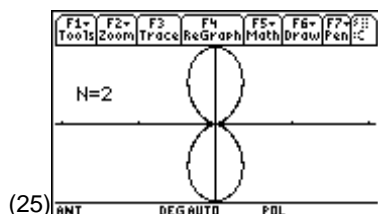
 $90 \rightarrow \beta$

5. With more elements in an array, the beamwidth becomes narrower and more focused, as shown in screens 25-27 by arrays with 2, 4, and 6 elements ($n=2, 4$, and 6) spaced with one-half wavelength ($d=1/2$) and with zero phase-shift ($\beta=0$). However, this improvement in beamwidth is accompanied by an undesirable increase in the number and amplitude of sidelobes.

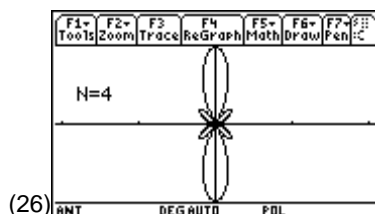
Edit $r1$ in the Y= Editor.

$\text{af}([n], [1] \div [2], [\theta], [0])$

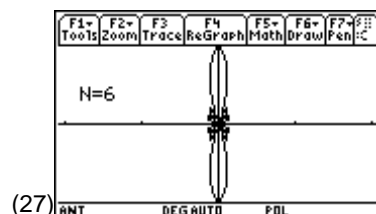
On the Home screen, assign the values of 2, 4, and 6 to n . Then graph each pattern.



(25)

 $2 \rightarrow n$ 

(26)

 $4 \rightarrow n$ 

(27)

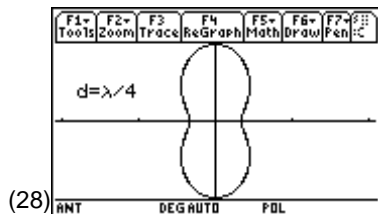
 $6 \rightarrow n$

6. Finally, the element spacing can narrow the beamwidth, but the wider spacing causes more and larger sidelobes. This effect is shown in screens 28-30 by 3 elements ($n=3$) with spacing of $\lambda/4$, $\lambda/2$, and λ ($d=\lambda/4$, $\lambda/2$, and λ) and $\beta=0$.

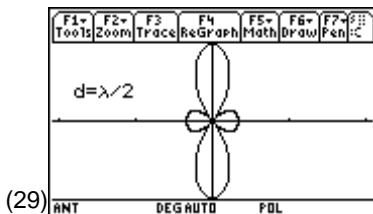
Edit **r1** in the Y= Editor.

af $\left(\left[3 \right] , d \left[\frac{\pi}{2} \right] , 0 \right)$

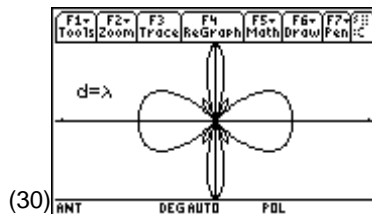
Assign the values for **d** on the Home screen. Graph the patterns.



$1/4 \rightarrow d$



$1/2 \rightarrow d$



$1 \rightarrow d$

Tips and Generalizations

These examples show how rather complex antenna and array equations can be better understood by making a few exploratory polar plots with the TI-89.

So far, only equations have been graphed. The next chapter shows that lab data also can be plotted.

1
NewFold diode

3. The values for the parameters can be entered now.

Enter **q** and **k**. Choose a typical value of **i0**=1.E-10 A and a temperature, **t**, of 273.16+25 K. The parameter **n** is not specified yet.

4. The usual way to plot the diode equation is to press \diamond [Y=] and enter the equation.

However, you can skip that step and enter the equation from the Home screen as shown in screen 3.

diodeeq \square **n** \square 1.2 **CATALOG** and **v** \square **x** **STO** \rightarrow **y1** \square **x** \square

5. Press \diamond [WINDOW] and set the window variable values as shown in screen 4.

6. Press \diamond [GRAPH] to see the plot (screen 5). It takes a couple of seconds for the graph to appear since the values on the left half of the screen are nearly 0 but must be plotted.

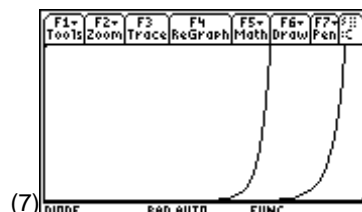
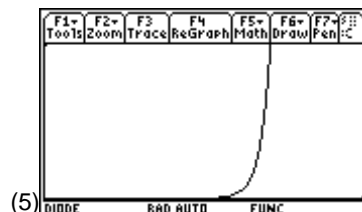
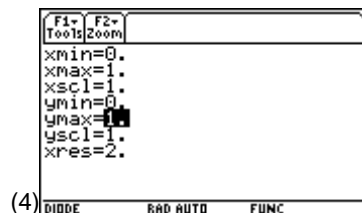
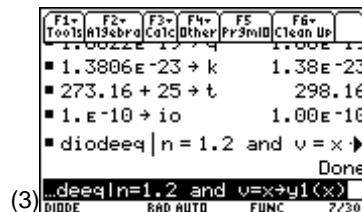
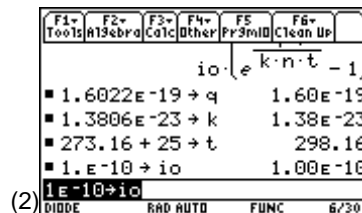
It looks like the proper curve for a diode. The effects of **n** can be seen by plotting a second equation with a different value of **n**.

7. In the Y= Editor, enter the equation for **y2** with **n**=1.6 as shown in screen 6.

diodeeq \square **n** \square 1.6 **CATALOG** and **v** \square **x**

8. Graph both equations.

Increasing **n** causes the curve to move to the right with a larger voltage drop across the diode in the “on” state (screen 7).



Topic 66: Lab Data

The Shockley equation attempts to model a diode, but how close is it? The only way to tell is to take some real data from a diode and compare it to the equation. Figure 1 shows the diagram of a circuit that was built to measure such diode characteristics. (The value for the resistor is the measured value of a resistor marked as 1k Ω .)

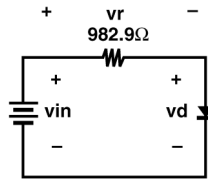


Figure 1. Diode measurement circuit

Several values for **vin** and **vr** were measured in the lab and entered into two files (called **vin.txt** and **vr.txt**, respectively) on a computer. These values can be downloaded to the TI-89 using the TI-GRAPH LINK™ cable and software. Alternatively, the data from **vin** and **vr** can be entered directly into the TI-89 by hand.

A single data value is entered on each line of the text file. The data presented here is inserted into a table to save space, **vin.txt** in Table 1 and **vr.txt** in Table 2. (Read across each row and then continue at the left of the next row.)

0.2002	0.2507	0.3003	0.3501
0.4008	0.4501	0.5014	0.5506
0.6001	0.6504	0.7006	0.7503
0.8008	0.8501	0.9	0.951
1.0005	1.1022	1.2029	1.3017
1.4013	1.5032	2.002	2.502
2.999	4.007	5.003	7.502
10.003			

Table 1. vin.txt data

0.0001	0.0004	0.0011	0.0031
0.008	0.0182	0.0368	0.062
0.0928	0.1282	0.1666	0.2063
0.2484	0.2902	0.3336	0.3778
0.4231	0.5158	0.6087	0.7007
0.7951	0.8912	1.369	1.855
2.341	3.331	4.315	6.79
9.274			

Table 2. vr.txt data

Method 1: Using the TI-GRAPH LINK™

Download the values from the computer into the TI-89 using TI-GRAPH LINK.

1. Start TI-GRAPH LINK on the PC and select **Tools:Import:ASCII Data**. A file selection box appears.
2. Select **vin.txt** on the computer and click **OK**. Naming the converted file as the default, **vin.9xl**, is fine, so click **OK**.
3. Repeat these steps for **vr.txt**.
4. Use the cable to connect the computer and the TI-89.
5. On the TI-89 screen, display the Home screen.
6. Now transfer these files to the TI-89 by selecting **Link:Send** on the computer.
7. Double-click on **vin.9xl** and **vr.9xl**, and then click **OK**. A window opens showing the progress of the file transfer. It shouldn't take long.
8. Click **OK** once the "*** Complete ***" message appears. The data is now stored in list form in the TI-89.
9. Check the values by entering **vin** and **vr** on the Home screen. The results should match screen 8.

(8)

F1→ Tools	F2→ Algebra	F3→ Calc	F4→ Other	F5→ PrgmID	F6→ Clean Up
■ diodeeq n = 1.2 and v = x → Done					
■ vin (.20 .25 .30 .35 .4)→					
■ vr (1.00E-4 4.00E-4 1.10E-4)→					
vr DIODE RAD AUTO FUNC 9/30					

Note: The default mode used in this book is "FIX 2"; therefore, only 2 digits are displayed even though 4 were entered.

Method 2: Entering the Data Directly

On the Home screen, enter the data directly as a list separated by commas.

[2nd] [t] .2002,4008,...,7.502 [)] [STO] vin

[2nd] [t] .0001,.008,...,6.79 [)] [STO] vr

1. Combine the two sets of data, **vin** and **vr**, into one data file by using **NewData** as shown in screen 9.

[CATALOG] NewData data [)] vin [)] vr

(9)

F1→ Tools	F2→ Algebra	F3→ Calc	F4→ Other	F5→ PrgmID	F6→ Clean Up
■ vin (.20 .25 .30 .35 .4)→					
■ vr (1.00E-4 4.00E-4 1.10E-4)→					
■ NewData data,vin,vr Done					
NewData data,vin,vr DIODE RAD AUTO FUNC 10/30					

- The contents of **data** do not appear on the Home screen, but they can be viewed by pressing [APPS] 6:Data/Matrix Editor 2:Open, specifying **Type** as **Data**, and selecting the variable **data**. (screen 10).

- It's a good idea to label each column of the data so that the two sets of data are not confused.

Press $\odot \odot$ and enter **vin**. Then press [ENTER] $\odot \odot$ **vr** [ENTER] (screen 11).

- The diode voltage, **vd=vin-vr**, is placed in column **c3** by pressing $\odot \odot$ **vd** [ENTER] **c1** \square **c2** [ENTER] (screen 12).

Note that the contents of the computed cells are "locked" to their computed values. This is indicated by the $\mathbf{\bar{I}}$ symbol preceding the cell name in the command line.

- The current through the diode is the same as the current through the resistor, which is **vr/982.9**.

Place this in column **c4** by pressing $\odot \odot \odot$ **id** [ENTER] **c2/982.9** [ENTER] (screen 13).

- Plot the diode current in **c4** versus the diode voltage in **c3** by pressing [F2] Plot Setup [F1] Define.
- Change **Mark** to **Box**. Then make sure the plot parameters are **c3** and **c4**, as shown in screen 14.

- Press \blacksquare [GRAPH] to see the plot (screen 15).

The squares on the bottom are the lab data. The solid curves are the plots from Topic 65. The plot range isn't right for the data.

- Press [F2] 9:ZoomData to scale the graph to the data (screen 16).

The left curve was for $n=1.2$, and the right was for $n=1.6$. The data looks like it fits a curve where $n \approx 1.5$. Try it.

(10)

F1 Tools	F2 Plot Setup	F3 Cell Header	F4 Cell	F5 Calc	F6 Util	F7 Stat
DATA						
		c1	c2	c3		
1		.20	1.0E-4			
2		.25	4.0E-4			
3		.30	1.1E-3			
4		.35	3.1E-3			
r1c1=.2002						
DIODE RAD AUTO FUNC						

(11)

F1 Tools	F2 Plot Setup	F3 Cell Header	F4 Cell	F5 Calc	F6 Util	F7 Stat
DATA						
		vin	vr			
		c1	c2	c3		
1		.20	1.0E-4			
2		.25	4.0E-4			
3		.30	1.1E-3			
4		.35	3.1E-3			
c2=						
DIODE RAD AUTO FUNC						

(12)

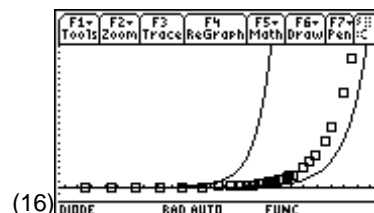
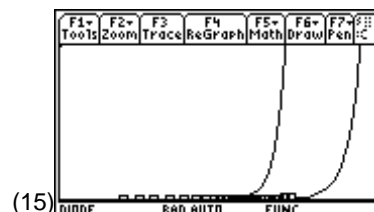
F1 Tools	F2 Plot Setup	F3 Cell Header	F4 Cell	F5 Calc	F6 Util	F7 Stat
DATA						
		vin	vr	vd		
		c1	c2	c3		
1		.20	1.0E-4	.20		
2		.25	4.0E-4	.25		
3		.30	1.1E-3	.30		
4		.35	3.1E-3	.35		
r1c3=.2001						
DIODE RAD AUTO FUNC						

(13)

F1 Tools	F2 Plot Setup	F3 Cell Header	F4 Cell	F5 Calc	F6 Util	F7 Stat
DATA						
		vr	vd	id		
		c2	c3	c4		
1		1.0E-4	.20	1.0E-7		
2		4.0E-4	.25	4.1E-7		
3		1.1E-3	.30	1.1E-6		
4		3.1E-3	.35	3.2E-6		
r1c4=1.0173974972022E-7						
DIODE RAD AUTO FUNC						

(14)

diode\data Plot 1						
Plot Type	Scatter \rightarrow					
Mark	Box \rightarrow					
X	c3					
Y	c4					
Axis, Back, > Back	A					
Freq and Categories?	NO \rightarrow					
Category 1						
Category 2						
Category 3						
Category 4						
Enter=SAVE ESC=CANCEL						
TYPE * [ENTER]=OK AND [ESC]=CANCEL						

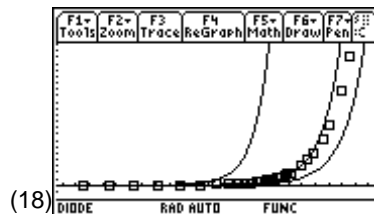
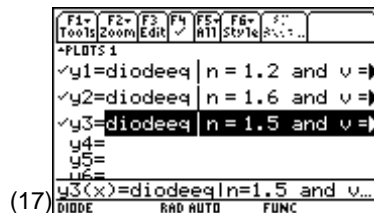


10. Press \diamond [Y=] and enter the diode equation as **y3** with **n=1.5** as shown in screen 17.

diodeeq [1] **n** [=] **1.5** [CATALOG] and **v** [=] **x**

11. Press \diamond [GRAPH] (screen 18).

The laboratory data closely fits the **n=1.5** curve. The actual value of **n** may be a little more than 1.5.



Tips and Generalizations

Any sort of lab data can be plotted using the techniques shown in this chapter. Plotting the data as it is taken in the lab could show quickly when anomalous data has been collected.

Chapter 15 shows how to compute what money is worth both now and in the future.

Features Used

seq(), SEQUENCE,
solve() \square , Σ (sum,
[ANS], NewProb

Setup

\square 1
NewFold econ

Financial Calculations

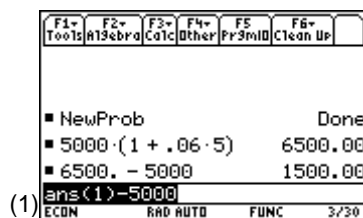
This chapter describes how to use the TI-89 to calculate interest, present worth, loan repayments, and so forth. These methods utilize the time-value-of-money.

Topic 67: Simple Interest

Money that is invested earns interest. The most basic form of interest is known as simple interest. An amount of money with present value P that is invested for N years at an annual interest rate of i has a future value F . For simple interest, the future value is calculated as $F = P + NPi = P(1 + iN)$. The future values can be converted back to present value as $P = F / (1 + iN)$.

1. Clear the TI-89 by pressin \square 2nd \square F6 2:NewProb \square ENTER.
2. Find the payment received after 5 years on a \$5000 investment at 6% simple interest (screen 1). The future value is given by $F = 5000(1 + 0.06 \cdot 5) = \6500 .

The total interest paid is $6500 - 5000 = \$1500$.



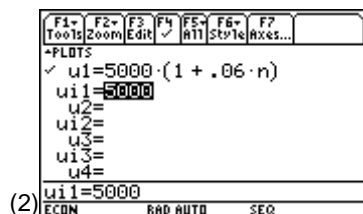
3. The TI-89 displays this type of sequential calculations in the **SEQUENCE** graphing mode.

Pres \square MODE and se tGraph mode t oSEQUENCE.

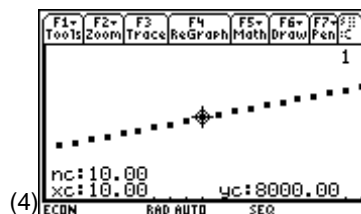
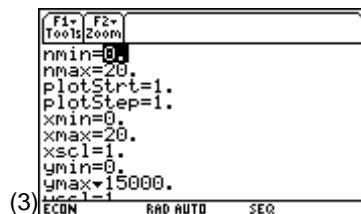
4. Pres \square \square [Y=] and enter the equation fo u_1 as a function of the payment period as shown in screen 2.

5000 \square 1 \square + \square .06 \square n \square

Also enter an initial value o $u_1 = 5000$.



5. Set the Window variable values in the Window Editor as shown in screen 3.
6. Press \square [GRAPH] to display the sequence for a 20-year period (screen 4). The future value at the 10th year is observed by pressing \square Trace and moving the cursor to the 10th year \square where the value is \$8,000.



Topic 68: Compound Interest

Compound interest is more common than simple interest and much better for the investor. The interest is calculated on the initial investment plus the interest earned to date. At the starting date, the value of the investment is $F(0)=P$. At the end of the first interest period, the value of the investment is $F(1)=P(1+i)$; at the end of the second period the value is $F(2)=P(1+i)^2$. The pattern is clear—the value after the n th period is $F(n) = P(1+i)^n$.

1. Clear the TI-89 by pressing \square [2nd] [F6] 2:NewProb [ENTER].
2. Screen 5 shows how to find the interest on the same \$5000 principle at 6% compound interest paid on a yearly basis for 5 years (screen 5). The future value is calculated by $F=5000(1+.06)^5 = \$6691.13$.

The total interest earned is $6691.13 - 5000 = \$1691.13$, more profitable for the investor than simple interest.

3. The most common method of interest payment is with monthly compounding. The monthly interest rate is $i_{\text{Month}} = i/12$.

Find the future value after 5 years for the \$5000 investment at 6% annual interest compounded monthly (screen 6): $F=5000(1+.06/12)^{(5*12)} = \6744.25 .

The interest earned is $\$1744.25$, an even more attractive investment.

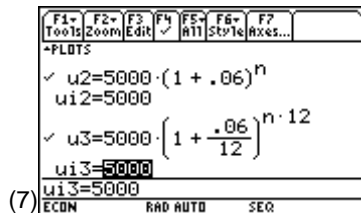
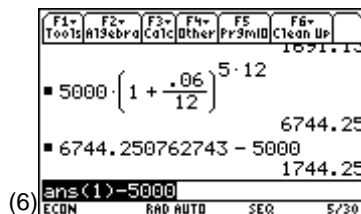
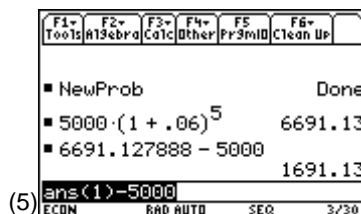
4. The two compound interest examples are compared graphically with the simple interest case by entering them in the Y= Editor.

$$u2(n) = 5000 (1 + .06)^n$$

with $ui2=5000$

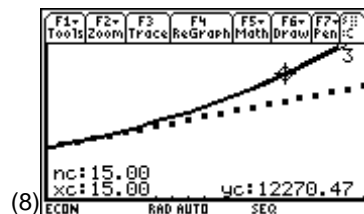
$$u3(n) = 5000 (1 + .06 \div 12)^{n \times 12}$$

with $ui3=5000$



5. Make the three graphs look different. Highlight the equation for **tu2** and press **[2nd] [F6] 1:Line**. Highlight **tu3** and press **[2nd] [F6] 4:Thick**. Then press **[G] [GRAPH]**.

Although the two compounding curves look the same, pressing **[gF3] Trac** shows that a **tc = 15** years the three graphs have the future worth of \$9500.00, \$11982.79, and \$12,270.47, for simple interest, yearly compounding, and monthly compounding, respectively.



(8)

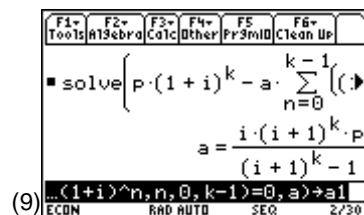
Note: Use \odot and \ominus to change from one graph to another.

Topic 69: Loans

The calculation of loan repayment schedules is of great interest in professional as well personal life. Typical loans require an equal periodic payment A made for k payment periods to repay an amount P borrowed at interest rate i per period. At the end of the first payment period, the amount owed is $P(1+i)$ (the principle plus interest for one period) minus one payment A , that is, $P(1+i)-A$. After the second payment, the remaining amount owed is $(P(1+i)-A)(1+i)-A = P(1+i)^2 - A(1+i)-A$. After the k th payment, the entire loan and interest is paid, $P(1+i)^k - A(1+i)^{k-1} - A(1+i)^{k-2} - \dots - A(1+i) - A = 0$. Use **solve()** to find the form of A .

1. Clear the TI-89 by pressing **[2nd] [F6] 2:NewProb [ENTER]**.
2. Enter the **solve()** command as shown in screen 9.

[CATALOG] solve (p [x] [1 + i] [^] k - a [x] [CATALOG] [Σ] ([1 + i] [^] n, n, 0, k - 1] = 0, a) [STO] a1

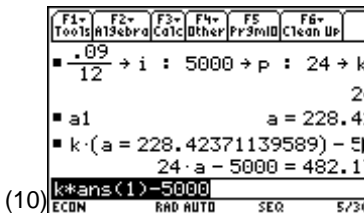


(9)

3. Calculate the total interest paid on a two-year, \$5000 auto loan at an annual interest rate of 9% repaid with monthly payments.

Enter the interest rate, loan amount, and number of payments (screen 10).

.09 [÷] 12 [STO] i [2nd] [:] 5000 [STO] p [2nd] [:] 24 [STO] k



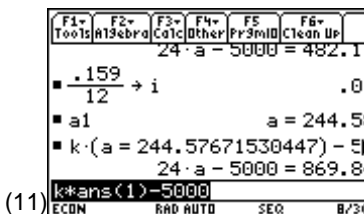
(10)

4. Display the payment amount (\$228.42) by entering **a1g**
5. Calculate the total interest paid.

k [x] [2nd] [ANS] [-] 5000

The total interest paid is \$482.17.

6. Find the payment for the same debt but with a typical credit card interest rate of 15.9% (screen 11). The monthly payment **a1** is \$244.58; and the total interest paid **sk * a - 5000 = \$869.84**, nearly twice the total interest for the smaller rate.



(11)

Topic 70: Annuities

An annuity is a financial process in which equal payments, A , are made to an account with an interest rate, i , for a fixed number of periods, k . Usually, the compounding takes place each period. This is often called a uniform series. The first payment earns compounded interest for $k-1$ periods with a future value of $A(1+i)^{k-1}$; the second payment has a future value of $A(1+i)^{k-2}$; the last payment that is made when the annuity is due has a future value of A . The sum of these terms gives the future value of the annuity, $F = \sum A(1+i)^n$ summed from 0 to $k-1$. To achieve a future value F , the periodic payment is $A = F \sum (1+i)^n$.

Example 1: Finding Monthly Payment Amount

- solve()** gives a closed form of solution for the computations, but first use **DelVar** so that previous values for i and k are deleted.

CATALOG DelVar i , k

- Use the **solve()** command to enter the annuities equation as shown in screen 12.

CATALOG solve ($a = f \div$ CATALOG $\Sigma((1+i)^n$, n
0, $k-1$), a) STO $a1$

- To calculate the monthly payments necessary to accumulate \$5000 in 5 years at 6% annual interest rate, enter the variable values as shown in screen 13.

- Find the monthly payment amount, the total amount paid, and the amount of interest earned (screen 14).

$a1$

\times [2nd] [ANS]

5000 $-$ [2nd] [ANS]

The monthly payment is \$71.66; the total amount paid is \$4299.84; the total interest earned is \$700.16.

(12)

$$a = \frac{f \cdot i}{(1+i)^k - 1}$$

(13)

$$i = .06/12$$

$$f = 5000$$

$$k = 60$$

$$a = 71.66$$

$$60 \cdot a = 4299.84$$

(14)

$$a = 71.66$$

$$60 \cdot a = 4299.84$$

$$5000 - 60 \cdot a = 700.16$$

Example 2: Finding Amount to Invest

The next example shows how to calculate the present amount to invest, P, required to receive equal periodic payments, A, over a fixed number of periods, k, from an account which earns a compound interest rate i. The equation is the same as the equation in Topic 69 with present value, $P = F/(1+i)^k$, substituted for future value.

1. Delete the values for \mathbf{k} , and \mathbf{dP} .

CATALOG DelVar i, k, p

- 2.** Enter the equation as shown in screen 15.

CATALOG solve (a \equiv i \times (i + 1) \wedge k \times p \div ((i + 1) \wedge k - 1), p) **STO►** p1

3. This process is the inverse of loan payments. Instead of receiving an amount of money and paying it back in equal payments, an amount of money is paid to an institution and the equal payments are received.

Calculate the amount to be paid in order to receive equal monthly payments of $a = \$100$ for $k = 2$ years = 24 months from an account that earns $i = 7\%$ interest per month (screen 16).

A deposit of \$2233.51 **p10** returns a total of \$2400 over the two-year period.

$$\begin{aligned} & \text{solve} \left\{ a = \frac{i \cdot (i+1)^k \cdot p}{(i+1)^k - 1}, p \right\} \Rightarrow \\ & p = \frac{a \cdot ((i+1)^k - 1) \cdot (i+1)^{-k}}{i} \\ & \text{ECM} \quad \text{RAD AUTO} \quad \text{SEQ} \quad 16/30 \end{aligned}$$

Calculator display showing the calculation of the present value (p) of an annuity. The formula entered is $p = \frac{a \cdot ((1 + i)^n - 1)}{i \cdot (1 + i)}$. The input values are $a = 0.07$, $i = 0.1$, and $n = 100$. The result displayed is $p = 2233.51$.

Tips and Generalizations

This chapter has shown how the TI-89 can easily derive and solve time-value-of-money problems. Consider using these examples before you apply for a loan or get a credit card.

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